Volatility Trading Workshop



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New York University

RiskMathics Trading and Risk Management

Mexico, June 2016/ Rio de Janeiro, November 2016

Trading Volatility: what does this mean?

Trading volatility means using equities and options to generate strategies which make or lose money when the market becomes more volatile or less volatile.

One can think of market volatility as being the actual (realized) volatility of equities or, alternatively, the volatility implied by option prices (implied volatility)

In current markets one can trade

-- realized volatility
-- implied volatility
-- implied versus realized
-- implied versus implied
-- realized versus realized
-- correlation between two or more assets
-- implied correlation
-- dispersion


IB Volatility Lab - Implied Volatility
201.57 -1.52 (-0.75%) U1133678
SPY ARCA Implied Volatility
Expiries: Nov Dec Jan'16
Time period: 6 months
212.50
210.00
207.50
205.00
201.57
200.00
197.50
195.00
192.50
190.00
187.50
26%
25%
24%
23%
22%
21%
20%
19%
18%
17%
16%
15.3%
14.2%
13%
5/1
6/1
7/1
8/3
9/1
10/1
SPY ARCA Multi-Expiry Skew
Expiries: Nov Jan'16
Strikes
Date: Oct 21st
NOV20
35%
30%
25%
20%
15%
10%
Strike: 170 180 190 201.57 210 220 230 240 250
SPY ARCA Time Lapse Skew
Days: Today Yesterday
Strikes
Expiry: Nov20 '15
Today
25%
22.5%
20%
17.5%
15%
12.5%
10%
Strike: 185 190 195 201.57 205 210 215 220 225
SPY ARCA Volatility Profile
HV
IV
10 30 50 75 100 150 200
25.0%
22.5%
20.0%
17.5%
15.0%
12.5%
10.0%
Nov06'15 Dec18'15 Jan15'16 Feb19'16 Mar31'16 Jur
SPY ARCA Open Interest
Expiries: Nov Jan'16
Strikes
Date: Oct 21st
Puts + Calls
400K
300K
200K
100K
0
Strike: 176.5 190.5 201.57 218.5 229
Implied Volatility
Historical Volatility
Industry Comparison
Time (Descending) + Action (Ascending)


IB Volatility Lab - Implied Volatility
15.20 -0.74 (-4.64%) U1133678
FCAU NYSE Implied Volatility
Expiries: Nov20 Jan15 '16
Time period: 6 months
16.50
16.00
15.50
15.20
15.00
14.50
14.00
13.50
13.00
12.50
12.00
52.5%
50%
47.5%
45.4%
42.5%
41.6%
40%
37.5%
35%
5/1
6/1
7/1
8/3
9/1
10/1
FCAU NYSE Multi-Expiry Skew
Expiries: Nov20
Strikes
Date: Oct 21st
NOV20
150%
125%
100%
75%
50%
Strike: 10.0 12.5 15.20 17.5 20.0 22.5 25.0
FCAU NYSE Time Lapse Skew
Days: Today Yesterday
Strikes
Expiry: Nov20 '15
Today
100%
90%
80%
70%
60%
50%
Strike: 12 13 14 15.20 16 17 18 19 20 21 22
FCAU NYSE Volatility Profile
HV
IV
10 30 50 75 100 150 200
52.5%
50.0%
47.5%
45.0%
42.5%
40.0%
Nov20'15
Jan15'16
Mar18'16
Jur
FCAU NYSE Open Interest
Expiries: Nov20 Jan15 '16
Strikes
Date: Oct 21st
Puts + Calls
30K
20K
10K
0
Strike: 9 11 12.5 15.20 17.5 19 21 22.5 24
Implied Volatility
Historical Volatility
Industry Comparison
Time (Descending) + Action (Ascending)

IB Risk Navigator(SM): My Portfolio Portfolio Edit Report Dimensions Metrics View Settings

My Portfolio — Risk by Underlying — Suspend

Net.Liq: 180,235 USD | Maint.Margin: 29,816 USD | VAR: 3,234 USD
P&L: 1,655 USD | Init.Margin: 32,799 USD | ES: 4,081 USD

Equity | Bond | Forex | Commodity | MoneyMarket | StructuredProducts | P&L | VAR | Margin Sensitivity

Underlying	Position	Price	P&L for Day	Unrealiz...	Delta (Δ)	Delta Dollars	Gamma Dollars	Gamma ...	Vega	Theta (Θ)
All Underlyings			**805**	**4,535**	**11**	**171**	**2**	**198**	**40**	**-11**
FCAU <NYSE>		**15.19**	**805**	**4,535**	**11**	**171**	**2**	**198**	**40**	**-11**
FCAU Stock (NYSE)	-892	15.19	698	-295	-892	-13,549				
2015-11-20			**15**	**1,289**	**229**	**3,478**	**-2**	**-141**	**-15**	**15**
FCAU NOV 20 '15 13 Put	-15	0.14	15	1,289	229	3,478	-2	-141	-15	15
2016-01-15			**50**	**1,276**	**164**	**2,498**	**-1**	**-114**	**-23**	**5**
FCAU JAN 15 '16 17.5 Call	-20	0.45	500	184	-527	-7,998	-3	-225	-49	11
FCAU JAN 15 '16 14 Call	10	2.10	-450	1,092	691	10,496	1	110	26	-7
2015-12-18			**42**	**2,265**	**510**	**7,744**	**5**	**453**	**79**	**-30**
FCAU DEC 18 '15 13 Call	15	3.17	36	3,129	1,237	18,784	2	137	26	-10
FCAU DEC 18 '15 14 Put	25	0.40	6	-864	-727	-11,039	4	316	53	-20

Range | Update | Zoom



Report: Risk by Underlying
Plot: Equity Portfolio Value Change
Underlying: FCAU <NYSE>
Currency: All Currencies
Last Trading Day: TOTAL
Date: Select
Drill:

Slicer: [[Underlying]=FCAU <NYSE>]

Position(s) not included

plot report both

U1133678 | Reference currency: USD | Included Equity Positions: 77 — Live update

Time (Descending) + Action (Ascending)

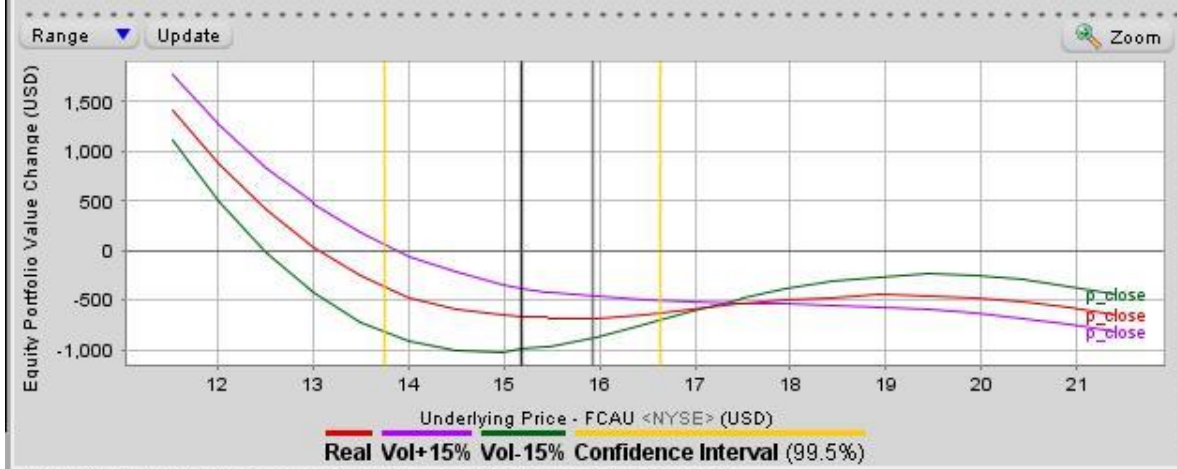
IB Risk Navigator(SM): My Portfolio Portfolio Edit Report Dimensions Metrics View Settings

My Portfolio — Risk by Underlying

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P&L: 1,655 USD | Init.Margin: 32,799 USD | ES: 4,081 USD

Equity | Bond | Forex | Commodity | MoneyMarket | StructuredProducts | P&L | VAR | Margin Sensitivity

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2016-01-15			**50**	**1,276**	**164**	**2,498**	**-1**	**-114**	**-23**	
FCAU JAN 15 '16 17.5 Call	-20	0.45	500	184	-527	-7,998	-3	-225	-49	
FCAU JAN 15 '16 14 Call	10	2.10	-450	1,092	691	10,496	1	110	26	
2015-12-18			**42**	**2,265**	**510**	**7,744**	**5**	**453**	**79**	
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FCAU DEC 18 '15 14 Put	25	0.40	6	-864	-727	-11,039	4	316	53	



Report: Risk by Underlying
Plot: Equity Portfolio Value Change
Underlying: FCAU <NYSE>
Currency: All Currencies
Last Trading Day: TOTAL
Date: Select
Drill:
Slicer: [[Underlying]=FCAU <NYSE>]
Position(s) not included
plot report both

U1133678 | Reference currency: USD | Included Equity Positions: 77 — Live upda

Time (Descending) + Action (Ascending)


IB Volatility Lab - Implied Volatility
118.90 -27.84 (-18.97%) U1133678
VRX NYSE Implied Volatility
Expiries: Nov Dec Jan'16
Time period: 6 months
104.2%
92.5%
250.00
225.00
200.00
175.00
150.00
125.00
118.90
100%
90%
80%
70%
60%
50%
40%
30%
5/1
6/1
7/1
8/3
9/1
10/1
VRX NYSE Multi-Expiry Skew
Expiries: Nov Jan'16
Strikes
Date: Oct 21st
NOV20
150%
125%
100%
75%
Strike: 10 118.90 150 175 200 225 250 275 300
VRX NYSE Time Lapse Skew
Days: Today Yesterday
Strikes
Expiry: Nov20 '15
Today
125%
100%
75%
Strike: 118.90 150 175 200 225
VRX NYSE Volatility Profile
HV IV
10 30 50 75 100 150 200
175.0%
150.0%
125.0%
100.0%
75.0%
Nov06'15
Dec18'15
Mar18'16
Jun1
VRX NYSE Open Interest
Expiries: Nov Jan'16
Strikes
Date: Oct 21st
Puts + Calls
30K
20K
10K
0
Strike: 109 118.90 129 139 149 197.5 247.5
Implied Volatility
Historical Volatility
Industry Comparison
Time (Descending) + Action (Ascending)

File Edit View History Bookmarks Tools Help
Valeant (VRX) Stock Plummets...
www.thestreet.com/story/13332670/1/valeant-pharmaceuticals-vrx-stock-tanks-citron-in
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NEW YORK (TheStreet) -- **Valeant Pharmaceuticals** (VRX - Get Report) stock is plummeting by 31.52% to $100.49 in midday trading on Wednesday, after **Citron Research** released a report accusing the company of creating a network of "phantom" pharmacies to falsify sales and avoid auditor scrutiny.



Pharmaceutical companies have determined a way to vastly overcharge patients for combinations of generic drugs that would cost much less when sold separately, the *New York Times* reported on Monday.

The manufacturers avoid insurers' and pharmacists' recommendations of generic drugs by imploring doctors to submit prescriptions directly to a mail-order pharmacy connected to the drug maker, the *Times* alleged.

Valeant and a specialty pharmacy called **Philidor Rx Services** have such a relationship, the *Times* reported, and Citron Research reiterated this in a report this morning.

The relationship came under scrutiny following Valeant's earnings report on Monday, in which the company announced it had purchased an option to acquire Philidor in late 2014.

TSLA Buy Sell $209.92 -1.50%

GET $350
with an eligible fully-funded Advance relationship and qualifying activities.
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Offer Details
HSBC


IB Volatility Lab - Implied Volatility
150.07 -0.38 (-0.25%) U1133678
AMGN NASDAQ.NMS Implied Volatility
Expiries: Nov Dec Jan'16
Time period: 6 months
175.00
170.00
165.00
160.00
155.00
150.07
145.00
140.00
135.00
42.5%
40%
37.5%
35%
33.7%
32.5%
30.7%
30%
27.5%
25%
22.5%
5/1
6/1
7/1
8/3
9/1
10/1
AMGN NASDAQ.NMS Multi-Expiry Skew
Expiries: Nov Jan'16
Strikes
Date: Oct 21st
NOV 20
70%
60%
50%
40%
30%
Strike: 100 125 150.07 175 200 225
AMGN NASDAQ.NMS Time Lapse Skew
Days: Today Yesterday
Strikes
Expiry: Nov20 '15
Today
55%
50%
45%
40%
35%
30%
25%
Strike: 120 130 140 150.07 160 170 180 190
AMGN NASDAQ.NMS Volatility Profile
HV IV
10 30 50 75 100 150 200
42.5%
40.0%
37.5%
35.0%
32.5%
30.0%
Nov06'15 Dec18'15 Jan15'16 Apr15'16
AMGN NASDAQ.NMS Open Interest
Expiries: Nov Jan'16
Strikes
Date: Oct 21st
Puts + Calls
15K
10K
5K
0
Strike: 113 120 127 134 141 150.07 180
Implied Volatility
Historical Volatility
Industry Comparison
Time (Descending) + Action (Ascending)


IB Volatility Lab - Implied Volatility
68.00 -0.91 (-1.32%) U1133678
XLV ARCA Implied Volatility
Expiries: Nov Dec Jan'16
Time period: 6 months
77.00
76.00
75.28
74.00
73.00
72.00
71.00
70.00
69.00
68.00
67.00
66.00
65.00
64.00
30%
27.5%
25%
22.5%
20.5%
17.5%
16.5%
15%
5/1
6/1
7/17/10
8/3
9/1
10/1
XLV ARCA Multi-Expiry Skew
Expiries: Nov Jan'16
Strikes
Date: Oct 21st
NOV20
60%
50%
40%
30%
20%
Strike: 55 60 6568.000 75 80 85 90
XLV ARCA Time Lapse Skew
Days: Today Yesterday
Strikes
Expiry: Nov20 '15
Today
45%
40%
35%
30%
25%
20%
15%
Strike: 60.0 62.5 65.0 68.0070.0 72.5 75.0 77.5 80.0
XLV ARCA Volatility Profile
HV IV
10 30 50 75 100 150 200
45.0%
40.0%
35.0%
30.0%
25.0%
20.0%
Nov06'15
Dec18'15 Jan15'16
Mar18'16
Jur
XLV ARCA Open Interest
Expiries: Nov Jan'16
Strikes
Date: Oct 21st
Puts + Calls
20K
10K
0
Strike: 55 58.5 62 65.68.00 72.5 76 79.5 83
Implied Volatility
Historical Volatility
Industry Comparison

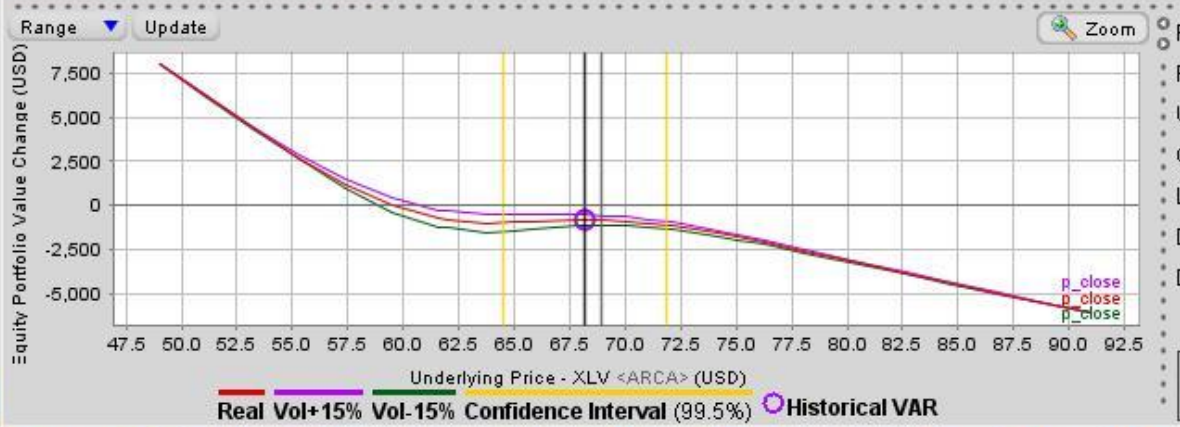
IB Risk Navigator(SM): My Portfolio Portfolio Edit Report Dimensions Metrics View Settings

My Portfolio — Risk by Underlying — Suspend

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P&L: 1,661 USD | Init.Margin: 32,799 USD | ES: 4,101 USD

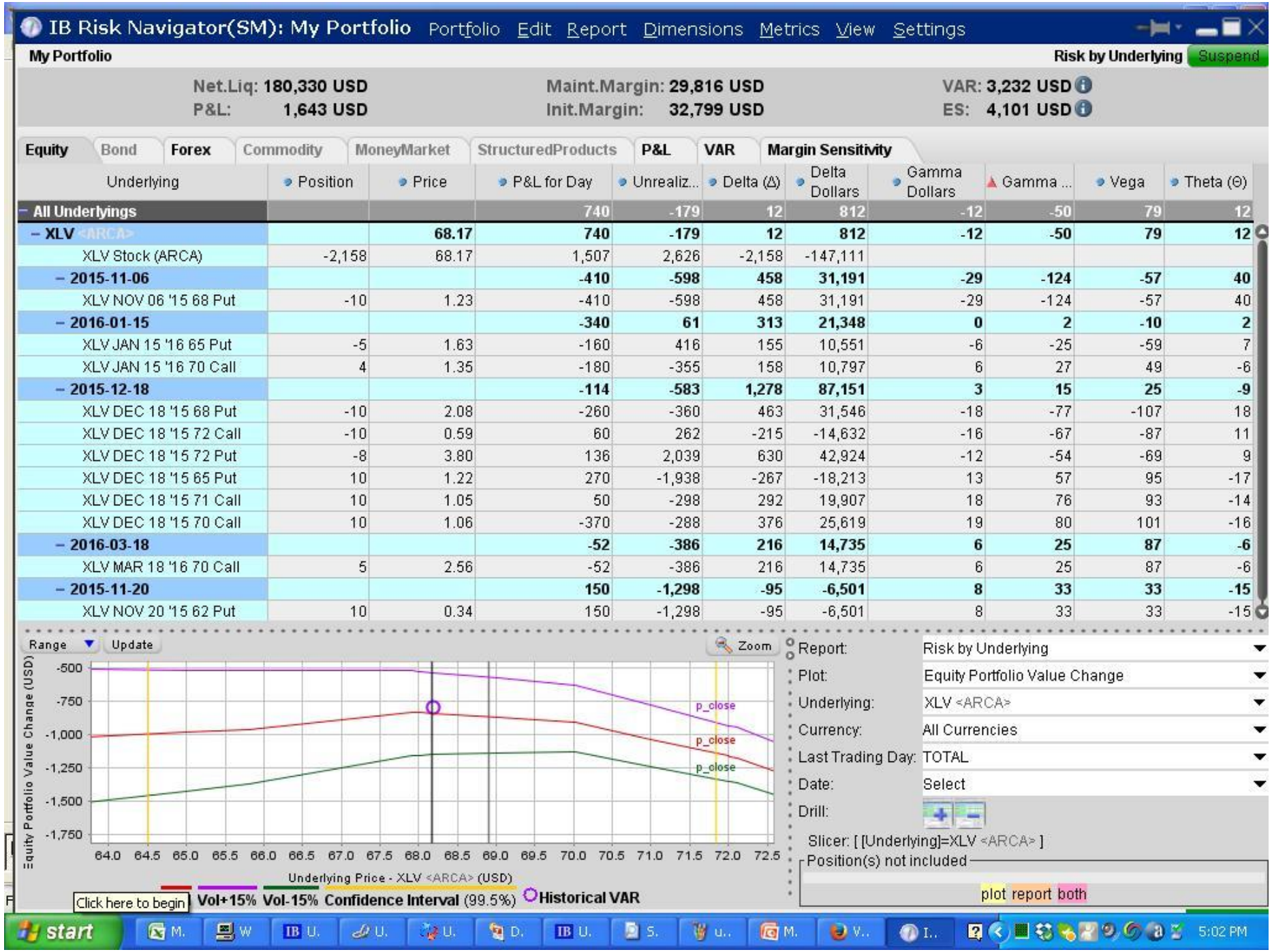
Equity | Bond | Forex | Commodity | MoneyMarket | StructuredProducts | P&L | VAR | Margin Sensitivity

Underlying	Position	Price	P&L for Day	Unrealiz...	Delta (Δ)	Delta Dollars	Gamma Dollars	Gamma ...	Vega	Theta (Θ)
All Underlyings			**740**	**-179**	**12**	**812**	**-12**	**-50**	**79**	**12**
XLV <ARCA>		**68.17**	**740**	**-179**	**12**	**812**	**-12**	**-50**	**79**	**12**
XLV Stock (ARCA)	-2,158	68.17	1,507	2,626	-2,158	-147,111				
2015-11-06			**-410**	**-598**	**458**	**31,191**	**-29**	**-124**	**-57**	**40**
XLV NOV 06 '15 68 Put	-10	1.23	-410	-598	458	31,191	-29	-124	-57	40
2016-01-15			**-340**	**61**	**313**	**21,348**	**0**	**2**	**-10**	**2**
XLV JAN 15 '16 65 Put	-5	1.63	-160	416	155	10,551	-6	-25	-59	7
XLV JAN 15 '16 70 Call	4	1.35	-180	-355	158	10,797	6	27	49	-6
2015-12-18			**-114**	**-583**	**1,278**	**87,151**	**3**	**15**	**25**	**-9**
XLV DEC 18 '15 68 Put	-10	2.08	-260	-360	463	31,546	-18	-77	-107	18
XLV DEC 18 '15 72 Call	-10	0.59	60	262	-215	-14,632	-16	-67	-87	11
XLV DEC 18 '15 72 Put	-8	3.80	136	2,039	630	42,924	-12	-54	-69	9
XLV DEC 18 '15 65 Put	10	1.22	270	-1,938	-267	-18,213	13	57	95	-17
XLV DEC 18 '15 71 Call	10	1.05	50	-298	292	19,907	18	76	93	-14
XLV DEC 18 '15 70 Call	10	1.06	-370	-288	376	25,619	19	80	101	-16
2016-03-18			**-52**	**-386**	**216**	**14,735**	**6**	**25**	**87**	**-6**
XLV MAR 18 '16 70 Call	5	2.56	-52	-386	216	14,735	6	25	87	-6
2015-11-20			**150**	**-1,298**	**-95**	**-6,501**	**8**	**33**	**33**	**-15**
XLV NOV 20 '15 62 Put	10	0.34	150	-1,298	-95	-6,501	8	33	33	-15

