An E-ARCH Model for the Term Structure of Implied Volatility of FX Options

by

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We construct a statistical model for the term-structure of implied volatilities of currency options based on daily historical data for 13 currency pairs over a 19-month period. We examine the joint evolution of 1 month, 2 month, 3 month, 6 month and 1 year at-the-money (50 Δ) options in all the currency pairs. We show that there exist three uncorrelated state variables (principal components) which account for the parallel movement, slope oscillation, and curvature of the term structure and which explain, on average, the movements of the term-structure of volatility to more than 95% in all cases. We test and construct an exponential ARCH, or E-ARCH, model for each state variable. One of the applications of this model is to produce confidence bands for the term structure of volatility.

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1 Introduction

With the rapid innovation and growth of the derivative securities, the management of volatility risk in its many forms has become an important topic for researchers and practitioners. This is due to the sensitivity of derivatives to market volatility and the need to manage this risk accurately and at low cost. There have been several approaches to this end, each with its own advantages and pitfalls. One consists in using "implied tree" models (Dupire, 1994; Rubinstein, 1994; Derman and Kani, 1994); a more traditional approach consists managing the Vegas corresponding to different maturities. Other models use the notion of stochastic volatility. Hull and White (1987), for example, treat the spot volatility as an exogenous random source, while Engle and collaborators (Engle and Noh, 1994; Engle and Mezrich, 1995; Engle and Rosenberg, 1995) analyze the volatility of the underlying process using heteroskedastic auto-regressive models (the Autoregressive Conditionally Heteroskedastic, or ARCH-GARCH family). Other approaches involve the use of confidence bands for future volatility movements (Avellaneda and Parás, 1996).

In this article, we contribute to the theoretical understanding of the volatility of option prices by studying empirically the dynamics of the *term-structure of implied volatilities* of currency options. We use a Principal Component Analysis (PCA) (Judge, 1988) combined with ARCH techniques to derive a statistical model for the evolution of the term structure of volatility. Thus, the present statistical analysis is not on the volatility of the underlying asset, as in traditional work (see Engle, 1994, 1995), but rather on the implied volatilities. The latter provide a "dimensionless" representation of the currency options market.

Using historical data on the implied volatility of options on 13 currency pairs for the period Jan. 1, 1995 to July 30, 1996, we develop a three-factor term-structure model which appears to be applicable to all the studied currency pairs. A similar methodology was used by other authors in the study of term structure of interest-rates (Litterman and Scheinkman, 1991). There are, however, important structural differences between interest rates and implied volatilities. The main difference is that the term-structure of volatility is a stochastic process which is *far from equilibrium.* As an illustration of this, Figure 1 shows an "equilibrium" AR model fitted using least squares and maximum likelihood techniques compared against real data. One clearly observes more structure in the real data than in the equilibrium model, and unlike the latter, the real implied volatility data exhibits a trend. A formal test for non-stationarity or "trend" of time series is the unit root test (Enders, 1995). We apply this test to the term-structure of volatility, and the results

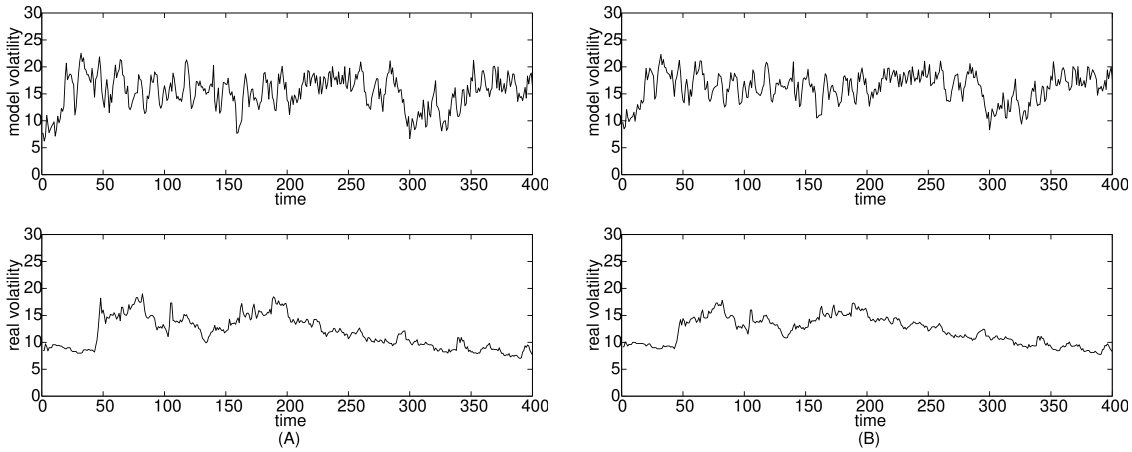

Figure 1: Simulated paths (one realization) of AR Model vs. Real Term Volatility for USD/JPY. A) 30 day implied Volatility , B) 60 day implied Volatility . The simulated paths do not reflect the structure of the real data.

fail to reject the null hypothesis of non-stationarity in all cases. On the other hand, we will show that in similar vein to interest rates, most of the variance on the term structure of volatility can be explained in terms of three factors: level movement ($\approx 90\%$), slope ($\approx 5\%$) and curvature ($\approx 1\%$).

The implied volatility processes exhibit strong heteroskedasticity, ie., the volatility of volatility is not constant. Therefore, we propose a class of 3-factor exponential ARCH, or E-ARCH, models to describe their dynamics. Based on the analysis of sections 2 and 3, the example in Figure 2 suggests that this model predicts the real movement of implied volatilities much better than naïve AR models.

As an application of this E-ARCH model, we present a method for calculating conditional confidence bands for the motion of the volatility curve. The method is illustrated with the aforementioned dataset. One possible application of such confidence bands could be for "statistical arbitrage", or alternatively, in the context of the Uncertain Volatility Model (Avellaneda and Parás, 1996), where option hedges are computed based on a proposed range for the future spot volatility.

This article is organized as follows. Section 2 discusses the three-factor model obtained by the Principal Component Analysis method applied to log-differences of the vector of implied volatilities. Section 3 discusses the three-factor E-ARCH model. Section 4 describes the construction of upper and lower confidence bands, for given time horizon, initial conditions and confidence level. The conclusions are presented in Section 5.