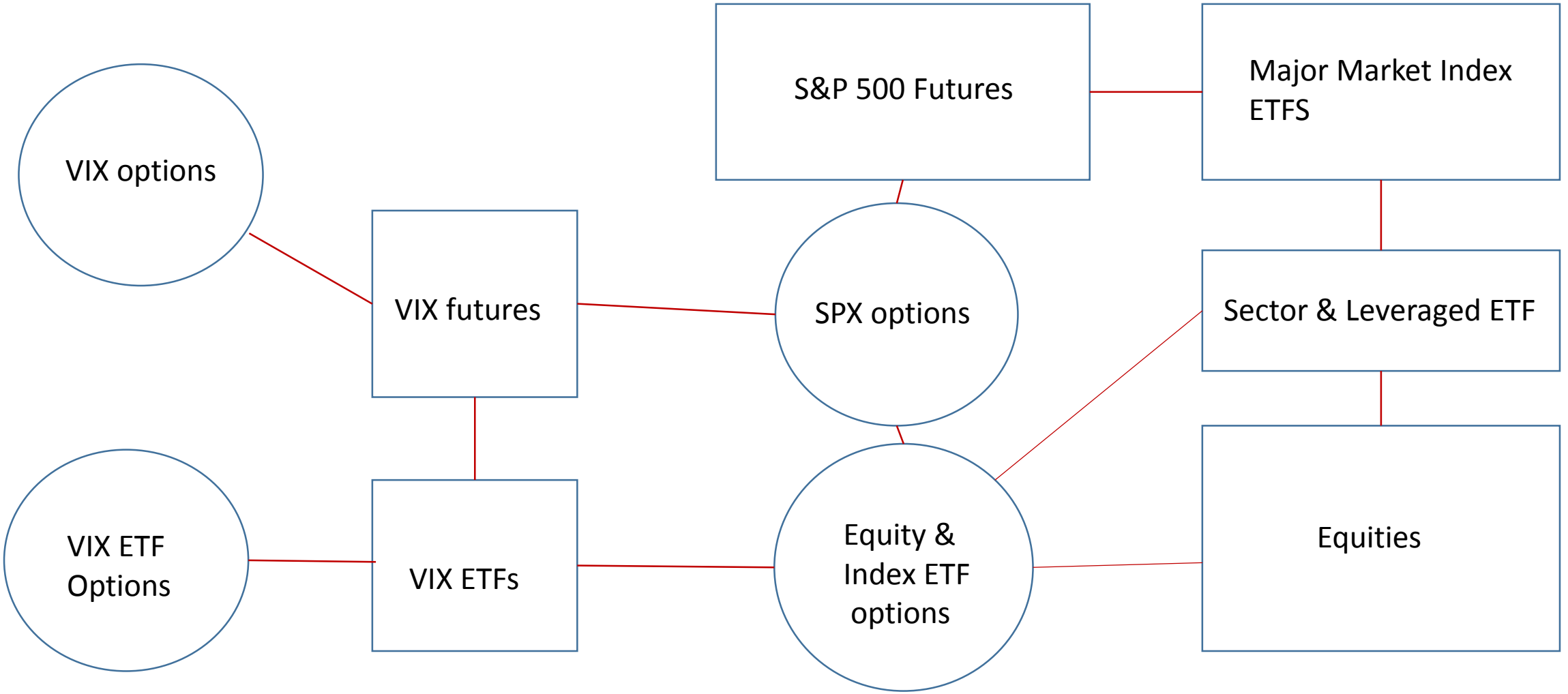


Report: Risk by Underlying
Plot: Equity Portfolio Value Change
Underlying: XLV <ARCA>
Currency: All Currencies
Last Trading Day: TOTAL
Date: Select
Drill:
Slicer: [[Underlying]=XLV <ARCA>]
Position(s) not included
plot report both

start M.. W.. IB U.. U.. U.. D.. IB U.. S.. V.. M.. V.. I.. 5:01 PM


IB Risk Navigator(SM): My Portfolio
Portfolio Edit Report Dimensions Metrics View Settings
My Portfolio
Risk by Underlying
Suspend
Net.Liq: 180,330 USD
P&L: 1,643 USD
Maint.Margin: 29,816 USD
Init.Margin: 32,799 USD
VAR: 3,232 USD
ES: 4,101 USD
Equity
Bond
Forex
Commodity
MoneyMarket
StructuredProducts
P&L
VAR
Margin Sensitivity
Underlying
Position
Price
P&L for Day
Unrealiz...
Delta (Δ)
Delta Dollars
Gamma Dollars
Gamma ...
Vega
Theta (Θ)
All Underlyings 740 -179 12 812 -12 -50 79 12
XLV <ARCA> 68.17 740 -179 12 812 -12 -50 79 12
XLV Stock (ARCA) -2,158 68.17 1,507 2,626 -2,158 -147,111
2015-11-06 -410 -598 458 31,191 -29 -124 -57 40
XLV NOV 06 '15 68 Put -10 1.23 -410 -598 458 31,191 -29 -124 -57 40
2016-01-15 -340 61 313 21,348 0 2 -10 2
XLV JAN 15 '16 65 Put -5 1.63 -160 416 155 10,551 -6 -25 -59 7
XLV JAN 15 '16 70 Call 4 1.35 -180 -355 158 10,797 6 27 49 -6
2015-12-18 -114 -583 1,278 87,151 3 15 25 -9
XLV DEC 18 '15 68 Put -10 2.08 -260 -360 463 31,546 -18 -77 -107 18
XLV DEC 18 '15 72 Call -10 0.59 60 262 -215 -14,632 -16 -67 -87 11
XLV DEC 18 '15 72 Put -8 3.80 136 2,039 630 42,924 -12 -54 -69 9
XLV DEC 18 '15 65 Put 10 1.22 270 -1,938 -267 -18,213 13 57 95 -17
XLV DEC 18 '15 71 Call 10 1.05 50 -298 292 19,907 18 76 93 -14
XLV DEC 18 '15 70 Call 10 1.06 -370 -288 376 25,619 19 80 101 -16
2016-03-18 -52 -386 216 14,735 6 25 87 -6
XLV MAR 18 '16 70 Call 5 2.56 -52 -386 216 14,735 6 25 87 -6
2015-11-20 150 -1,298 -95 -6,501 8 33 33 -15
XLV NOV 20 '15 62 Put 10 0.34 150 -1,298 -95 -6,501 8 33 33 -15
Range
Update
Zoom
Equity Portfolio Value Change (USD)
-500
-750
-1,000
-1,250
-1,500
-1,750
p_close
p_close
p_close
64.0 64.5 65.0 65.5 66.0 66.5 67.0 67.5 68.0 68.5 69.0 69.5 70.0 70.5 71.0 71.5 72.0 72.5
Underlying Price - XLV <ARCA> (USD)
Click here to begin
Vol+15% Vol-15% Confidence Interval (99.5%) Historical VAR
Report: Risk by Underlying
Plot: Equity Portfolio Value Change
Underlying: XLV <ARCA>
Currency: All Currencies
Last Trading Day: TOTAL
Date: Select
Drill:
Slicer: [[Underlying]=XLV <ARCA>]
Position(s) not included
plot report both
start
5:02 PM

The Equity Volatility World

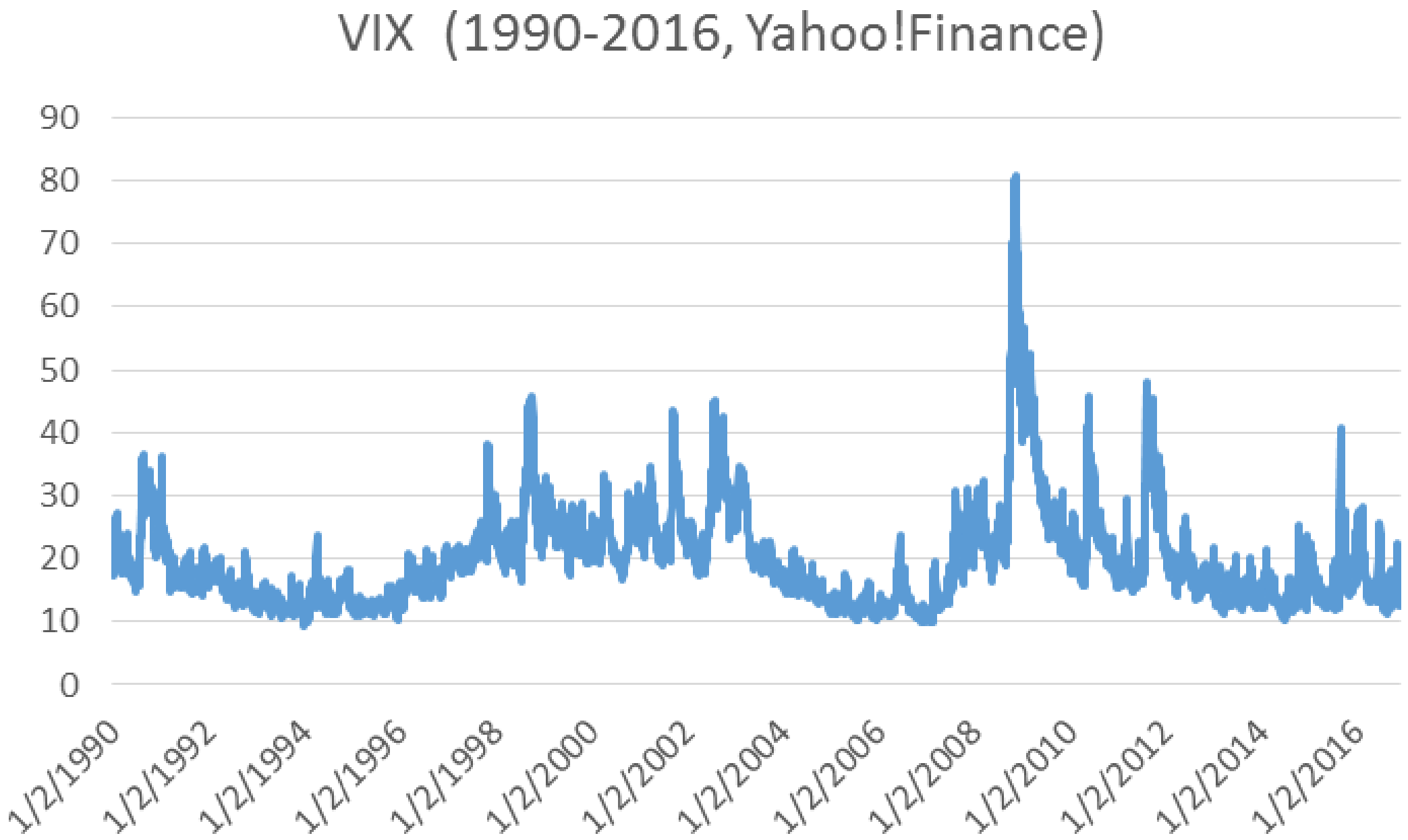


2. Focus on VIX futures and ETNs

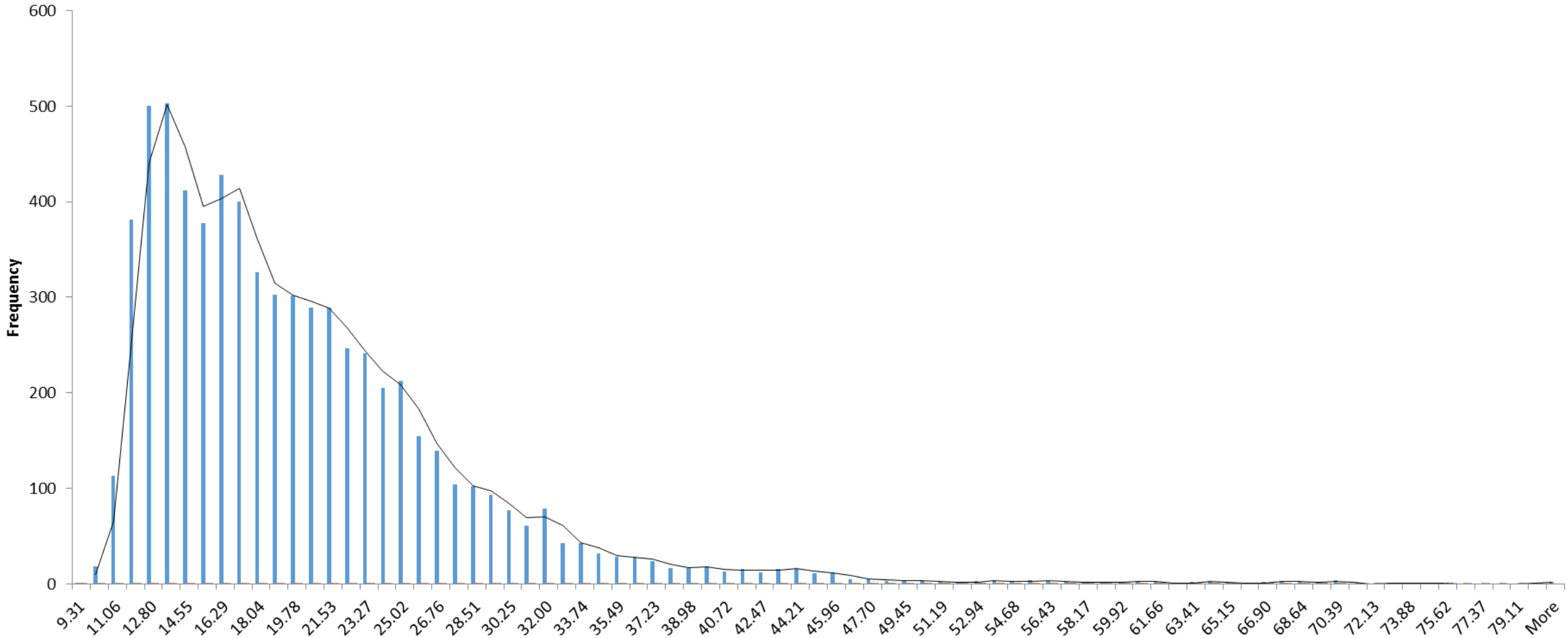
What is VIX?

- VIX is an index which tracks the implied volatilities of first- and second- month expiration of SPX options.
- Old VIX (since 1992): a weighted average of ATM implied volatilities
- New VIX (since 2000): based on **Variance Swap formula,** representing a basket traded options. Hence New VIX is an ``investable index''.
- VIX Futures started trading in 2004
- VIX Options started trading in 2006
- VIX ETNs started trading in 2009

VIX is one of the most important indices in US equity trading, representing the ``fear gauge'' of the Market.


VIX (1990-2016, Yahoo!Finance)
90
80
70
60
50
40
30
20
10
0
1/2/1990
1/2/1992
1/2/1994
1/2/1996
1/2/1998
1/2/2000
1/2/2002
1/2/2004
1/2/2006
1/2/2008
1/2/2010
1/2/2012
1/2/2014
1/2/2016

Histogram VIX since inception (6779 days)



Statistics of VIX Levels

Long term mean

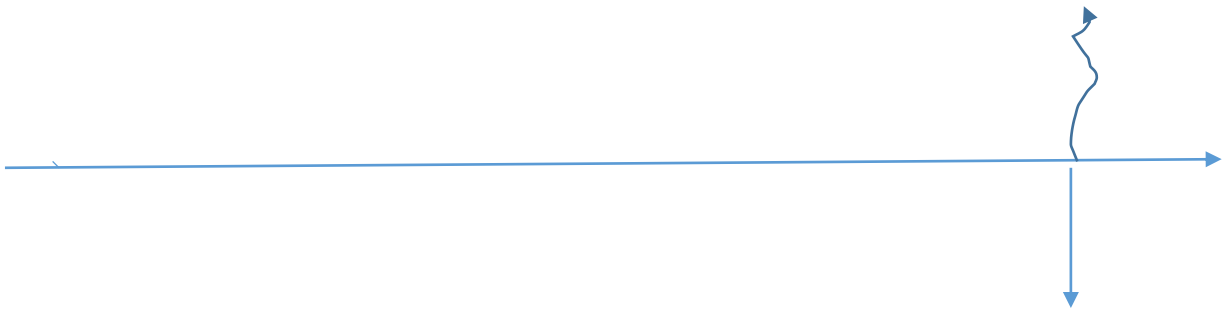
Most likely level

Fat tails

Upside risk

VIX Descriptive Stats	
Mean	19.70256232
Standard Error	0.095343095
Median	17.84
Mode	12.42
Standard Deviation	7.850043462
Sample Variance	61.62318236
Kurtosis	7.753084648
Skewness	2.111767067
Range	71.550001
Minimum	9.31
Maximum	80.860001
Count	6779
Largest(10)	69.25
Smallest(10)	10.02
Confidence Level(95.0%)	0.186902407

Variance Swap



Floating leg = Variance

Fixed leg = Fixed amount (VSV)

$$Floating\ Leg = \sum_{i=1}^{N} R_i^2$$

$$Fixed\ Leg\ = E\left\{\sum_{i=1}^{N} R_i^2\right\}$$

$$R_i = \frac{S_{i\Delta t}}{S_{(i-1)\Delta t}} - 1, i = 1, \dots, N, \qquad N\Delta t = T$$

Calculation of Fixed Leg (I)

F= forward price of SPX

$$d\ln F = \frac{dF}{F} - \frac{1}{2}\left(\frac{dF}{F}\right)^2 + \mathrm{o}(\Delta t)$$

$$\ln\left(\frac{F_T}{F_0}\right) = \sum_{i=1}^{N}\frac{dF}{F} - \frac{1}{2}\sum_{i=1}^{N}\left(\frac{dF}{F}\right)^2 + \mathrm{o}(1)$$

$$\mathrm{E}\left(\ln\left(\frac{F_T}{F_0}\right)\right) = E\left(\sum_{i=1}^{N}\frac{dF}{F}\right) - \frac{1}{2}E\left(\sum_{i=1}^{N}\left(\frac{dF}{F}\right)^2\right) + \mathrm{o}(1)$$

$$\mathrm{E}\left(\ln\left(\frac{F_T}{F_0}\right)\right) = -\frac{1}{2}E\left(\sum_{i=1}^{N}\left(\frac{dF}{F}\right)^2\right) + \mathrm{o}(1)$$

Calculation of Fixed Leg of VS (II)

$$\frac{dF}{F} = \frac{dS}{S} + (r - d)\Delta t$$

$$\left(\frac{dF}{F}\right)^2 = \left(\frac{dS}{S}\right)^2 + o(\Delta t)$$

$$\sum_{i=1}^{N}\left(\frac{dS}{S}\right)^2 = \sum_{i=1}^{N}\left(\frac{dF}{F}\right)^2 + o(1)$$

$$E\left(\sum_{i=1}^{N}\left(\frac{dS}{S}\right)^2\right) = E\left(\sum_{i=1}^{N}\left(\frac{dF}{F}\right)^2\right) + o(1) = 2E\left(ln\left(\frac{F_0}{F_T}\right)\right) + o(1)$$

Fixed Leg of VS (III)

As $\Delta t \to 0$,

$$Fixed\ Leg = 2\,E\left[ln\left(\frac{F_0}{F_T}\right)\right] = 2\,E\left[ln\left(\frac{F_0}{S_T}\right)\right]$$

Taylor Expansion:

$$ln\frac{S}{F} = \frac{1}{F}(S-F)\ -\int_F^S (S-V)\frac{dV}{V^2}$$

$$ln\frac{S}{F} = \frac{1}{F}(S-F) - \int_F^\infty (S-V)^+\frac{dV}{V^2} - \int_0^F (V-S)^+\frac{dV}{V^2}$$

$$Fixed\ Leg = 2\,E\left[ln\left(\frac{F_0}{S_T}\right)\right] = 2\int_{F_0}^\infty E(S_T-V)^+\frac{dV}{V^2} + 2\int_0^{F_0} E(V-S_T)^+\frac{dV}{V^2}$$

$$E(S-F)=0$$

The VIX Formula

- Variance Swap Volatility

$$\sigma_T^2 = \frac{2e^{rT}}{T}\int_0^{\infty} OTM(K,T,S)\frac{dK}{K^2}$$

- Here $OTM(K,T,S)$ represents the value of the OTM (forward) option with strike K, or ATM if S=F.

- CBOE created a discrete version of the VSV in which the sum replaces the integral and the maturity is 30 days. Since there are no 30 day options, VIX uses first two maturities*

$$VIX = \sqrt{w_1\sum_{i=1}^{n} OTM(K_i,T_1,S)\frac{\Delta K}{K_i^2} + w_2\sum_{i=1}^{n} OTM(K_i,T_2,S)\frac{\Delta K}{K_i^2}}$$

* My understanding is that recently they could have added more maturities using weekly options as well.


VOLATILITY S&P 500 (^VIX)
Watchlist
14.13
-0.56(-3.81%)
Chicago Board Options Exchange - As of 3:10PM EDT
Beat the market
Get the app
Indicator
Comparison
1d
5d
1m
3m
6m
YTD
1y
2y
5y
10y
Max
Linear
Go To Symbol
Open 16.16
Close 12.59
Low 12.49
High 16.16
Vol 0.00
% Chg -27.77%
^VIX 12.59
VIX Last 5 years
US Downgrade
60.0
50.0
40.0
30.0
20.0
10.0
Jan 2 '12
Mar 4 '13
Jan 6 '14
Jan 5 '15
Jan 4 '16


VOLATILITY S&P 500 (^VIX)
Watchlist
14.21
-0.48(-3.27%)
Chicago Board Options Exchange - As of 03:22pm EDT
Beat the market
Get the app
Indicator
Comparison
1d
5d
1m
3m
6m
YTD
1y
2y
5y
10y
Max
Linear
Go To Symbol
^VIX 14.20
Open 14.55
Close 14.20
Low 13.95
High 15.42
Vol 0.00
% Chg 8.56%
VIX: Last 10 years
Lehman
Greece
US Downgrade
China deval
YAHOO! FINANCE
100.0
75.0
50.0
25.0
0.0
14.2
Jan 1 '10
Jan 1 '16

VIX as a weighted average of volatilites

Emphasizing the dependence on the implied vol:

$$\sigma_T^2 = \frac{2e^{rT}}{T}\int_0^\infty OTM(K,T,\sigma(K,T))\frac{dK}{K^2}$$

and that, trivially,

$$\sigma_T^2 = \frac{2e^{rT}}{T}\int_0^\infty OTM(K,T,\sigma_T)\frac{dK}{K^2}$$

conclude that

$$0 = \int_0^\infty [OTM\big(K,T,\sigma(K,T)\big) - \ OTM(K,T,\sigma_T)]\frac{dK}{K^2}$$

VIX as weighted average of IVOLS

$$0 = \int_0^\infty [OTM\big(K,T,\sigma(K,T)\big) - \ OTM(K,T,\sigma_T)]\frac{dK}{K^2}$$

From last slide

$$0 = \int_0^\infty Vega\big(K,T,\sigma^*(K,T)\big)\,(\sigma(K,T) - \sigma_T)\frac{dK}{K^2}$$

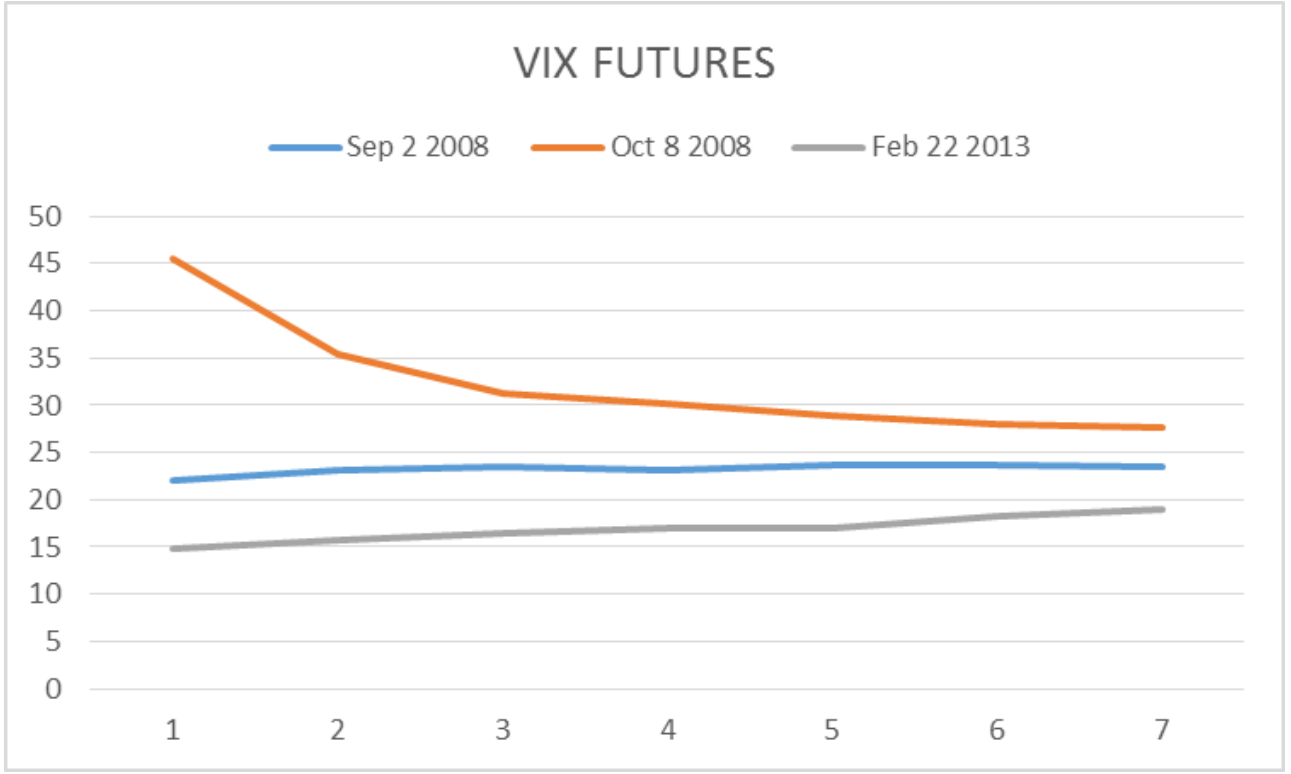
Mean value thm.

$$\sigma_T = \frac{\int_0^\infty \sigma(K,T)\,Vega\big(K,T,\sigma^*(K,T)\big)\frac{dK}{K^2}}{\int_0^\infty Vega\big(K,T,\sigma^*(K,T)\big)\frac{dK}{K^2}}$$

This formula can be simplified considerably by passing to log-moneyness variables and using the explicit form of Vega. This is left as an exercise to the reader. Due to the concentration of Vega around the 50-delta strikes and that options are short-dated, **this weighted average is very concentrated on near the money options.** Intuitively, think of VIX at 30-day ATM implied volatility of S&P 500.

VIX Futures (CBOE, symbol VX)

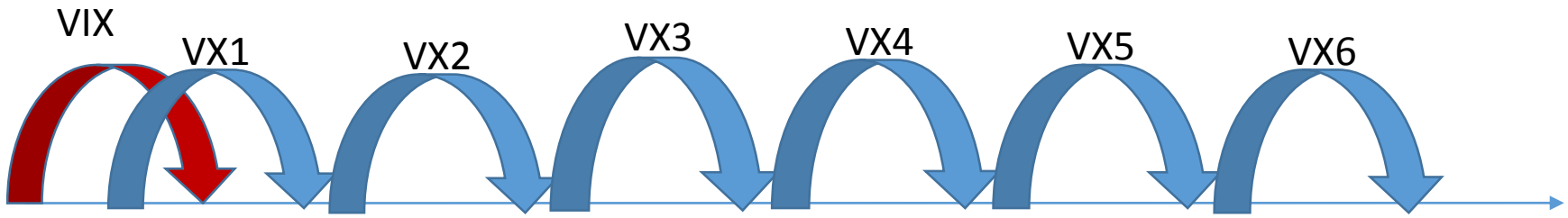
- Monthly contracts with settlements up to 1 year
- Weekly expirations in the 2 front months
- Each contract notional amount is for 1000× VIX at settlement. This means each contract has vega≅ $1000



Full information on VX
On the CBOE.com site.

Tick size, margin, hours,
etc, etc.

VIX futures as a ``forward volatility'' forecast



- Each VIX futures covers 30 days of volatility after the settlement date.
- Settlement dates are 1 month apart.
- Recently, weekly settlements have been added in the first two months.

Heuristically, we think of Vxi as a risk-neutral forecast of the forward 30-day volatility in I months, so we expect

$$\sigma^2_{n\,months} = \frac{1}{n}\sum_{i=0}^{n-1}(VXi)^2$$

to hold at any settlement date. This is not the case, because VX has no strike.
However, this formula suggests a strong link between SPX ATM IVOLS and VIX.

Example of an application of VIX futures for hedging

- A trader has 3-month Vega exposure of USD 300,000 and wants to hedge it.
- Each VIX contract has a Vega of USD 1,000.
- He assumes that the volatilities move in parallel, but nevertheless wants to capture the volatility of volatility (short dated vol is more volatile).
- A logical hedge would be to hedge 300,000 Vegas with weeklies, and the first **two** monthly contracts.

Buy 100 contracts of VX2, 100 contracts of VX1 and 100 contracts split between the weeklies from now to the first settlement. If you don't want to use weekly futures, then split the 300 contracts between VX1 and VX2, with a slight overweight on VX1 (eg. 170 VX1, 130 VX2.)

VIX as the ``commodity'' of equity markets

- VIX Futures = Futures contracts with monthly expirations settling on spot VIX. 10 monthly expirations are actively traded with considerable liquidity in the 3 front months.

- VX term structure is generally in upward-sloping (like index option volatility)

- In a trending market, option volatility is higher for longer maturities than for shorter ones

- During a market dislocation, traders ``bid up'' near term volatility producing Backwardation in the VIX futures

- Volatility is ``stored'' in SPX options and variance contracts, but is not easily converted to VIX and vice versa. **No easy cost-of-carry formula!!**


VIX Dec'16 @CFE
1 wk/5 min bars
File
Edit
View
Alert
Buy
Sell
L: 13.3500 CH: +0.2500 CH%: 1.91%
LONG 2 @ 15.0023 P&L: 500
15.5000
15.2500
15.0000
14.7500
14.5000
14.2500
14.0000
13.7500
13.5000
13.3500
13.2500
13.0000
12.7500
10:00
12:00
Dec 2 10:00
12:00
Dec 5 10:00
12:00
Dec 6 10:00
12:00
Dec 7 10:00
12:00
14:00
'Sep 12
'Sep 26
'Oct 10
'Oct 24
'Nov 7
'Nov 21
'Dec 5

Description File Configure

Contract Description

CBOE Volatility Index

Security Type	**FUT**
Underlying	**VIX IND**
Contract Month	**DEC16**
Expiration	**DEC 21 '16 08:00 CST**
Currency	**USD**
Multiplier	**1000**
Exchange	**CFE**
Trading Class	**VX**
Symbol	**VXZ6**
Sector	**Indices**
Industry	**Volatility Index**
Category	*
Product Type	**Equity Index Volatility**
Settlement Method	**Cash**

Trading Hours: December 7, 2016 Calendar

Regular Trading Session	08:30 CST - 15:15 CST (09:30 EST - 16:15 EST)
Total Available Hours	*15:30 CST* - 15:15 CST (*16:30 EST* - 16:15 EST)

Note: times in italics are on the calendar date preceding trade date

Current Margin Requirements

Intraday Initial	**11180.00**
Intraday Maintenance	**8944.00**
Overnight Initial	**11180.00**
Overnight Maintenance	**8944.00**

Theoretical Futures Pricing

- Cash & carry costs include transportation and storage and ``convenience'' of having oil to be able to deliver it and replace it later (think crude oil)

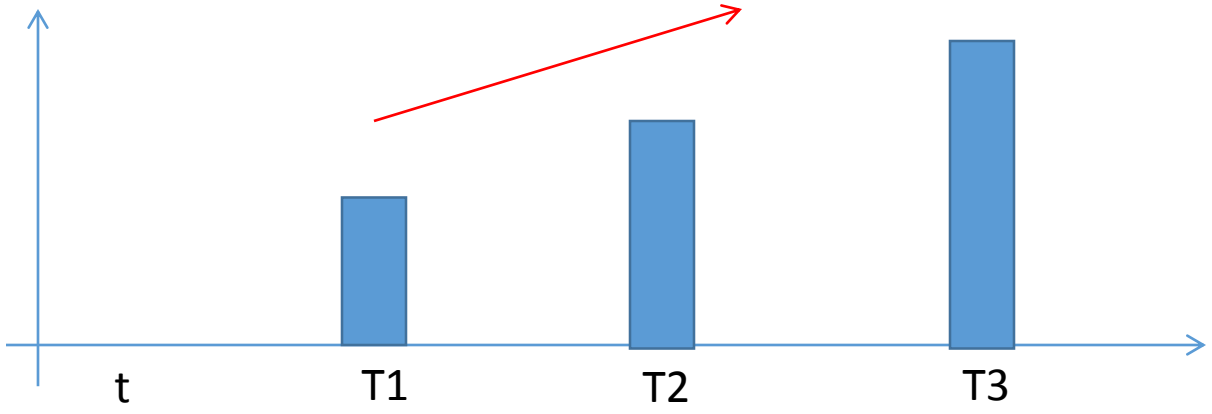
$$q = (\text{convenience yield}) - (\text{transportation}) - (\text{storage costs})$$

$$r = \text{term rate}$$

$$F_{0,T} = S_0 e^{(r-q)T}$$

- The shape of the forward curve depends on the supply/demand commodity & forecasts.

Commodity Futures: Contango

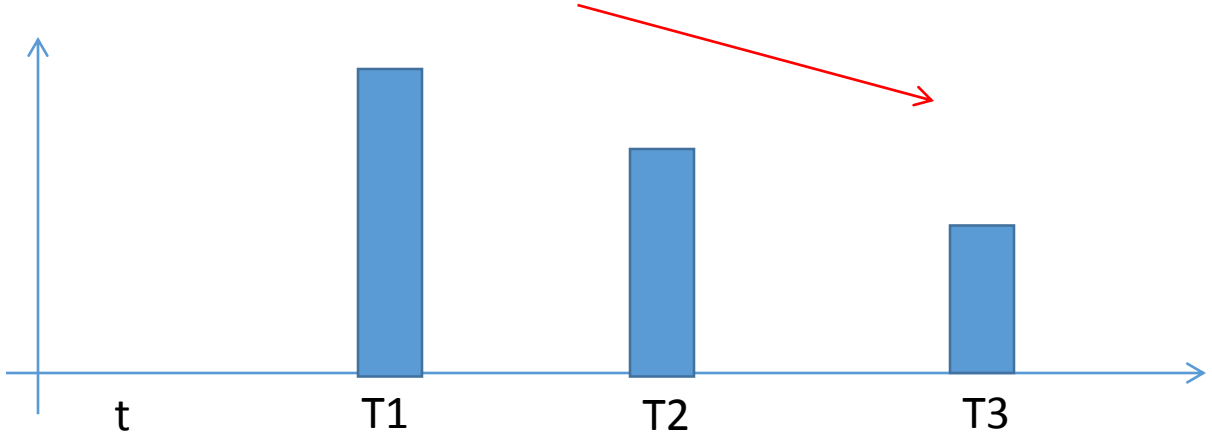


Futures are said to be in **contango** if the futures price increases with the time-to-delivery (futures is higher than spot)

If the futures are in contango, this means that the ``convenience yield'' is low and the cost of storing and financing make the forward delivery more expensive as time passes.

For VIX futures its all about demand for hedging products – there is no possibility of ``storage''… how do we price it!!

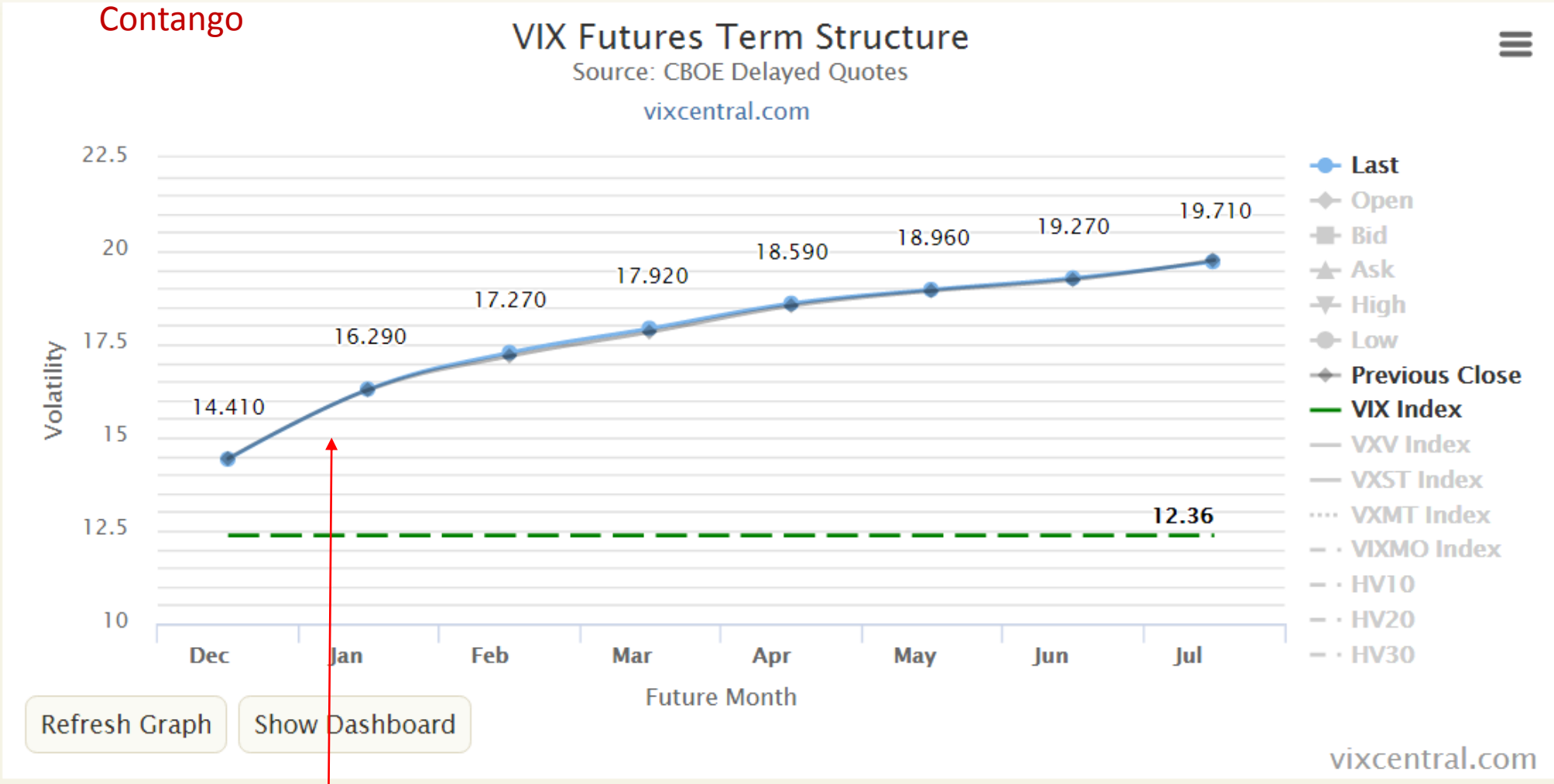
Commodity Futures: Backwardation



Futures are said to be in **backwardation** if the futures price decreases with the time-to-delivery (futures is lower than spot)

Associated with a high convenience yield. For example, rental for gold could be expensive, etc.

Contango

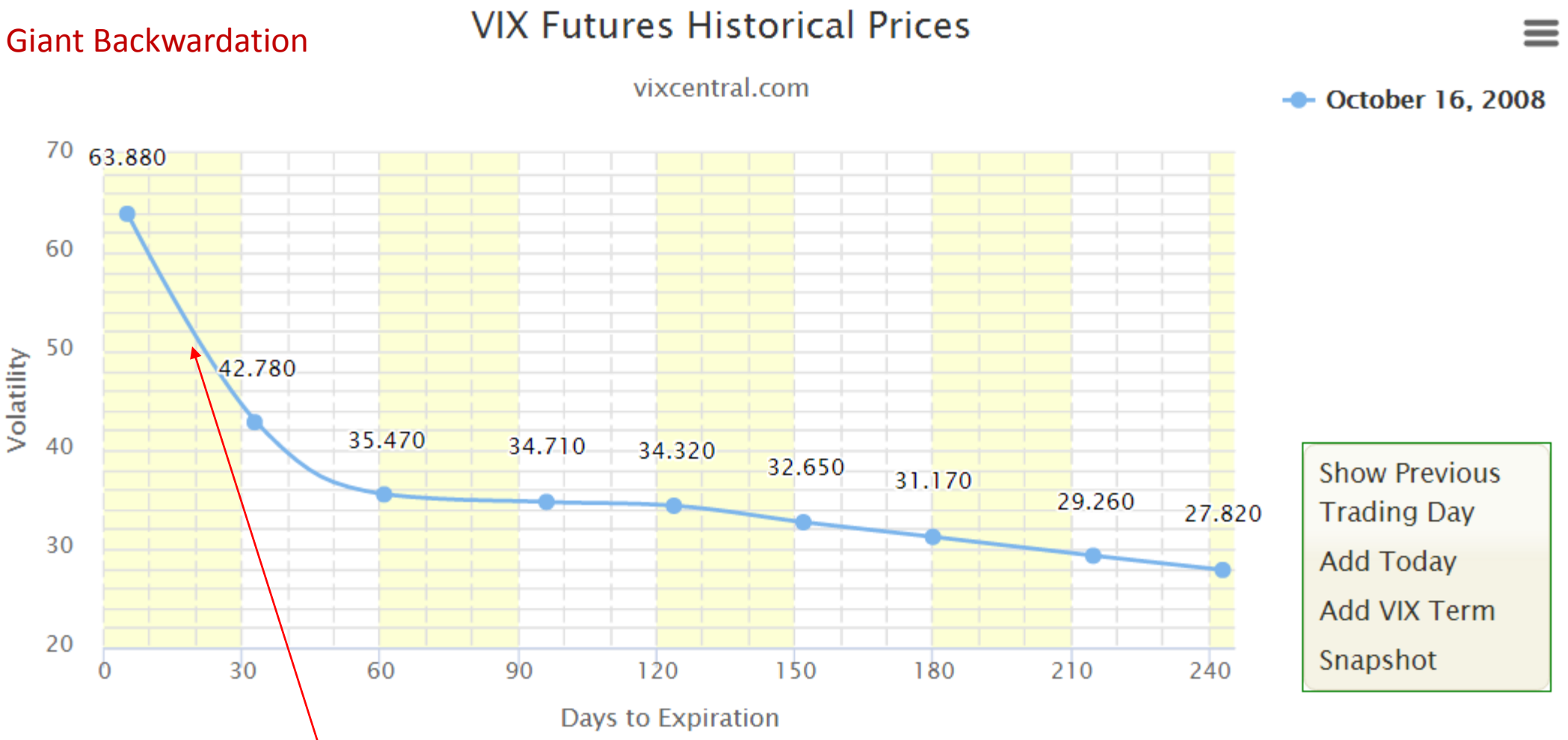


% Contango	**1**	13.05%	**2**	6.02%	**3**	3.76%	**4**	3.74%	**5**	1.99%	**6**	1.64%	**7**	2.28%	**8**
Difference	**1**	1.88	**2**	0.98	**3**	0.65	**4**	0.67	**5**	0.37	**6**	0.31	**7**	0.44	**8**

Month 7 to 4 contango	7.53%	2.51%

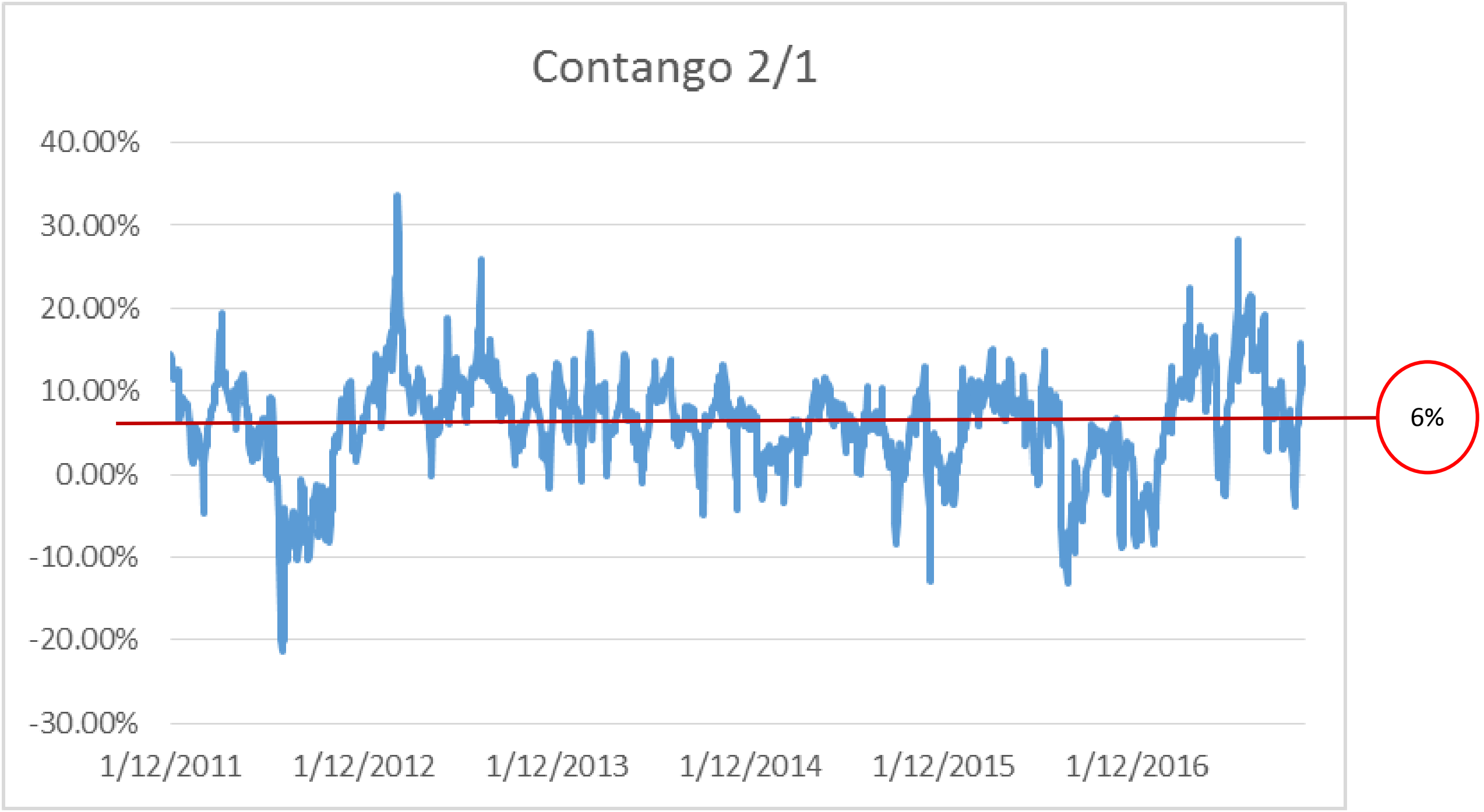
VIX futures curve on November 23, 2016, 4:20 pm

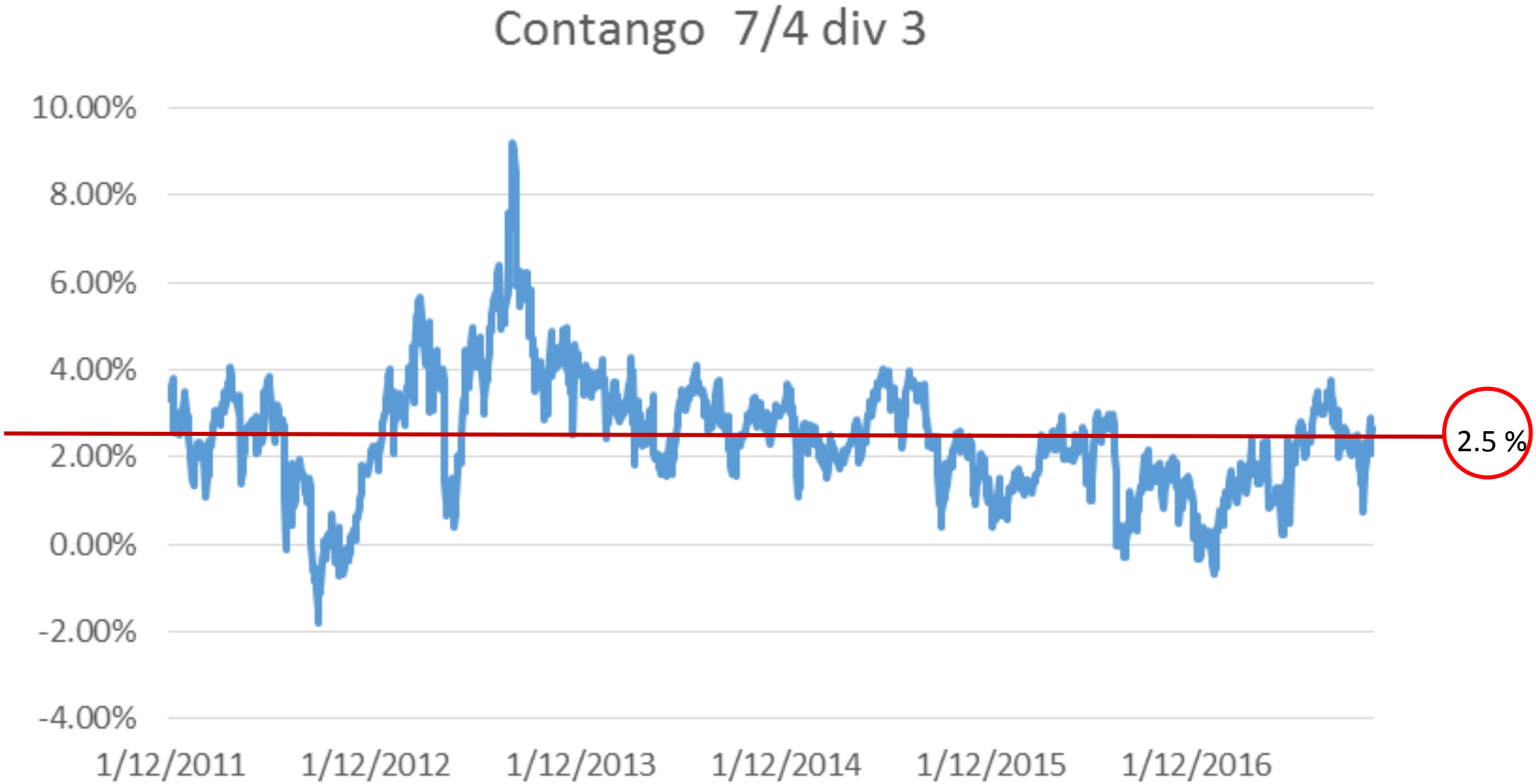
Giant Backwardation

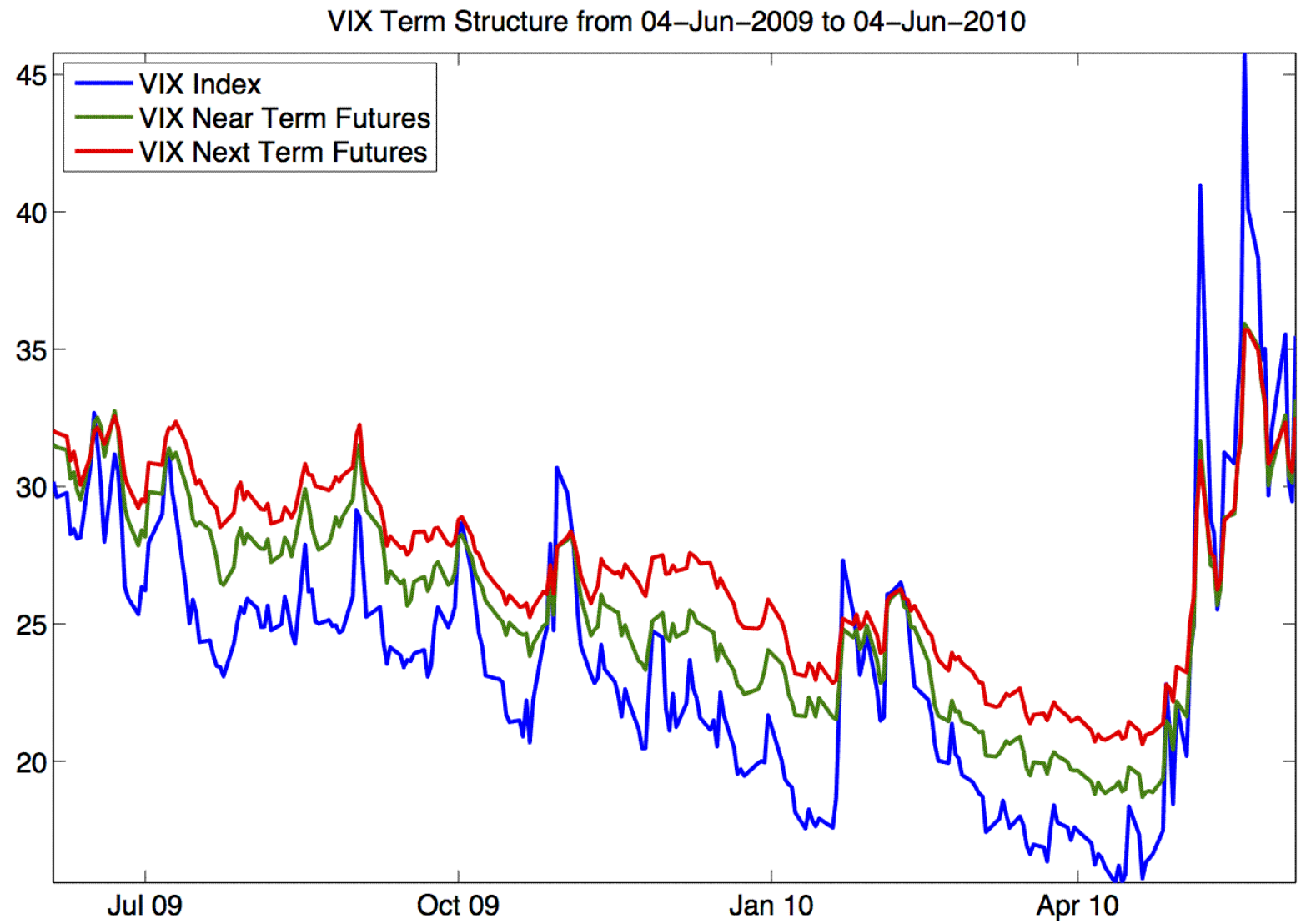


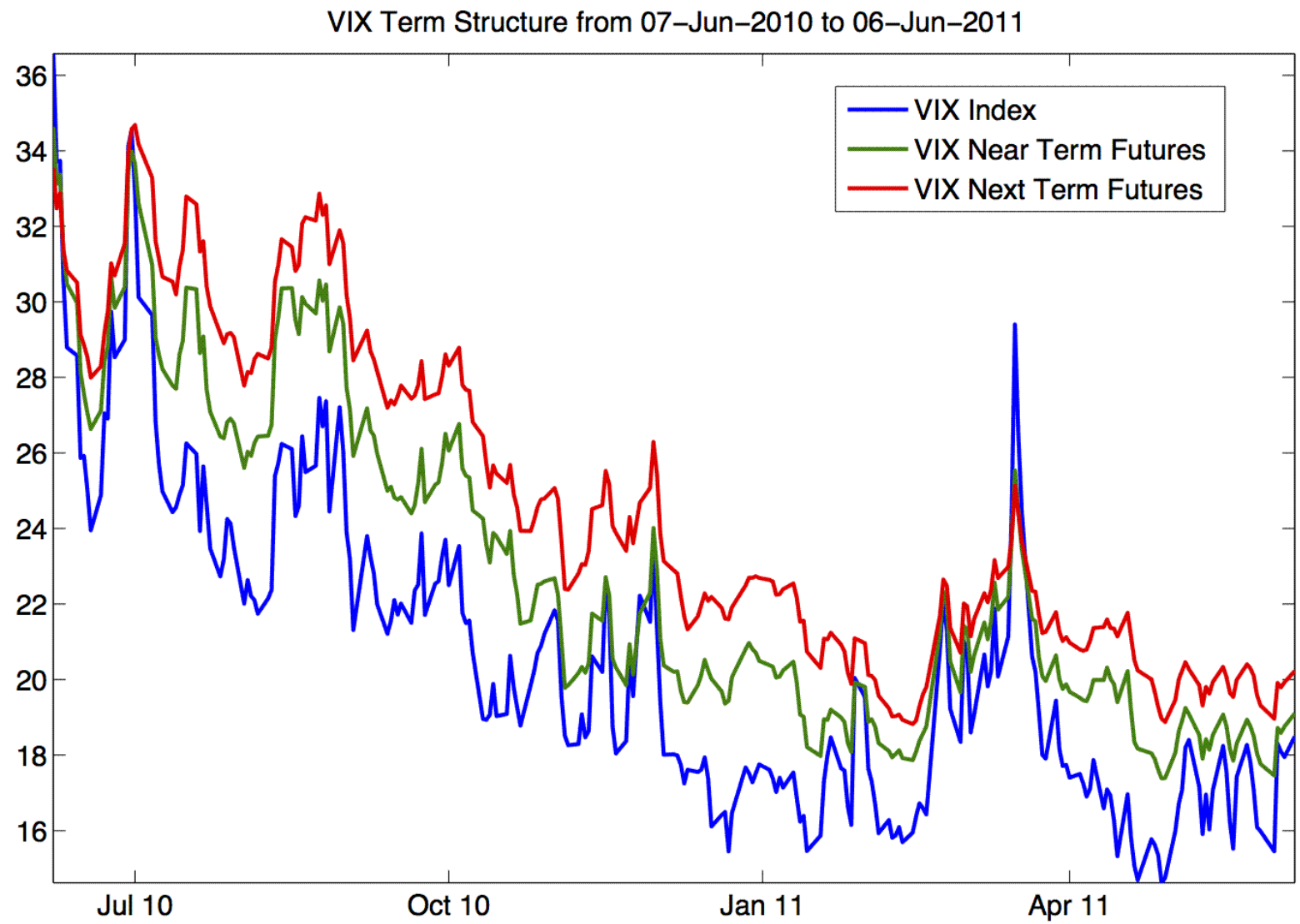
% Contango	**1**	-33.03%	**2**	-17.09%	**3**	-2.14%	**4**	-1.12%	**5**	-4.87%	**6**	-4.53%	**7**	-6.13%	**8**
Difference	**1**	-21.10	**2**	-7.31	**3**	-0.76	**4**	-0.39	**5**	-1.67	**6**	-1.48	**7**	-1.91	**8**

Month 7 to 4 contango	-10.20%	-3.40%

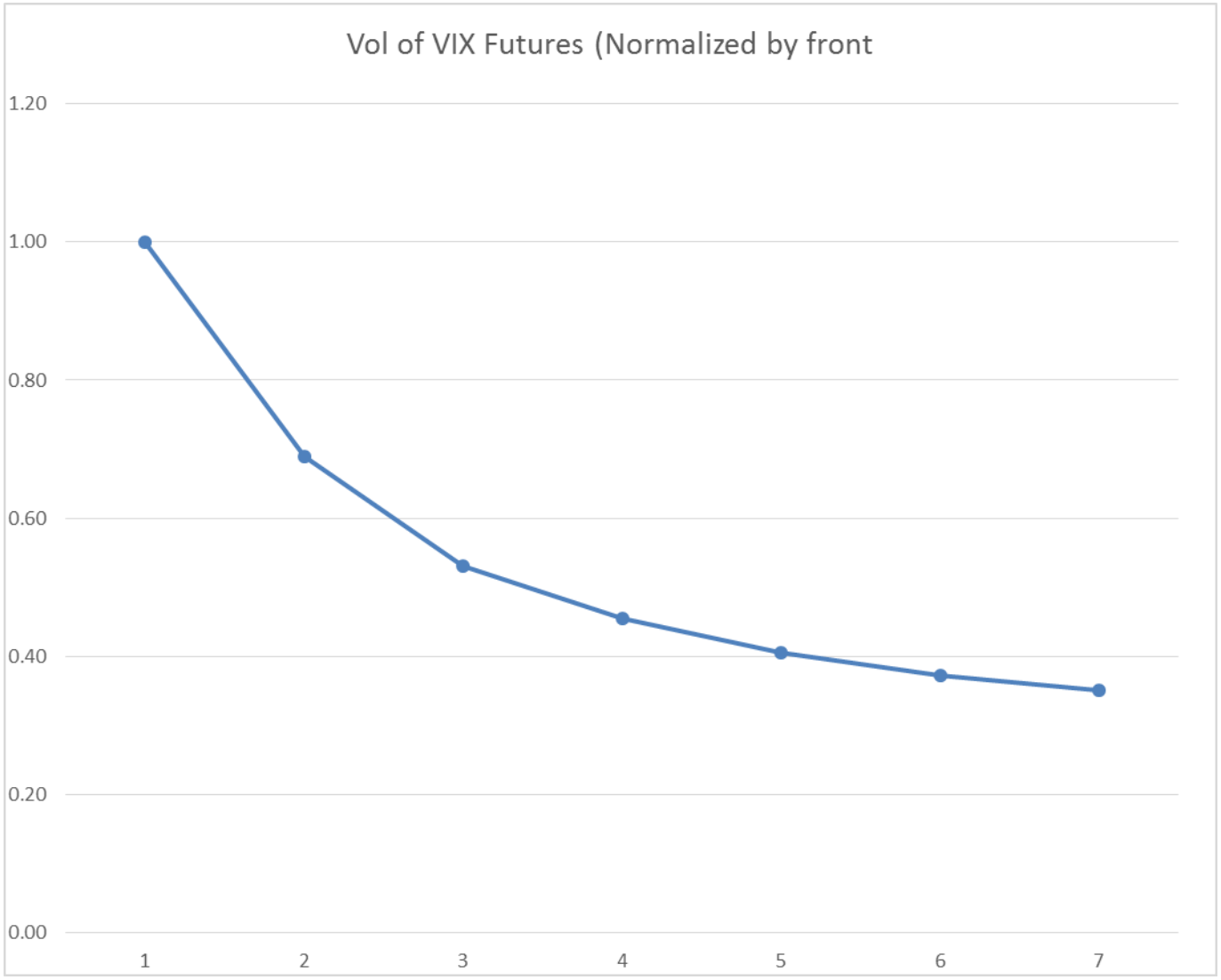

Contango 2/1
40.00%
30.00%
20.00%
10.00%
0.00%
-10.00%
-20.00%
-30.00%
1/12/2011
1/12/2012
1/12/2013
1/12/2014
1/12/2015
1/12/2016
6%


Contango 7/4 div 3
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
-2.00%
-4.00%
1/12/2011
1/12/2012
1/12/2013
1/12/2014
1/12/2015
1/12/2016
2.5 %


VIX Term Structure from 04–Jun–2009 to 04–Jun–2010
VIX Index
VIX Near Term Futures
VIX Next Term Futures
45
40
35
30
25
20
Jul 09
Oct 09
Jan 10
Apr 10


VIX Term Structure from 07–Jun–2010 to 06–Jun–2011
36
34
32
30
28
26
24
22
20
18
16
Jul 10
Oct 10
Jan 11
Apr 11
VIX Index
VIX Near Term Futures
VIX Next Term Futures

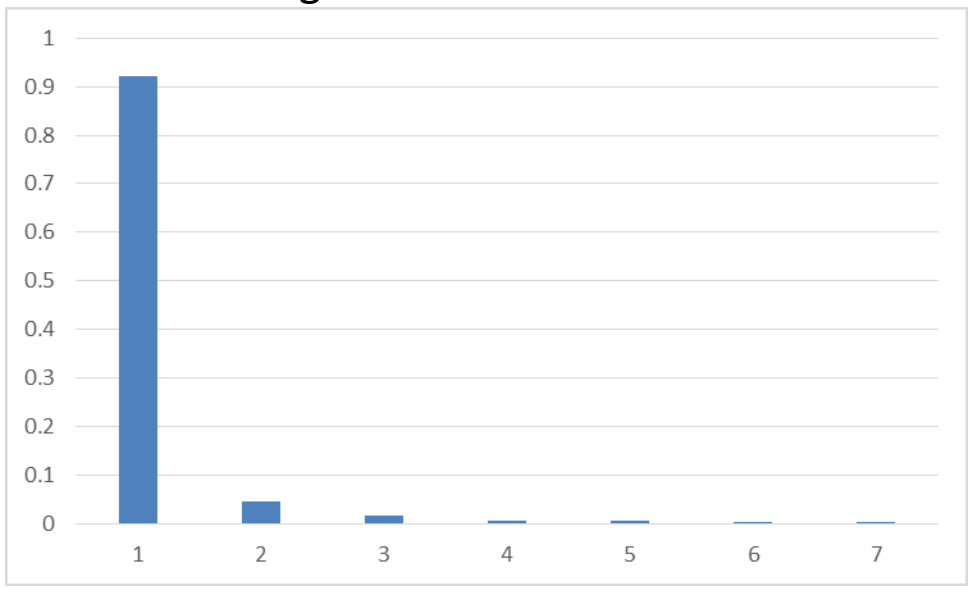
Understanding volatility of VIX futures



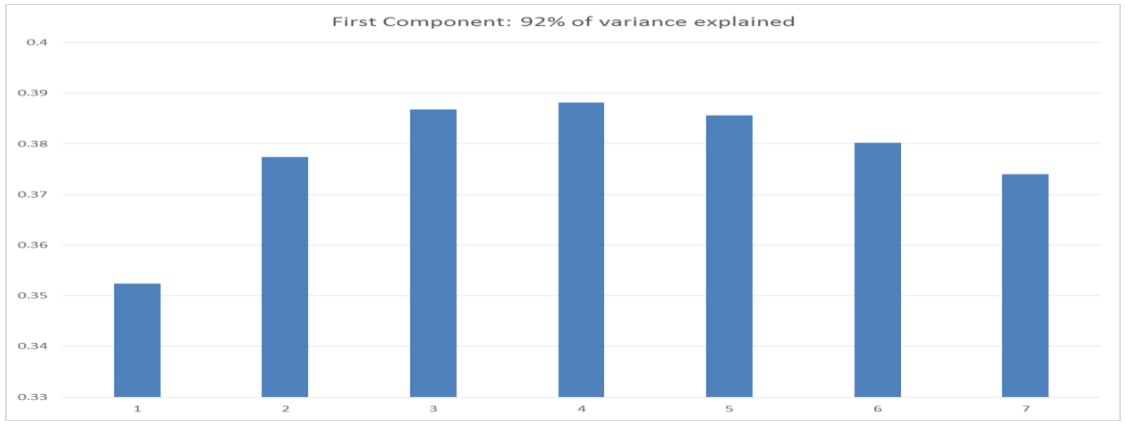
- Front month VIX futures volatility= 91% (Historical, since inception)
- Chart on the left shows volatility ratios with front contract (back ~ 50% of front)
- Data: inception (Feb 2009) until present

Understanding correlations of VIX futures: PCA

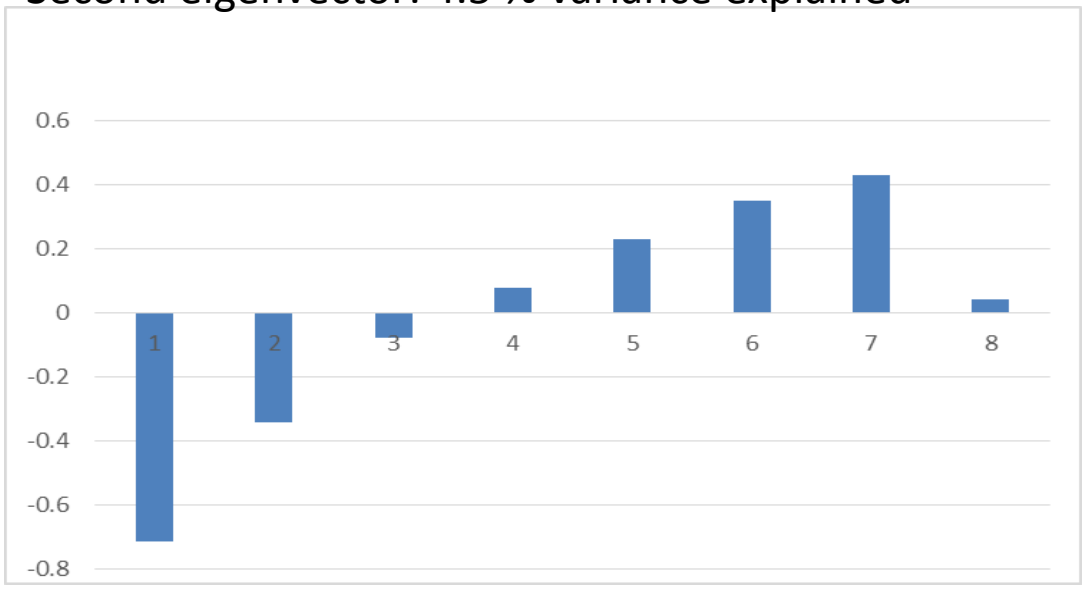
Normalized eigenvalues



First eigenvector: 92 % variance explained



Second eigenvector: 4.5 % variance explained



The VXX and VXZ ETNs

VXX: iShares ETN which tracks short term VIX futures (months 1 and 2)
target maturity 30 days; continuous roll

VXZ: iShares ETN, tracks mid-term VIX futures (months 4 through 7);
target maturity 120 days; continuous roll

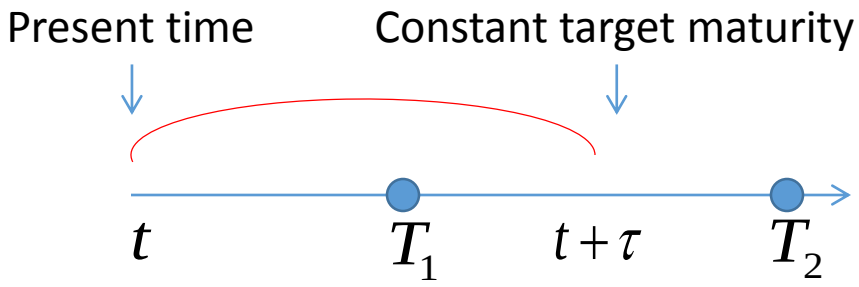
Both securities have negative drift and are correlated to the same underlying asset.

This gives rise to the possibility of arbitrage by building a long-short position

Rolling with constant maturity τ

Discrete rolling (USO, UNG)

$$a(t) = \begin{cases} 1, \text{ if } t+\tau < \frac{1}{2}(T_1+T_2) \\ 0, \text{ if } t+\tau \geq \frac{1}{2}(T_1+T_2) \end{cases}$$



Continuous rolling (VXX, VXZ)

$$a(t) = \frac{T_2 - (t+\tau)}{T_2 - T_1}$$

Typically, tau > T2-T1

Contango implies futures drop towards spot

Use simple model for F

$$F_t^{(i)} = S_t e^{(r_i - d_i)(T_i - t)} \qquad \text{contango} \Rightarrow r_i - d_i > 0$$

$S_t = \text{spot price}$

$r_i = \text{ rate for expiration } T_i$

$d_i = \text{convenience yield - storage cost for mat. } T_i$

$$\frac{dF_t^{(i)}}{F_t^{(i)}} = \frac{dS_t}{S_t} - \left(r_i - d_i\right)dt,$$

In a low interest rate environment, contango means that convenience yields are negative. ($d_i < 0$)

Consequence for futures-based ETFs

$$\frac{dI_t}{I_t} = a(t)\frac{dF_t^{(1)}}{F_t^{(1)}} + (1-a(t))\frac{dF_t^{(2)}}{F_t^{(2)}} + rdt$$

$$= \frac{dS_t}{S_t} - [a(t)(r_1 - d_1) + (1-a(t))(r_2 - d_2)]dt + rdt$$

$$= \frac{dS_t}{S_t} + [a(t)d_1 + (1-a(t))d_2]dt$$

Negative drift relative to spot px if convenience yield is negative

iPath S&P 500 VIX ST Futures ETN (VXX)

Watchlist

15.35 **+0.51 (+3.44%)** NYSEArca - As of 4:00PM EDT

After Hours: **15.48** +0.13 (+0.85%) 5:00PM EDT

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36.78



Trading the decay in VIX ETFs

- Since VIX ETFS are based on futures rolls and thus decay, is it possible to short them and hedge with futures or other ETFs?

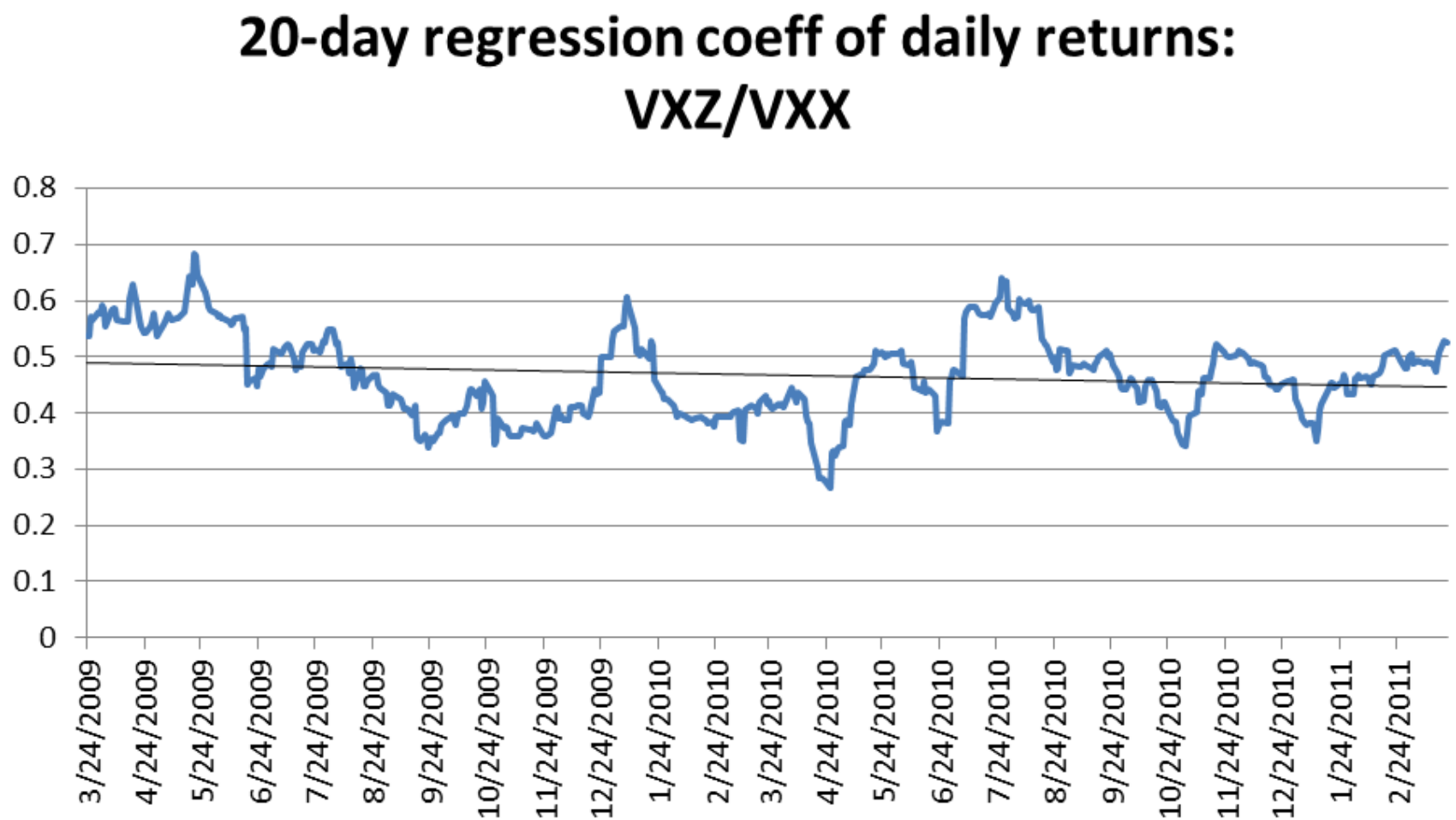
- VXX is based on futures rolls (it is based on a theoretical index built from futures).

- We can try to hedge the short volatility exposure using other volatility products in different segments of the term structure.

- Statistical analysis (e.g. C Alexander, but also other authors) indicate that statistically the main curve movement is parallel shift, so hedging along the curve where there is less decay may give profitable strats due to high correlation (in principle….)


SHORT VXX
Sharpe= 1.3
Sell VXX and reinvest profits of sale
But: incredibly destructive drawdowns: Greek crisis (-50%), U.S. downgrade (-75%)
This is just selling volatility using the VX1 / VX 2 roll
U.S. Downgrade
By rating agencies
Greek crisis
1800
1600
1400
1200
1000
800
600
400
200
0
2/2/2009
3/2/2009
4/2/2009
5/2/2009
6/2/2009
7/2/2009
8/2/2009
9/2/2009
10/2/2009
11/2/2009
12/2/2009
1/2/2010
2/2/2010
3/2/2010
4/2/2010
5/2/2010
6/2/2010
7/2/2010
8/2/2010
9/2/2010
10/2/2010
11/2/2010
12/2/2010
1/2/2011
2/2/2011
3/2/2011
4/2/2011
5/2/2011
6/2/2011
7/2/2011
8/2/2011
9/2/2011
10/2/2011
11/2/2011
12/2/2011
1/2/2012
2/2/2012
3/2/2012
4/2/2012
5/2/2012
6/2/2012
7/2/2012
8/2/2012
9/2/2012
10/2/2012
11/2/2012
12/2/2012
1/2/2013
2/2/2013
3/2/2013
4/2/2013
5/2/2013

Connecting the volatilities of VXX and VXZ



Long-Short portfolio of VXX and VXZ

$X_t = \text{daily return of VXX}$

$Y_t = \text{daily return of VXZ}$

$Y_t = \beta X_t + \varepsilon_t, \ \beta = 0.5$

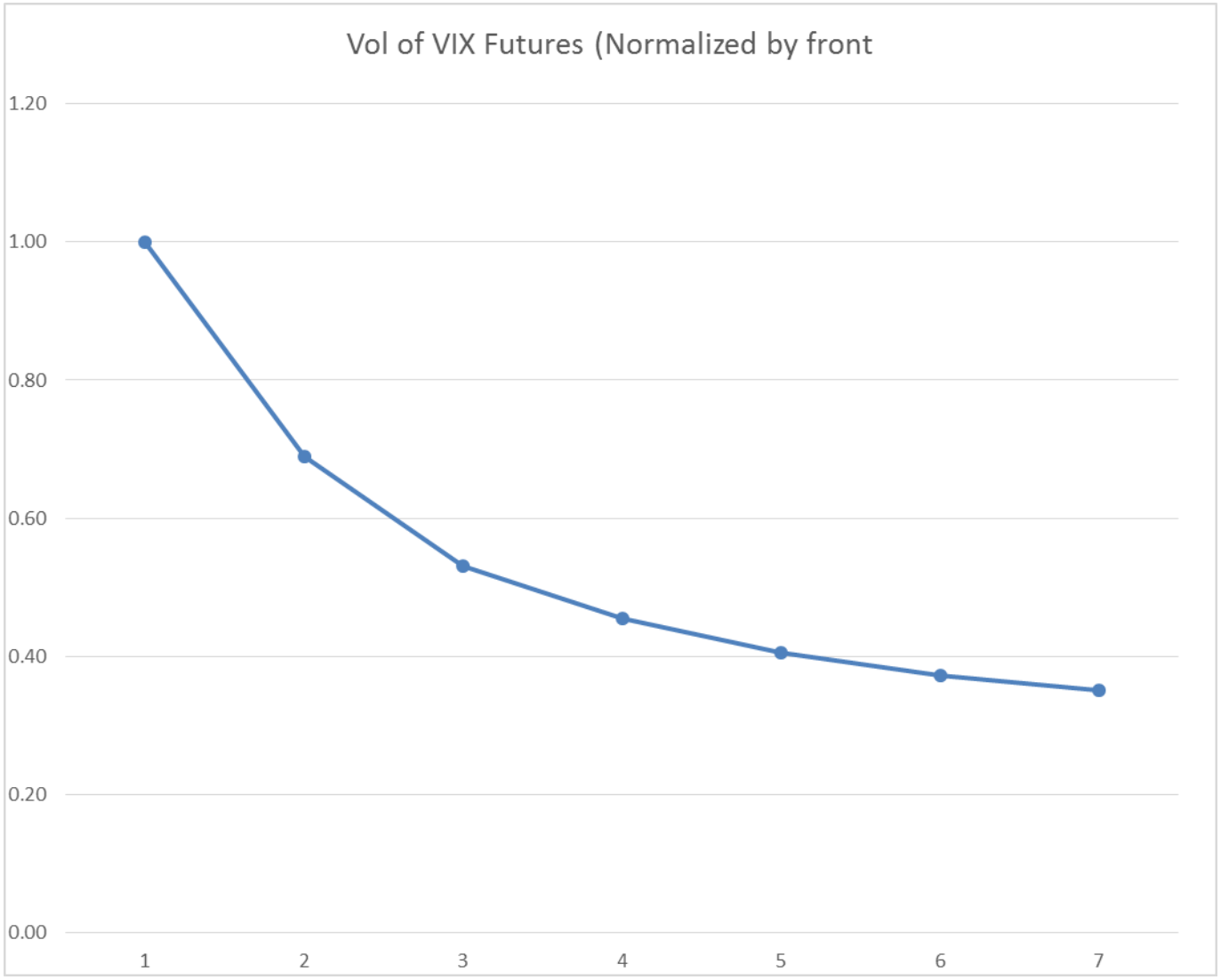
Long - short portfolio :

sell \$1 of VXX, buy \$2 of VXZ

$$PnL = \prod_{t=1}^{T} (1 - X_t + 2Y_t - r\Delta t + r_s \Delta t)$$

R_s (VXX)=-2.505% at present (financing of short position is included)

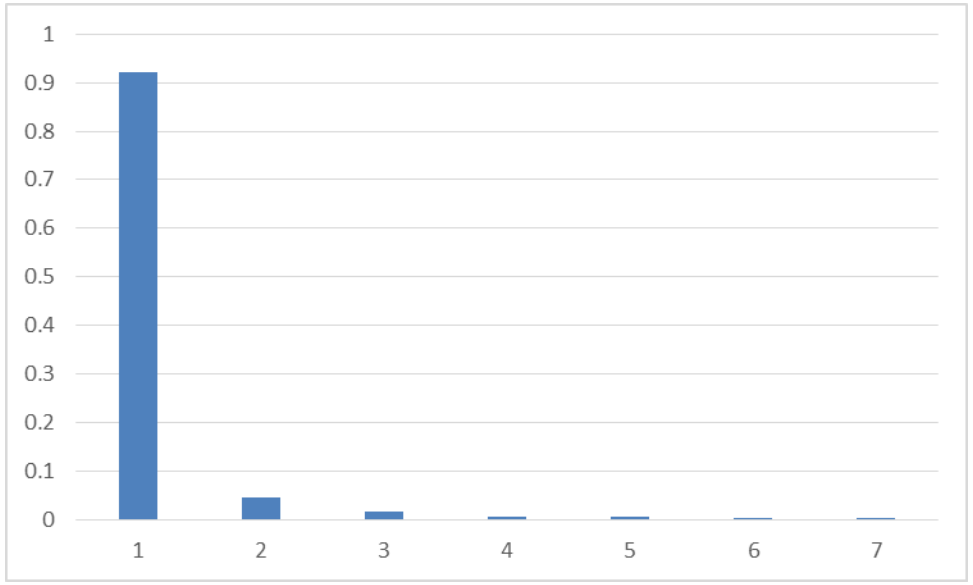
Understanding volatility of VIX futures



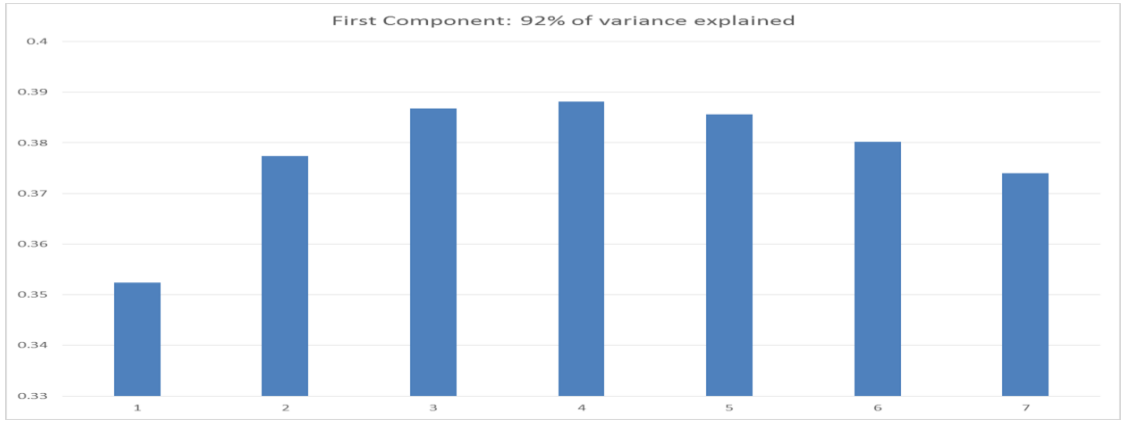
- Front month VIX futures volatility= 91%
- Chart on the left shows volatility ratios with front contract (back ~ 50% of front)
- Data: inception (Feb 2009) until present

Understanding correlations of VIX futures: PCA

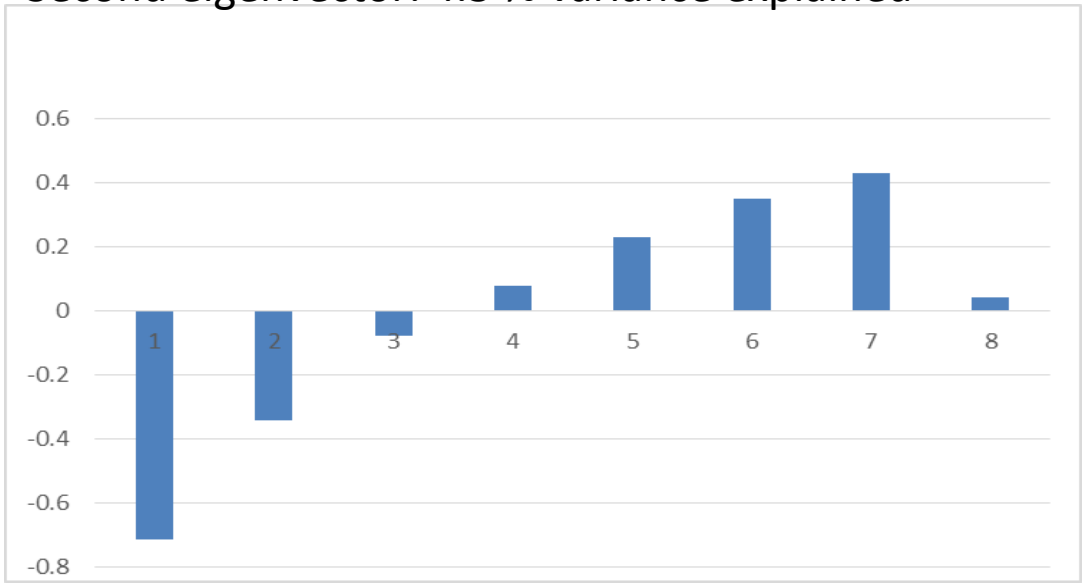
Normalized eigenvalues



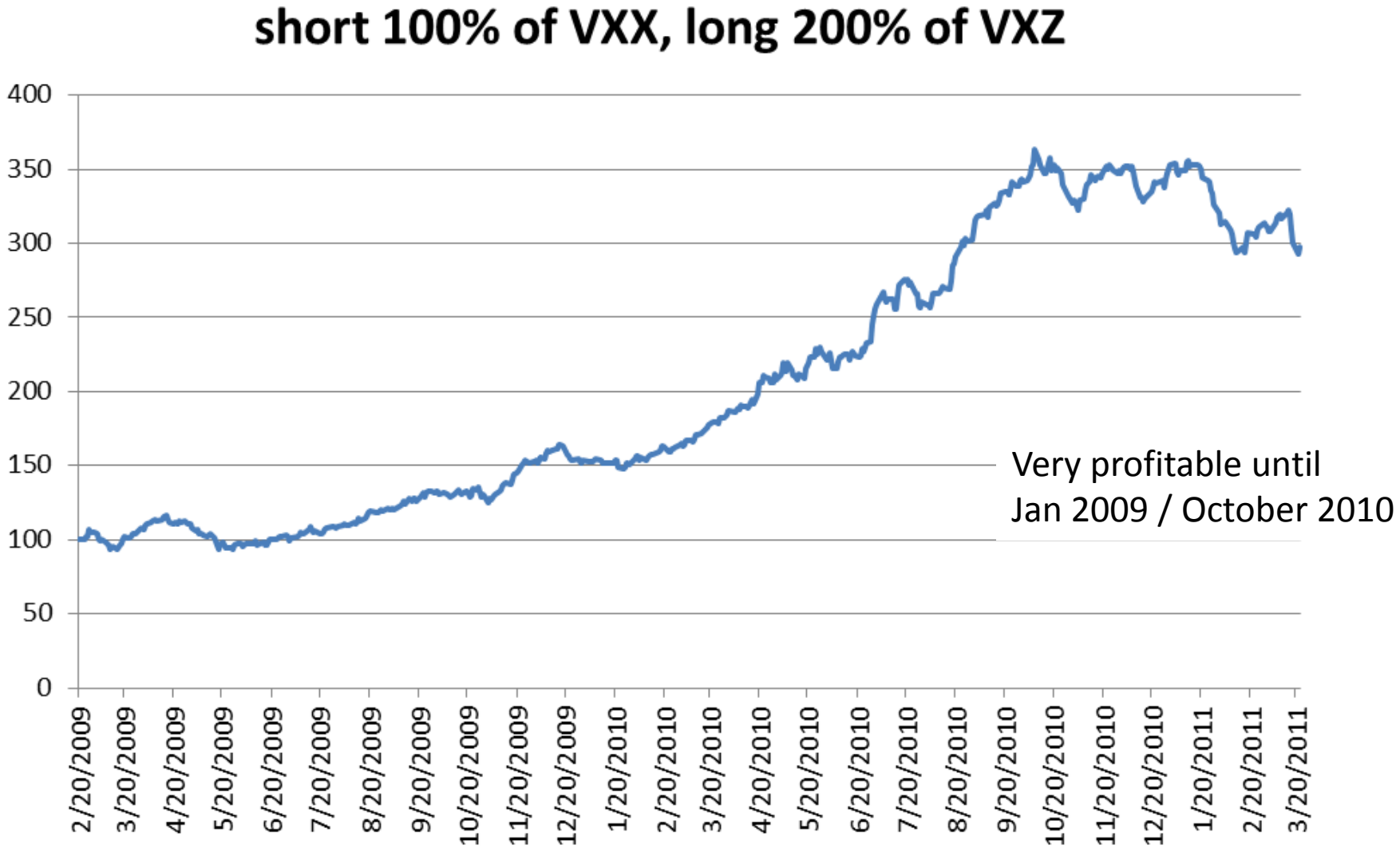
First eigenvector: 92 % variance explained



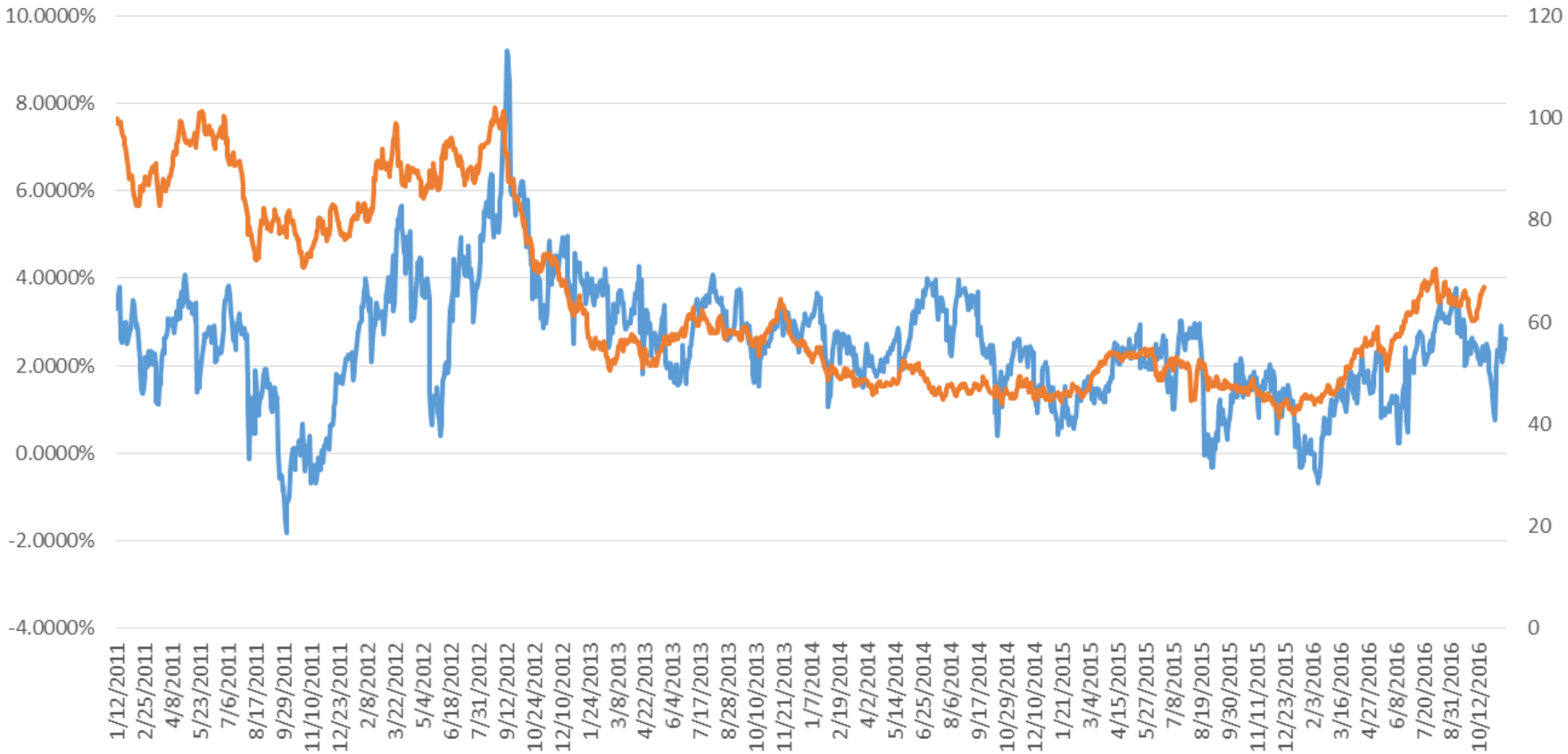
Second eigenvector: 4.5 % variance explained



Short the front-month ETN, long the back-month × 2 (since inception till 3/2011)




Short VXX long 2*VXZ
Not globally profitable.
Can we investigate more?
450
400
350
300
250
200
150
100
50
0
2/23/2009
2/23/2010
2/23/2011
2/23/2012
2/23/2013
2/23/2014
2/23/2015
2/23/2016


Left Axis (blue)=front contango-2*(back contango)= expected roll-yield for portfolio
Right axis (orange) = PNL for short VXX, long 2*VXZ (dollar terms, rebalanced daily)
10.0000%
8.0000%
6.0000%
4.0000%
2.0000%
0.0000%
-2.0000%
-4.0000%
120
100
80
60
40
20
0
1/12/2011
2/25/2011
4/8/2011
5/23/2011
7/6/2011
8/17/2011
9/29/2011
11/10/2011
12/23/2011
2/8/2012
3/22/2012
5/4/2012
6/18/2012
7/31/2012
9/12/2012
10/24/2012
12/10/2012
1/24/2013
3/8/2013
4/22/2013
6/4/2013
7/17/2013
8/28/2013
10/10/2013
11/21/2013
1/7/2014
2/19/2014
4/2/2014
5/14/2014
6/25/2014
8/6/2014
9/17/2014
10/29/2014
12/10/2014
1/21/2015
3/4/2015
4/15/2015
5/27/2015
7/8/2015
8/19/2015
9/30/2015
11/11/2015
12/23/2015
2/3/2016
3/16/2016
4/27/2016
6/8/2016
7/20/2016
8/31/2016
10/12/2016

Strategies with VIX futures to hedge short VXX

- Use the futures curve to diminish volatility (hedge) in a short VXX basic strategy
- Use minimum variance and a fixed futures position rolled monthly
- Market-neutral or not-market neutral (to hedge parallel shift), constrained >0 or unconstrained
- Minimize tracking error

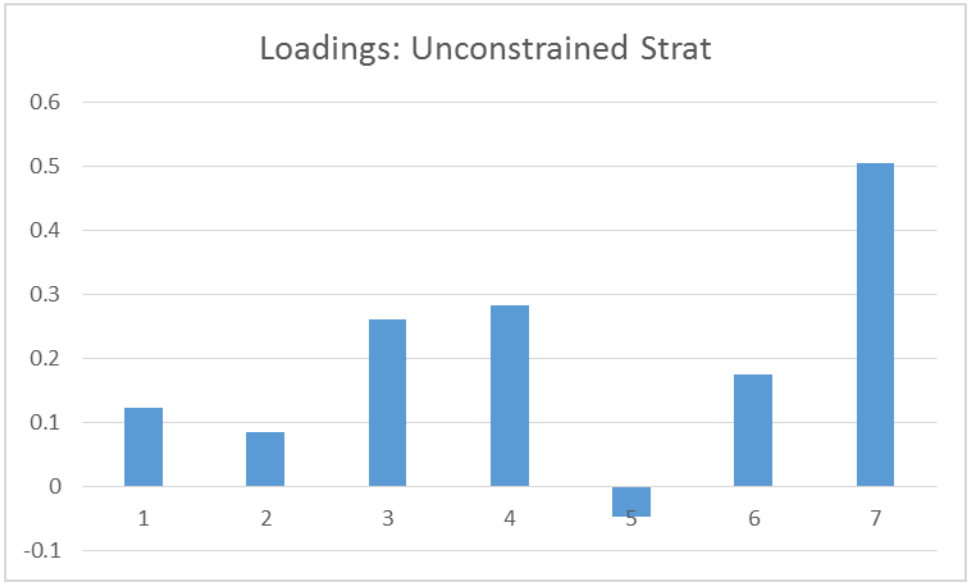
$$\min_{\beta} \sum_{t=1}^{T} \left(R_{VXX,t} - \sum_{k=1}^{7} \beta_k R_{VX\ k,t} \right)^2$$

- Market Neutrality $\sum_{k=1}^{7} \beta_k = 1$
- Positivity $\beta_k \geq 0$

Unconstrained Strategy (no MN, no positivity)

Futures (VX)	1	2	3	4	5	6	7
Beta	0.12	0.09	0.26	0.28	-0.05	0.17	0.51
Futures Notional							1.39

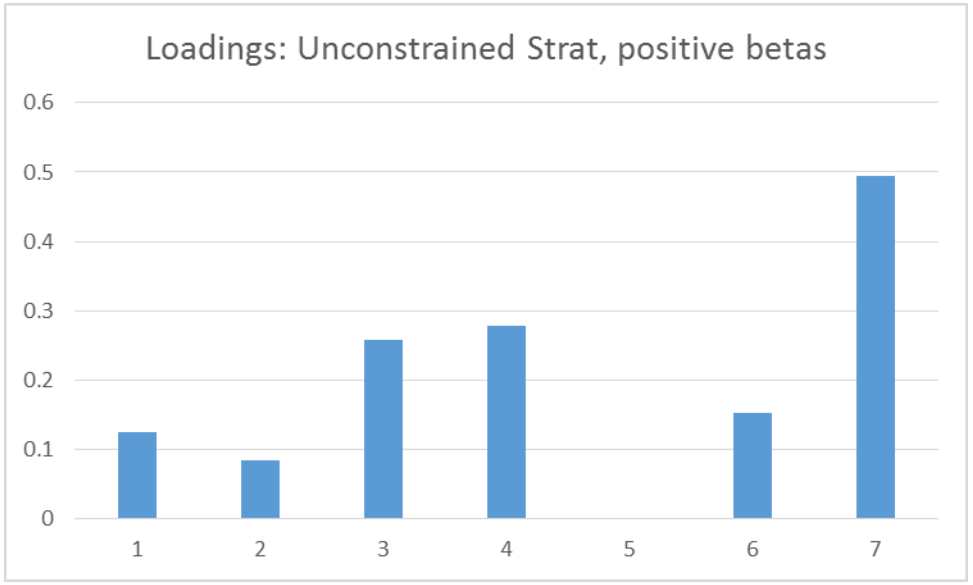




Positively Constrained, Not Market-Neutral

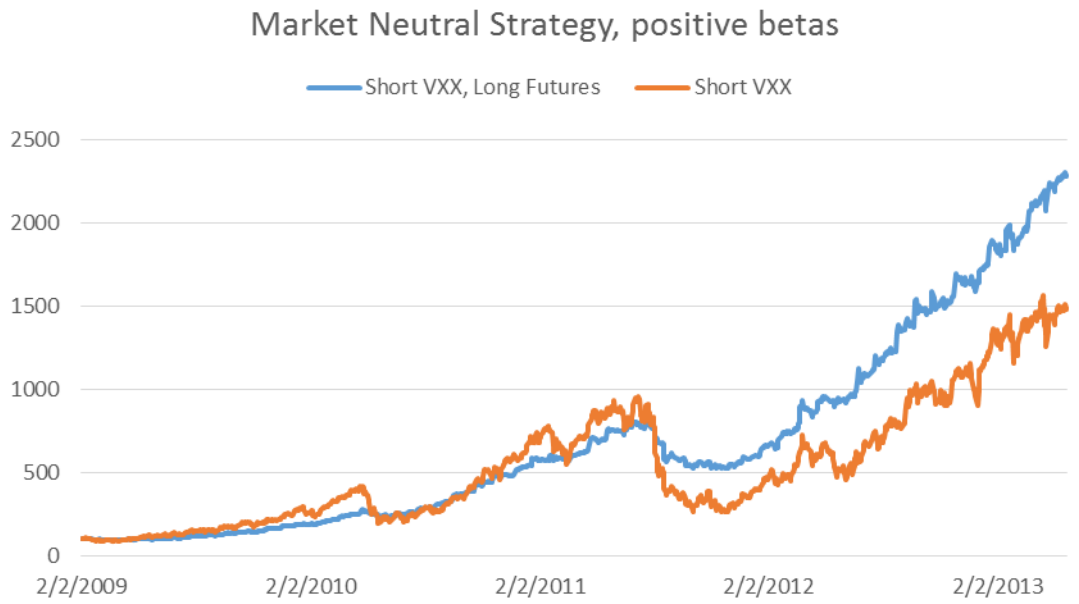
Futures (VX)	1	2	3	4	5	6	7
Beta	0.12	0.08	0.26	0.28	0.00	0.15	0.49
Futures Notional							1.39

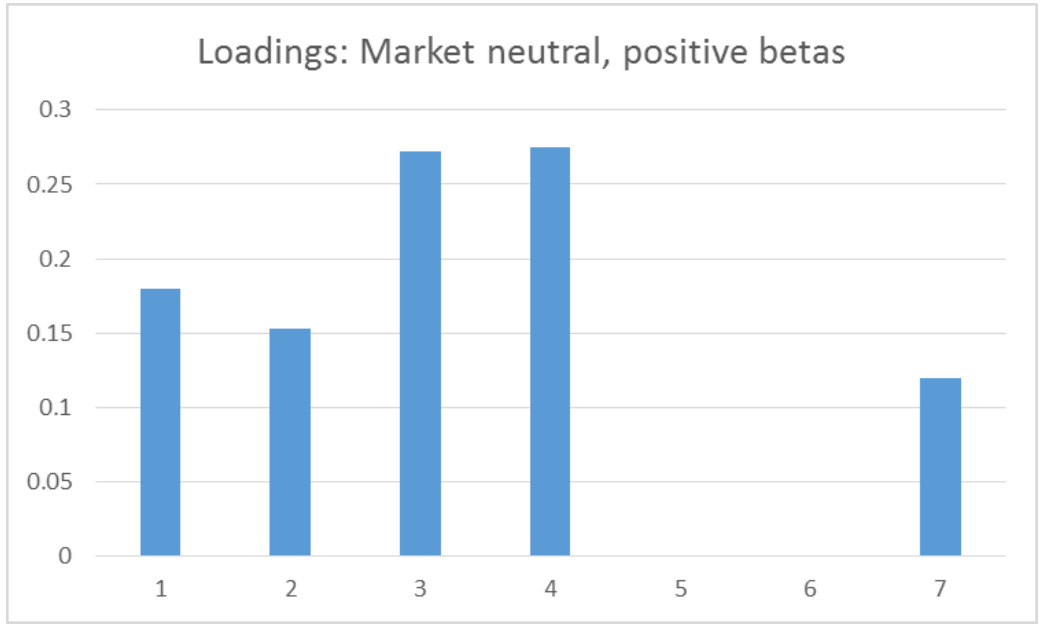




Market Neutral, Positive Betas

Futures (VX)	1	2	3	4	5	6	7
Beta	0.18	0.15	0.27	0.27	0.00	0.00	0.12
Futures Notional							1.00





3. VIX Options

VXX Options

- Options on VIX futures settle and are priced based on VIX futures
- (i.e. the ``forward price'' is the co-terminal VIX price). Black 76 works.

- What about options on VXX? **Contango plays a role** in the evolution of VXX prices, and thus the fair value of an option (regardless of volatility).

- Our point: The term-structure of VIX is volatile, so the **forward pricing of VXX is not be that straight-forward.**

- **Strategy A:** derive a better ``physical measure'' (or forecast) for VXX based on econometric analysis of VIX futures curves and roll formula.
  Trading strategy: buy cheap options and don't delta hedge

- **Strategy B**: use the physical measure as prior and build a risk-neutral measure for trading VXX options relative to VIX options.
  Trading strategy: trade VXX options vs. VIX options (delta-neutral)

Joint work with Shinshun Jiang, NYU

Build a good physical measure for VXX

- VXX is based on an index, which represents a theoretical roll between the two front VIX contracts.

- Fit the term-structure of futures to a multivariate GARCH (1,1) model.

  Step 1: Perform PCA on the futures curve, parameterized by constant maturity

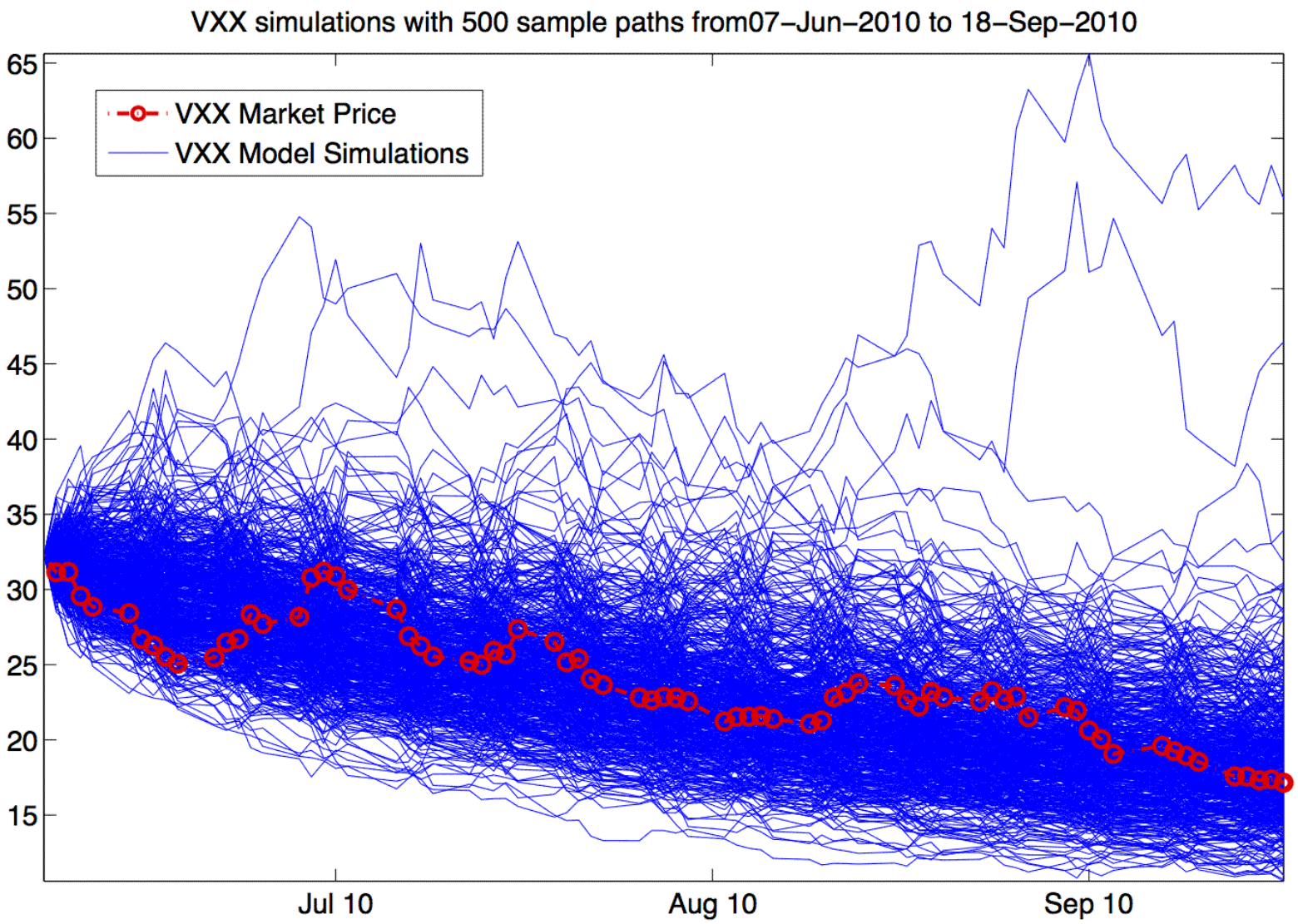
  Step 2: Derive significant Principal Components for VIX term-structure

  Step 3: Estimate the evolution of the Principal Component loadings as uncorrelated GARCH (1,1) processes

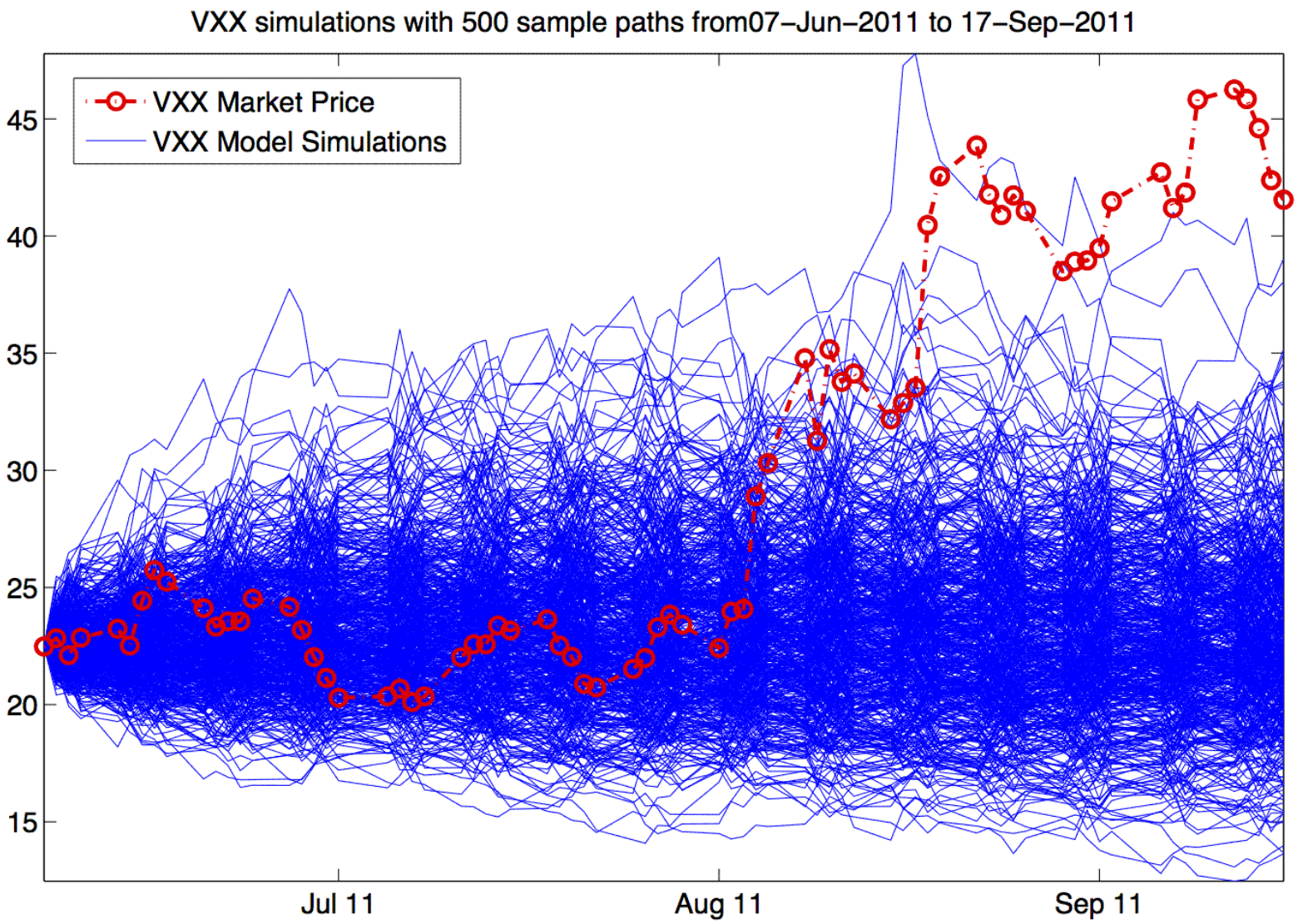
This gives a tool for **forecasting VXX.**

For PCA: see Carol Alexander and Dmitris Korovilas (2002)
For GARCH on term structures: see Avellaneda and Zhou (1996)

Actual VXX is within the forecast prices (out of sample simulation)



Effect of the Downgrade of the US Treasury by S&P in August 2011



VXX Option Valuation

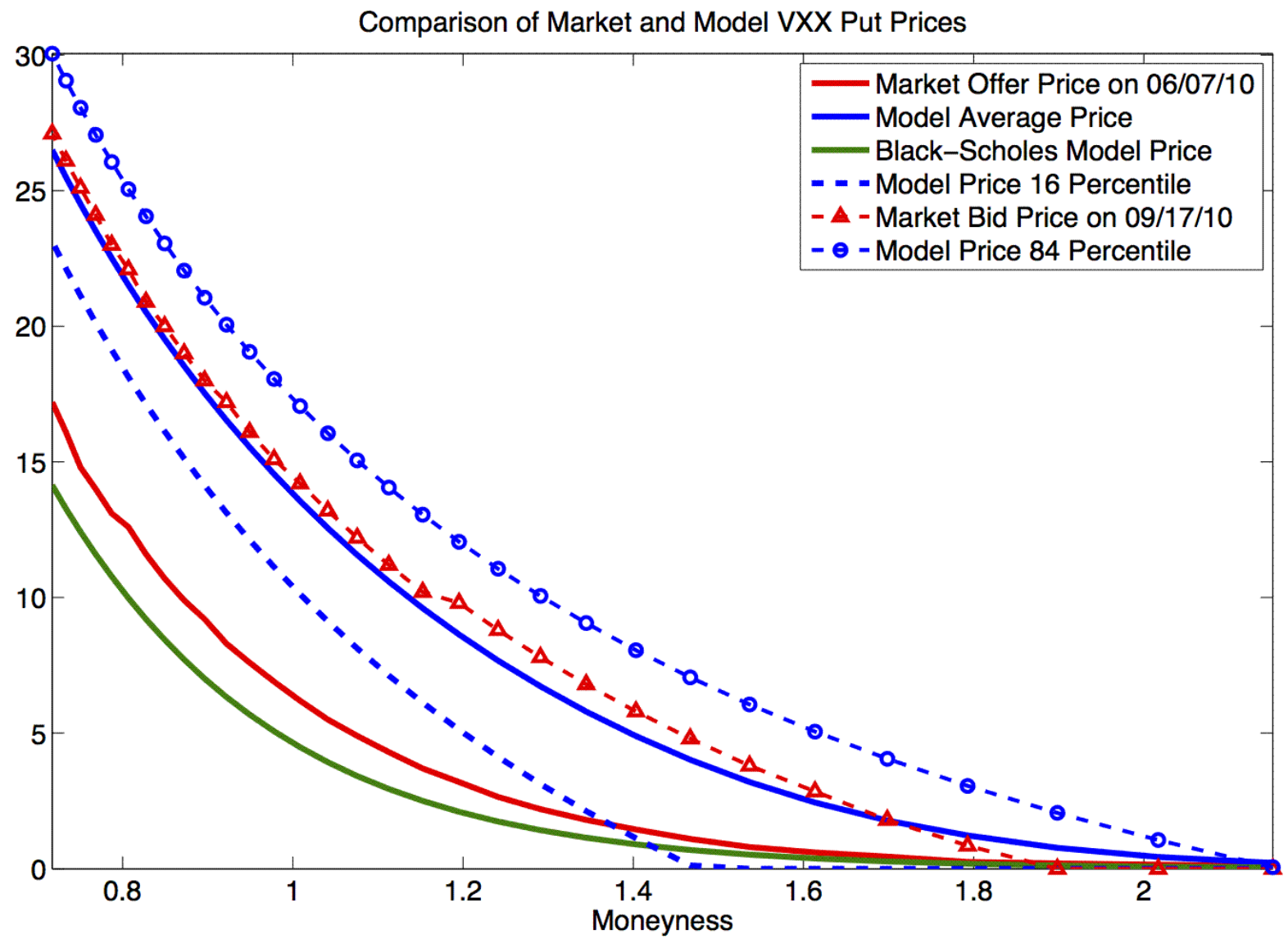
- **Market value:** this is the value of the VXX option from the market, on the pricing date.

- **Black Scholes with historical volatility:** based on an estimation window in the past, use the BS formula with historical volatility to determine a subjective price for VXX options on the pricing date.

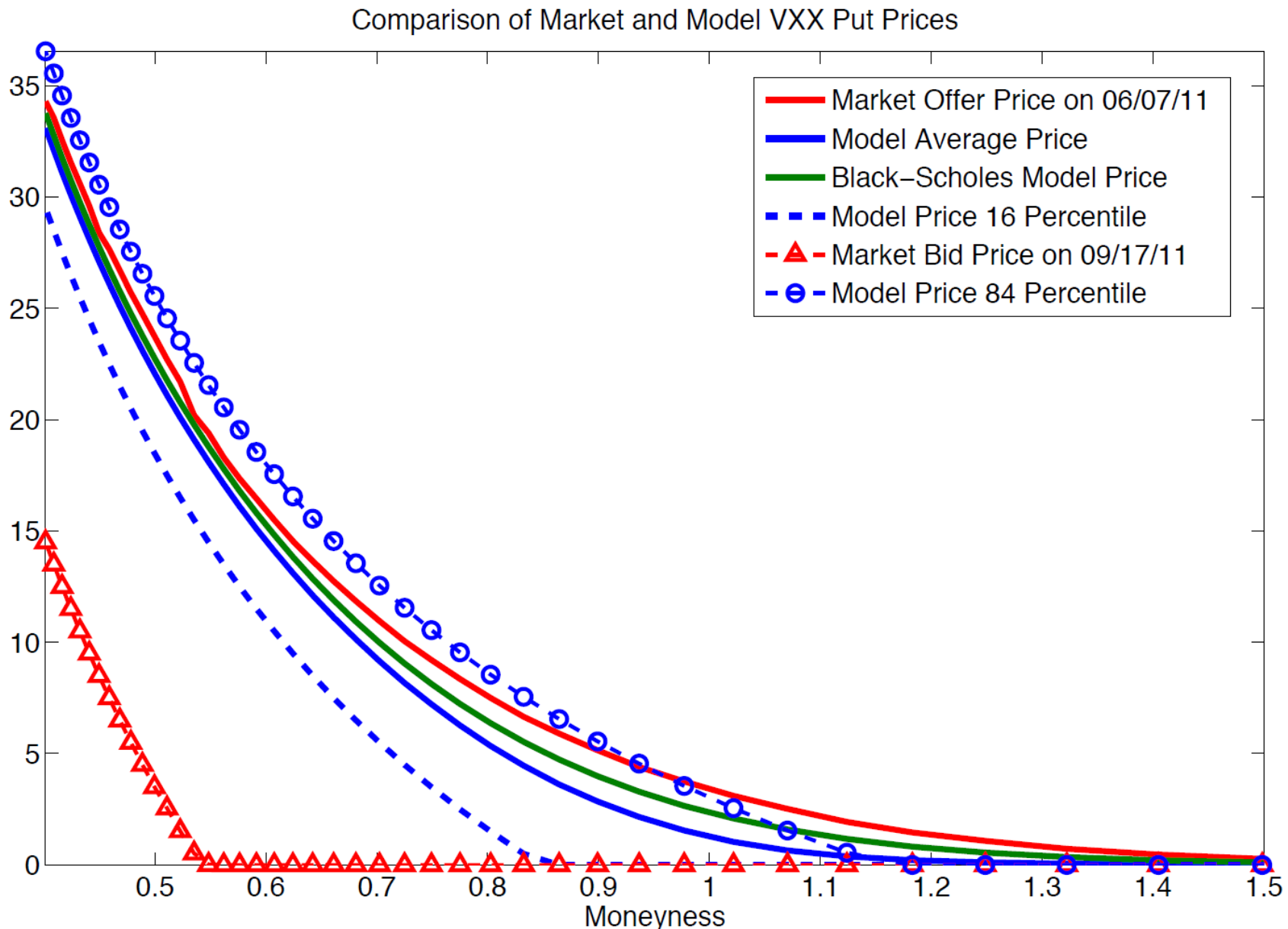
- **New Model:** Use GARCH (1,1) estimation of the movements of the VIX futures curve to simulate scenarios of evolution of the VXX beyond the pricing date. Price options by averaging final payoff over GARCH statistics.

Negative drift of VXX suggests opportunities in buying puts/selling calls.

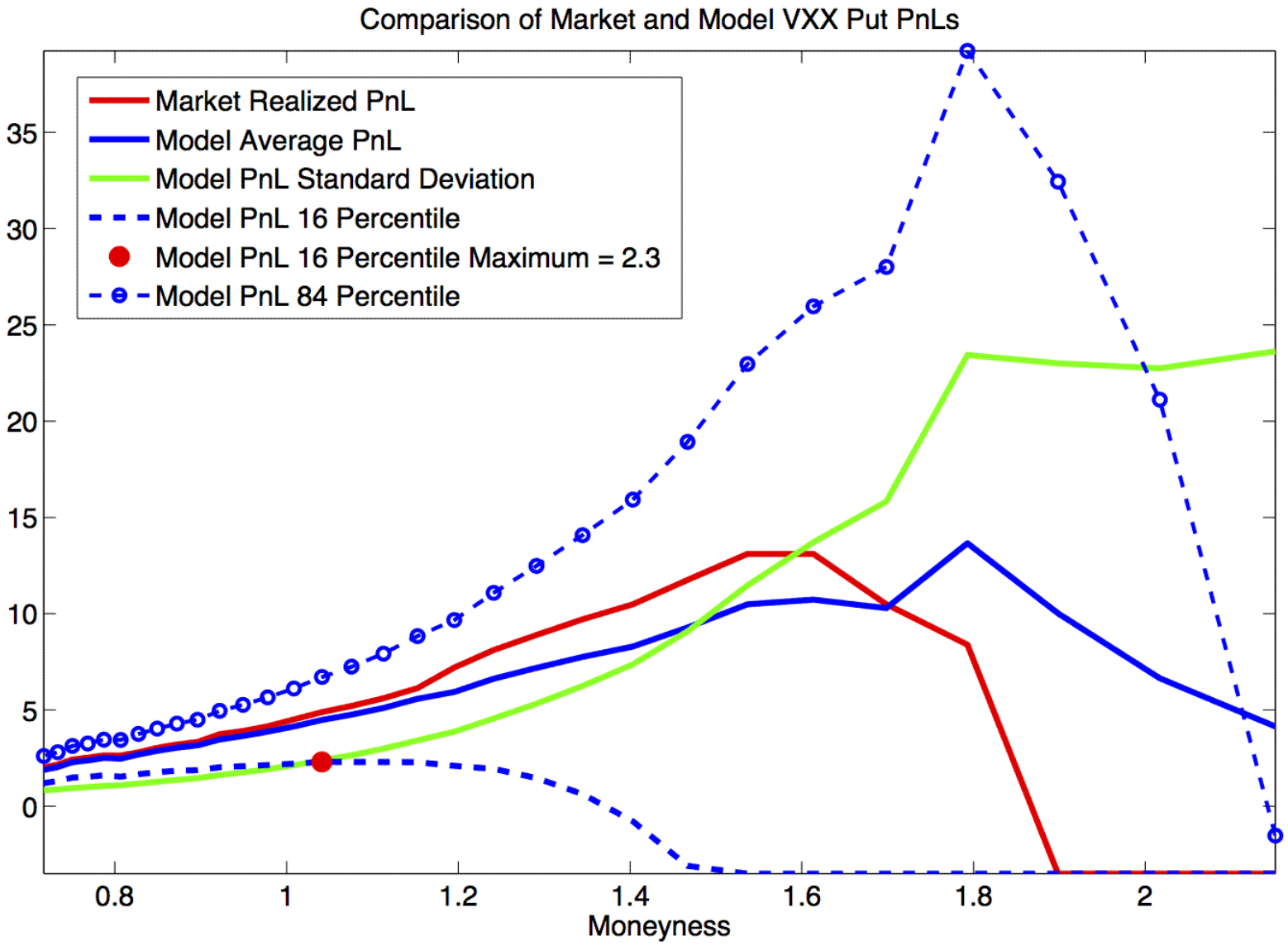
Price Date: June 7, 2010



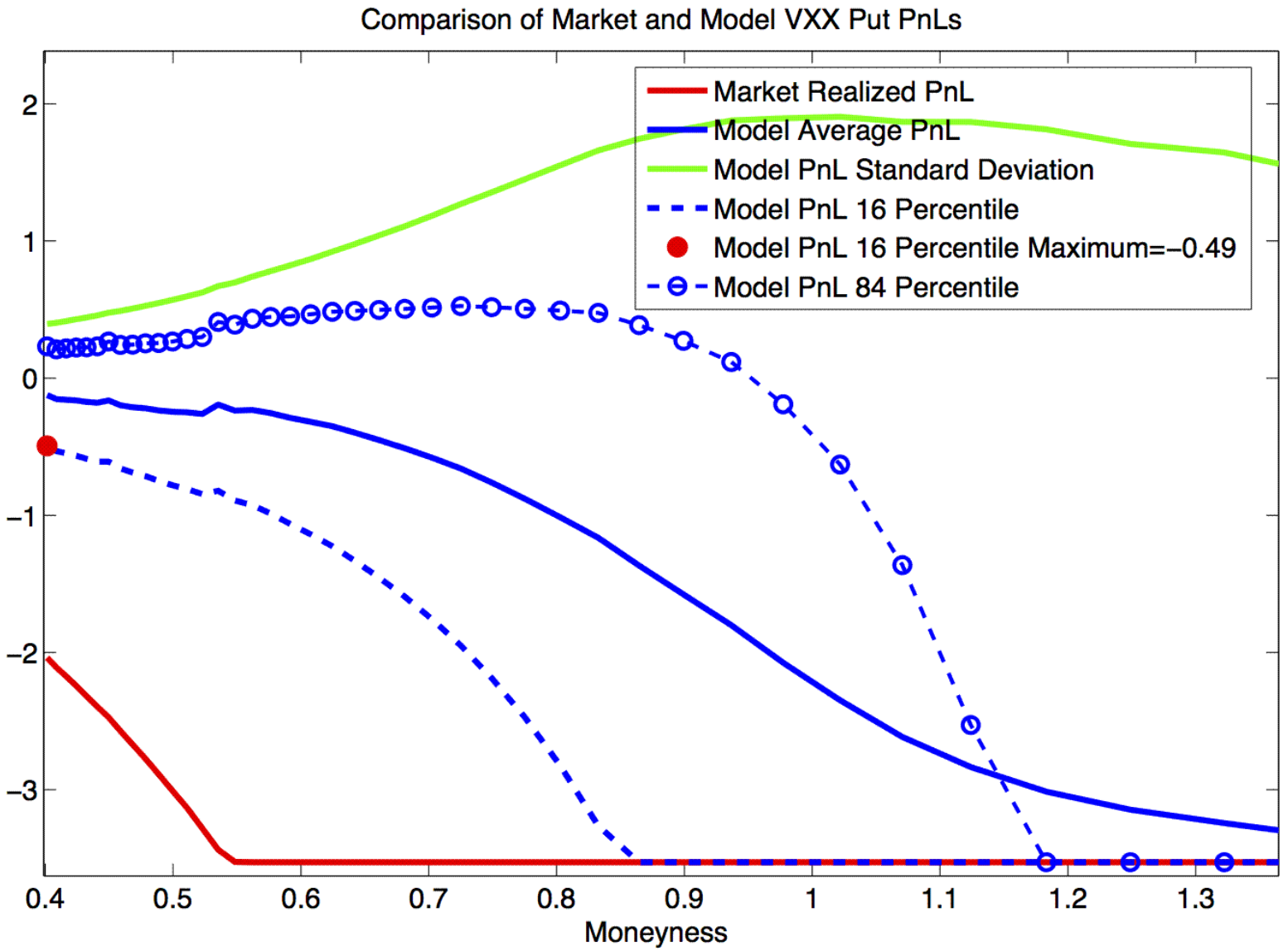
Price Date: June 7, 2011



PNL analysis of Put-buying strategy (6/7/2010)



PNL analysis of Put-buying strategy 6/7/2011



Backtesting Strategy A: May 2008, Oct 2012

99 Weeks, one Pricing Date per week.

White= profit

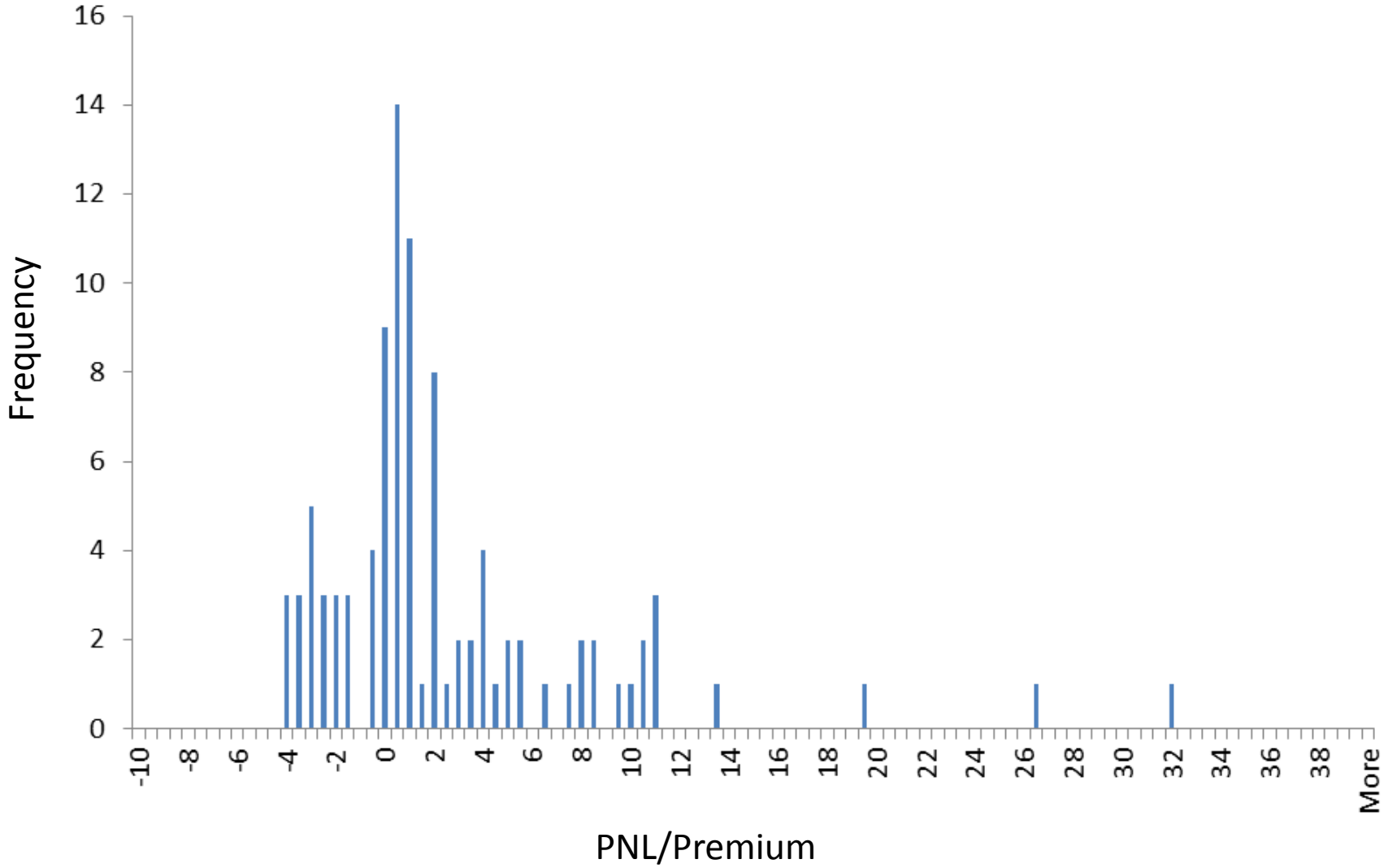
Red = loss

PNL

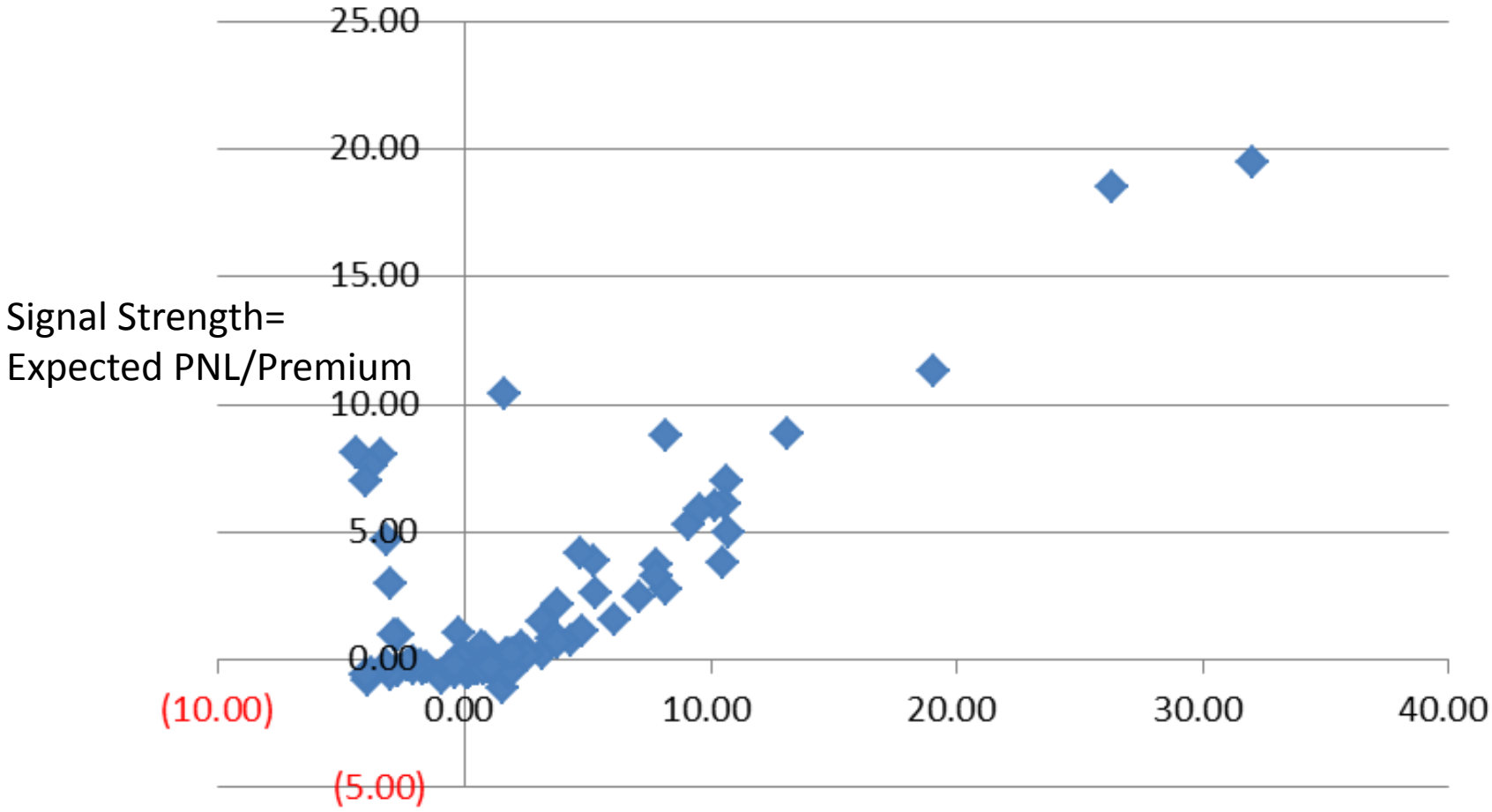
Moneyness

cDate	exDate	PnL	prcMax	Moneyness	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
5/28/2010	9/18/2010	8.16	8.78	1.43	1.50	1.16	1.58	1.94	2.59	3.46	4.83	6.30	7.01	7.90	7.32
6/7/2010	9/18/2010	13.31	8.88	1.54	32.15	7.43	1.96	2.64	3.38	4.41	5.46	7.31	9.04	10.44	12.56
6/14/2010	9/18/2010	10.65	6.98	1.29	4.10	1.52	1.95	2.61	3.62	4.72	6.32	8.70	10.75	12.41	11.17
6/21/2010	9/18/2010	10.59	6.10	1.21	1.89	1.41	1.83	2.58	3.55	5.74	7.77	10.27	11.54	7.92	(4.04)
6/28/2010	9/18/2010	19.02	11.33	1.28	7.16	2.00	2.70	4.05	5.70	8.13	11.60	15.42	19.19	20.59	17.17
7/6/2010	9/18/2010	26.27	18.53	1.30	11.06	2.54	3.42	4.81	6.92	9.37	14.84	20.82	26.06	29.39	30.15
7/13/2010	9/18/2010	10.21	6.00	1.09	2.04	1.93	2.62	3.66	4.99	7.33	10.54	12.75	14.36	8.37	(5.37)
7/20/2010	9/18/2010	32.00	19.45	1.26	2.66	2.36	3.24	4.69	6.79	11.03	16.10	24.56	31.11	19.16	(6.00)
7/27/2010	9/18/2010	9.11	5.25	1.03	1.32	1.83	2.59	3.47	5.72	8.07	11.44	11.88	(0.75)	(6.79)	(6.79)
8/3/2010	9/18/2010	8.16	2.76	0.98	1.22	1.71	2.55	4.22	5.88	8.81	13.25	10.76	(7.83)	(7.83)	(7.83)
11/10/2010	1/22/2011	1.51	(1.08)	0.64	(4.30)	0.55	2.30	3.07	3.99	5.03	5.78	5.85	2.83	(4.88)	(4.93)
11/17/2010	1/22/2011	3.13	0.21	0.59	1.03	2.04	3.30	5.02	6.78	9.69	12.80	15.90	17.85	11.71	(5.26)
11/24/2010	3/19/2011	0.45	0.02	0.62	10.80	0.68	0.62	0.58	0.65	0.22	(0.70)	(2.57)	(3.13)	(3.13)	(3.13)
12/2/2010	3/19/2011	0.52	(0.06)	0.61	(2.01)	0.51	0.69	0.65	0.78	0.40	(0.57)	(2.64)	(3.36)	(3.36)	(3.36)
12/9/2010	3/19/2011	0.15	(0.55)	0.57	(2.88)	0.17	0.13	(0.04)	(0.50)	(1.64)	(3.59)	(3.60)	(3.60)	(3.60)	(3.60)
12/16/2010	3/19/2011	0.18	(0.12)	0.55	(2.38)	0.29	0.32	0.25	(0.22)	(1.33)	(3.66)	(3.87)	(3.87)	(3.87)	(3.87)
12/23/2010	3/19/2011	(0.12)	(0.06)	0.56	(2.27)	0.10	0.11	(0.03)	(0.69)	(2.18)	(4.18)	(4.19)	(4.19)	(4.19)	(4.19)
12/31/2010	3/19/2011	0.05	(0.02)	0.53	(0.60)	0.14	0.13	(0.02)	(0.71)	(2.44)	(4.61)	(4.62)	(4.62)	(4.62)	(4.62)
1/7/2011	3/19/2011	(0.06)	(0.14)	0.55	(0.50)	(0.12)	(0.21)	(0.56)	(1.67)	(4.29)	(5.07)	(5.07)	(5.07)	(5.07)	(5.07)
1/14/2011	3/19/2011	(0.80)	(0.51)	0.44	(0.89)	(1.30)	(1.90)	(3.24)	(5.61)	(5.63)	(5.63)	(5.63)	(5.63)	(5.63)	(5.63)
1/24/2011	3/19/2011	(0.92)	(0.57)	0.44	(1.00)	(1.50)	(2.21)	(3.81)	(6.64)	(6.67)	(6.67)	(6.67)	(6.67)	(6.67)	(6.67)
1/31/2011	3/19/2011	(0.67)	(0.36)	0.45	(0.65)	(1.03)	(1.58)	(2.74)	(5.59)	(7.28)	(7.66)	(7.66)	(7.66)	(7.66)	(7.66)
2/7/2011	6/18/2011	0.12	(0.51)	0.41	(0.02)	0.17	0.09	0.00	(0.16)	(0.77)	(2.00)	(2.75)	(2.75)	(2.75)	(2.75)
2/14/2011	6/18/2011	0.09	(0.53)	0.40	0.06	0.06	0.06	(0.10)	(0.48)	(1.28)	(2.85)	(2.90)	(2.90)	(2.90)	(2.90)
2/22/2011	5/21/2011	1.80	(0.50)	0.65	(104.74)	(4.53)	2.03	2.12	3.57	4.63	4.89	2.86	4.02	(2.23)	(4.09)
3/1/2011	5/21/2011	1.96	(0.44)	0.65	15.71	2.74	2.35	3.15	4.14	5.33	6.32	7.00	5.54	(2.15)	(4.44)
3/8/2011	6/18/2011	0.47	0.03	0.45	0.35	0.68	0.89	1.13	1.21	1.32	0.57	(1.25)	(3.53)	(3.53)	(3.53)
3/15/2011	6/18/2011	0.79	0.11	0.49	0.53	0.79	1.43	1.85	2.30	3.13	3.59	3.49	1.44	(3.79)	(3.79)
3/22/2011	6/18/2011	0.64	(0.07)	0.46	0.26	0.98	1.30	1.65	2.02	2.62	2.38	0.76	(4.09)	(4.09)	(4.09)
3/29/2011	6/18/2011	0.43	(0.36)	0.42	0.51	0.67	0.84	1.06	1.19	0.83	(0.53)	(4.44)	(4.44)	(4.44)	(4.44)
4/5/2011	6/18/2011	0.25	(0.40)	0.40	0.14	0.38	0.63	0.66	0.58	(0.28)	(3.48)	(4.86)	(4.86)	(4.86)	(4.86)
4/12/2011	6/18/2011	0.34	(0.19)	0.41	0.09	0.16	0.71	0.86	1.15	0.52	(2.08)	(5.37)	(5.37)	(5.37)	(5.37)
4/19/2011	6/18/2011	0.10	(0.53)	0.38	0.12	0.16	0.07	0.05	(0.65)	(2.72)	(5.00)	(6.00)	(6.00)	(6.00)	(6.00)
4/27/2011	6/18/2011	(0.37)	(0.54)	0.38	(0.76)	(1.11)	(1.56)	(2.81)	(5.58)	(6.92)	(6.92)	(6.92)	(6.92)	(6.92)	(6.92)
5/4/2011	6/18/2011	(0.29)	(0.40)	0.35	(0.49)	(0.67)	(0.96)	(1.81)	(4.04)	(8.00)	(8.00)	(8.00)	(8.00)	(8.00)	(8.00)
5/11/2011	9/17/2011	(1.57)	(0.33)	0.42	(2.15)	(2.79)	(2.79)	(2.79)	(2.79)	(2.79)	(2.79)	(2.79)	(2.79)	(2.79)	(2.79)
5/18/2011	9/17/2011	(1.67)	(0.36)	0.41	(2.39)	(2.95)	(2.95)	(2.95)	(2.95)	(2.95)	(2.95)	(2.95)	(2.95)	(2.95)	(2.95)
5/25/2011	9/17/2011	(1.79)	(0.23)	0.41	(2.59)	(3.13)	(3.13)	(3.13)	(3.13)	(3.13)	(3.13)	(3.13)	(3.13)	(3.13)	(3.13)
6/2/2011	8/20/2011	(3.96)	(0.80)	0.49	(4.18)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)
6/9/2011	9/17/2011	(2.08)	(0.38)	0.39	(3.16)	(3.60)	(3.60)	(3.60)	(3.60)	(3.60)	(3.60)	(3.60)	(3.60)	(3.60)	(3.60)
6/16/2011	9/17/2011	(2.06)	(0.10)	0.46	(2.44)	(3.58)	(3.87)	(3.87)	(3.87)	(3.87)	(3.87)	(3.87)	(3.87)	(3.87)	(3.87)
6/23/2011	9/17/2011	(2.34)	(0.25)	0.42	(3.23)	(4.19)	(4.19)	(4.19)	(4.19)	(4.19)	(4.19)	(4.19)	(4.19)	(4.19)	(4.19)
6/30/2011	9/17/2011	(2.65)	(0.15)	0.38	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)	(4.56)
7/8/2011	9/17/2011	(3.03)	(0.57)	0.36	(5.07)	(5.07)	(5.07)	(5.07)	(5.07)	(5.07)	(5.07)	(5.07)	(5.07)	(5.07)	(5.07)
7/15/2011	9/17/2011	(3.16)	(0.24)	0.41	(4.47)	(5.63)	(5.63)	(5.63)	(5.63)	(5.63)	(5.63)	(5.63)	(5.63)	(5.63)	(5.63)
7/22/2011	9/17/2011	(3.76)	(0.55)	0.37	(6.27)	(6.32)	(6.32)	(6.32)	(6.32)	(6.32)	(6.32)	(6.32)	(6.32)	(6.32)	(6.32)
7/29/2011	9/17/2011	(4.03)	(0.60)	0.42	(5.60)	(7.20)	(7.20)	(7.20)	(7.20)	(7.20)	(7.20)	(7.20)	(7.20)	(7.20)	(7.20)
8/5/2011	12/17/2011	(2.69)	1.01	0.92	(0.75)	(1.34)	(2.00)	(2.69)	(2.69)	(2.69)	(2.69)	(2.69)	(2.69)	(2.69)	(2.69)
8/12/2011	12/17/2011	(2.83)	1.00	0.90	(0.83)	(0.86)	(1.36)	(2.08)	(2.83)	(2.83)	(2.83)	(2.83)	(2.83)	(2.83)	(2.83)
8/19/2011	12/17/2011	(3.00)	2.96	1.18	52.35	1.51	(0.23)	(0.55)	(1.07)	(1.88)	(2.97)	(3.00)	(3.00)	(3.00)	(3.00)
8/26/2011	12/17/2011	(3.19)	4.71	1.32	(0.13)	(0.14)	(0.35)	(0.73)	(1.35)	(2.39)	(3.19)	(3.19)	(3.19)	(3.19)	(3.19)
9/2/2011	12/17/2011	(3.40)	8.05	1.38	(0.11)	(0.10)	(0.27)	(0.72)	(1.34)	(2.26)	(3.40)	(3.40)	(3.40)	(3.40)	(3.40)
9/12/2011	12/17/2011	(3.70)	7.59	1.35	0.41	0.45	0.37	0.14	(0.56)	(1.82)	(3.71)	(3.70)	(3.70)	(3.70)	(3.70)
9/19/2011	12/17/2011	(4.04)	6.96	1.26	0.06	0.11	0.16	(0.09)	(0.60)	(1.77)	(3.92)	(4.04)	(4.04)	(4.04)	(4.04)
9/26/2011	12/17/2011	(4.39)	8.13	1.28	9.47	0.90	1.07	1.21	1.22	0.85	(0.14)	(2.53)	(4.39)	(4.39)	(4.39)
10/3/2011	12/17/2011	1.59	10.44	1.35	(86.74)	(0.87)	2.35	2.68	3.71	4.48	4.94	4.79	3.25	(0.99)	(4.80)
10/10/2011	1/21/2012	5.23	3.84	1.14	0.99	1.41	1.94	2.66	3.42	4.16	4.97	5.57	5.92	5.30	3.45
10/17/2011	1/21/2012	4.71	4.18	1.15	0.92	1.08	1.91	2.67	3.32	3.97	4.52	4.74	4.28	2.84	(0.12)
10/24/2011	1/21/2012	3.78	2.20	1.03	0.82	1.25	1.73	2.44	3.02	3.60	3.93	3.66	2.21	(1.19)	(4.04)
10/31/2011	1/21/2012	3.18	1.52	0.91	0.81	1.18	1.84	2.31	3.01	3.36	3.65	3.35	0.98	(3.24)	(4.39)
11/7/2011	1/21/2012	5.28	2.62	1.00	1.20	1.82	2.39	3.44	4.35	5.32	6.15	6.63	6.04	3.10	(3.34)
11/14/2011	1/21/2012	7.80	3.71	1.06	1.31	2.02	2.92	4.19	5.37	6.74	8.32	9.90	10.64	9.70	4.14
11/21/2011	3/17/2012	9.56	5.29	1.36	1.34	1.90	2.93	3.82	4.44	6.19	7.86	8.77	9.82	10.84	10.84
11/29/2011	3/17/2012	10.69	5.01	1.31	1.42	1.94	2.55	3.27	4.13	5.44	7.23	8.72	10.51	12.15	13.71
12/6/2011	3/17/2012	7.76	3.31	1.10	1.30	1.83	2.44	3.28	4.78	6.14	7.83	9.64	11.66	13.57	14.62
12/13/2011	3/17/2012	10.46	3.82	1.16	1.35	2.06	2.75	3.66	5.37	7.08	9.05	11.44	14.18	16.99	19.45
12/20/2011	3/17/2012	7.09	2.43	1.01	1.27	1.80	2.51	3.87	5.28	6.97	9.05	11.41	13.73	15.11	14.68
12/28/2011	3/17/2012	6.09	1.55	0.90	1.46	1.87	2.78	4.53	6.14	8.24	10.94	14.43	18.24	21.02	21.14
1/5/2012	3/17/2012	2.33	0.50	0.69	1.14	1.77	2.62	4.19	5.71	7.74	10.27	12.86	14.02	10.73	(2.82)
1/12/2012	3/17/2012	2.55	0.33	0.64	1.21	1.69	3.12	4.26	5.83	7.79	10.50	12.76	12.64	0.65	(5.54)
1/20/2012	3/17/2012	1.69	0.20	0.57	0.80	1.53	2.72	3.71	4.82	6.08	7.15	6.20	(2.70)	(6.32)	(6.32)
1/27/2012	4/21/2012	0.44	(0.01)	0.31	0.92	1.34	1.97	3.04	4.25	5.81	7.61	9.49	10.18	5.43	(3.59)
2/3/2012	4/21/2012	0.34	(0.09)	0.32	0.47	1.41	2.09	2.74	3.17	4.47	5.26	4.83	(0.18)	(4.57)	(4.57)
2/10/2012	4/21/2012	0.39	0.11	0.39	1.13	2.01	3.13	4.51	5.11	6.38	10.08	14.13	18.16	19.17	14.99
2/17/2012	4/21/2012	3.55	0.79	0.74	1.22	1.81	2.95	4.08	5.66	7.48	9.84	12.08	13.30	8.82	(3.14)
2/27/2012	5/19/2012	(0.21)	1.04	0.84	0.12	0.29	0.22	(0.04)	(0.20)	(1.20)	(3.36)	(4.39)	(4.39)	(4.39)	(4.39)
3/5/2012	6/16/2012	0.84	0.47	0.67	0.41	0.81	0.85	1.22	1.32	1.25	0.81	(0.36)	(2.72)	(3.49)	(3.50)
3/12/2012	6/16/2012	0.69	0.55	0.70	0.19	0.26	0.69	0.70	0.54	0.10	(0.97)	(3.16)	(3.75)	(3.75)	(3.75)
3/19/2012	6/16/2012	(0.10)	0.26	0.55	0.08	(0.02)	0.12	(0.13)	(0.68)	(1.81)	(3.76)	(4.04)	(4.04)	(4.04)	(4.04)
3/26/2012	6/16/2012	(0.61)	(0.28)	0.30	(0.93)	(1.43)	(2.21)	(3.49)	(4.39)	(4.39)	(4.39)	(4.39)	(4.39)	(4.39)	(4.39)
4/2/2012	6/16/2012	(0.49)	(0.35)	0.26	(0.85)	(0.97)	(1.57)	(2.64)	(4.40)	(4.80)	(4.80)	(4.80)	(4.80)	(4.80)	(4.80)
4/10/2012	6/16/2012	(0.09)	(0.01)	0.41	(0.05)	0.12	0.37	0.10	(0.61)	(2.01)	(4.63)	(5.37)	(5.37)	(5.37)	(5.37)
4/17/2012	6/16/2012	(0.35)	(0.24)	0.28	(0.52)	(0.54)	(1.02)	(1.94)	(3.66)	(5.98)	(5.98)	(6.00)	(6.00)	(6.00)	(6.00)
4/24/2012	7/21/2012	0.93	(0.14)	0.44	0.93	1.23	1.59	1.94	2.32	2.54	2.42	1.54	(1.03)	(4.00)	(4.00)
5/1/2012	7/21/2012	0.50	(0.38)	0.43	0.60	0.85	1.08	1.26	1.24	0.97	(0.27)	(3.62)	(4.43)	(4.44)	(4.44)
5/8/2012	7/21/2012	0.84	(0.31)	0.46	0.97	1.32	1.70	2.11	2.56	2.77	2.46	0.58	(4.35)	(4.85)	(4.86)
5/15/2012	7/21/2012	2.51	0.16	0.65	0.89	2.06	2.91	3.67	5.02	6.49	8.02	9.35	9.51	4.86	(4.90)
5/22/2012	8/18/2012	3.72	0.92	0.76	2.38	2.31	3.11	4.09	5.39	7.00	9.02	11.27	14.27	17.28	19.26
5/30/2012	8/18/2012	4.35	0.74	0.79	1.72	2.37	3.22	4.47	5.86	7.65	10.04	12.73	15.98	19.02	20.64
6/6/2012	8/18/2012	3.76	0.71	0.70	1.95	2.74	3.74	5.02	6.73	9.31	12.44	16.52	21.69	27.01	31.95
6/13/2012	9/22/2012	4.77	1.13	0.82	1.84	2.53	3.38	4.46	5.81	7.53	9.83	12.66	16.17	20.27	25.19
6/20/2012	9/22/2012	1.74	0.20	0.53	1.56	2.15	2.93	3.90	5.26	7.06	9.38	12.58	16.21	20.66	21.18
6/27/2012	9/22/2012	1.95	0.20	0.53	1.71	2.44	3.37	4.52	5.99	8.04	10.48	13.79	17.85	22.10	25.10
7/5/2012	9/22/2012	0.91	(0.14)	0.35	1.52	2.43	3.47	4.83	6.55	8.90	11.50	14.01	17.25	20.42	19.50
7/12/2012	9/22/2012	0.98	(0.27)	0.35	1.68	2.40	3.32	4.52	6.13	8.34	11.57	15.05	18.51	19.59	14.19
7/19/2012	9/22/2012	0.80	(0.36)	0.33	1.54	2.14	2.95	3.99	5.59	7.92	10.45	12.22	11.02	3.36	(3.32)
7/26/2012	9/22/2012	0.97	(0.23)	0.32	1.99	2.82	3.94	5.43	7.82	11.23	16.31	23.45	33.97	32.81	22.74
8/2/2012	9/22/2012	1.09	(0.30)	0.32	2.23	3.13	4.45	6.27	9.09	13.33	19.21	27.39	38.10	36.76	25.83
		2.27			(0.60)	0.34	0.70	1.06	1.44	1.96	2.58	3.24	3.43	2.60	0.81

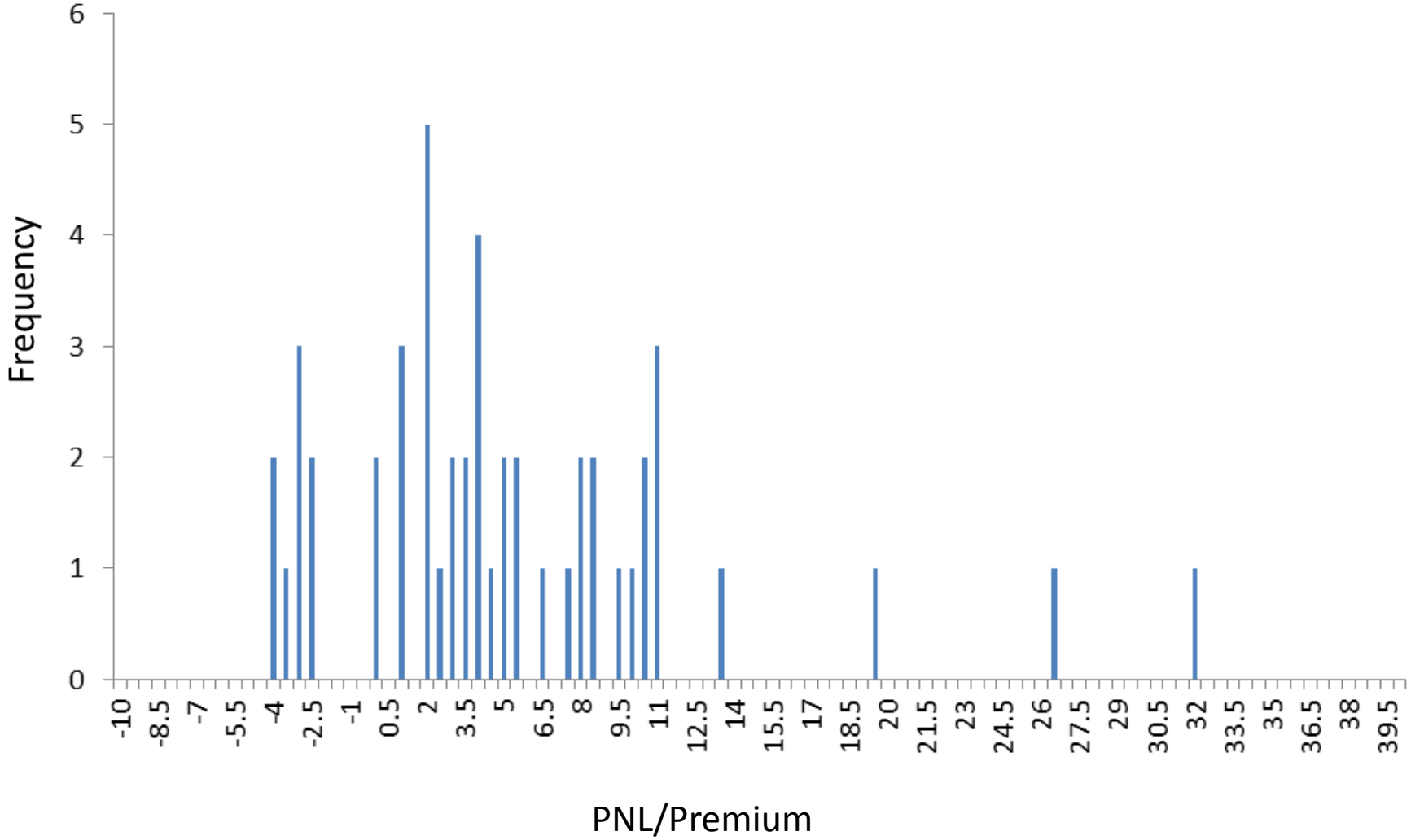
Histogram of 99 trades
(Best trade selected on each date based on model)



X-Y Plot (signal strength/PNL)



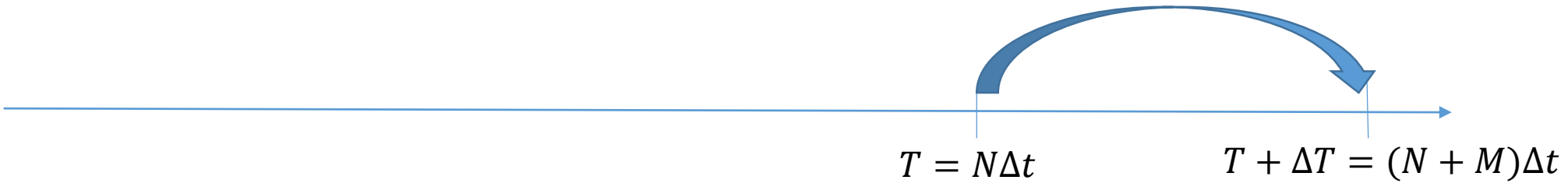
Histogram of best trades with signal threshold (model 16% quantile > 1.10; 49 trades)



Options on VIX Futures and Convexity Adjustment between Variance and Futures

Note: This is a difficult subject, but very interesting to explore, as it relates VX futures, VIX options and SPX options

Variance Swap Volatility and VIX Futures



Forward Variance Swap from T to $T + \Delta T$

$$E\left(\sum_{i=N}^{N+M}\left(\frac{dS}{S}\right)^2\right) = E\left\{E_T\left(\sum_{i=N}^{N+M}\left(\frac{dS}{S}\right)^2\right)\right\}$$

$$= E\{VIX_T^2\}\,\Delta T$$

Forward Variance vs. Futures Squared

$$VX_T = E\{VIX_T\}$$

VIX futures with settlement date T is the expected value of VIX_T

$$E\{VIX_T^2\} = (E\{VIX_T\})^2 + E\{VIX_T^2\} - (E\{VIX_T\})^2$$

$$= \ VX_T^2 + \text{Variance}(VIX_T)$$

Forward variance is futures squared + the VIX variance

The Variance of VIX as the value of a portfolio of VX futures

$$\text{Variance}(VIX_T) = E(VIX_T - VX_T)^2 = \int_0^\infty (v - \bar{v})^2 f(v) dv \qquad \bar{v} = VX_T$$

But $$(v - \bar{v})^2 = 2\int_0^{\bar{v}} (k - v)^+ \, dk + 2\int_{\bar{v}}^\infty (v - k)^+ \, dk$$

So

$$\text{Variance}(VIX_T) = 2e^{rT} \int_0^\infty OTM(VX_T, k, T) \, dk$$

Relation between Variance, VIX futures and VIX vol

$$E\{VIX_T^2\} = VX_T^2 + \text{Variance}(VIX_T)$$

$$= VX_T^2 + 2e^{rT}\int_0^\infty OTM_{vix}(VX_T, k, T)\, dk$$

Adding all forward variances

$$T\sigma_{T,vs}^2 = \Delta T \sum_{n=0}^{N-1} VX_{n\Delta T}^2 + 2\Delta T \sum_{n=0}^{N-1} e^{rn\Delta T}\int_0^\infty OTM_{vix}(VX_{n\Delta T}, k, n\Delta T)\, dk$$

Final link between SPX options, VIX futures and VIX options

$$2e^{rT}\int_0^{\infty} OTM_{spx}(SPX_T, Q, T)\frac{dQ}{Q^2} = \frac{1}{N}\sum_{n=0}^{N-1} VX_{n\Delta T}^2 + \frac{2}{N}\sum_{n=0}^{N-1} e^{rn\Delta T}\int_0^{\infty} OTM_{vix}(VX_{n\Delta T}, k, n\Delta T)\,dk$$

SPX (S&P 500) options

VIX futures

VIX options

Open question: how can this be converted into an arbitrage strategy?? (Too many moving parts!)

Compact formula using instantaneous futures (`weeklies')

$$\int_0^\infty OTM_{spx}(SPX_T, Q, T)\frac{dQ}{Q^2} = \frac{1}{2T}\int_0^T VX_t^2\,dt + \int_0^T e^{-r(T-t)}\int_0^\infty OTM_{vix}(VX_t, k, t)\,dk\,dt$$

The fair value of the variance of S&P 500 options is equal to the sum of

-- the variance implied by VIX futures contracts (strip of futures squared)

-- a term depending on the volatility of volatility (involving all strikes/maturities before T.

Conclusions on VIX trading

- VIX is an index representing an average of short-term volatilities of S&P500 index options. It has become a widely accepted ``fear gauge'' in markets.
- VIX is traded via variance swaps (OTC) and futures (CBOE). Futures extend to 1.5 years, with liquid first 7 settlement month
- There are also VIX options
- There are also many VIX-linked ETFs, the most important being VXX. Most of the ETFs have options.
- The arbitrage between SPX vol and VIX is complicated due to liquidity and due to the great number of options involved in getting an ``exact relation''.
- Term-structure arb via futures and futures/ETF trades are feasible and show interesting signals.
- VIX trading continues to evolve. For instance Brexit (UK referendum) has affected VIX in various ways.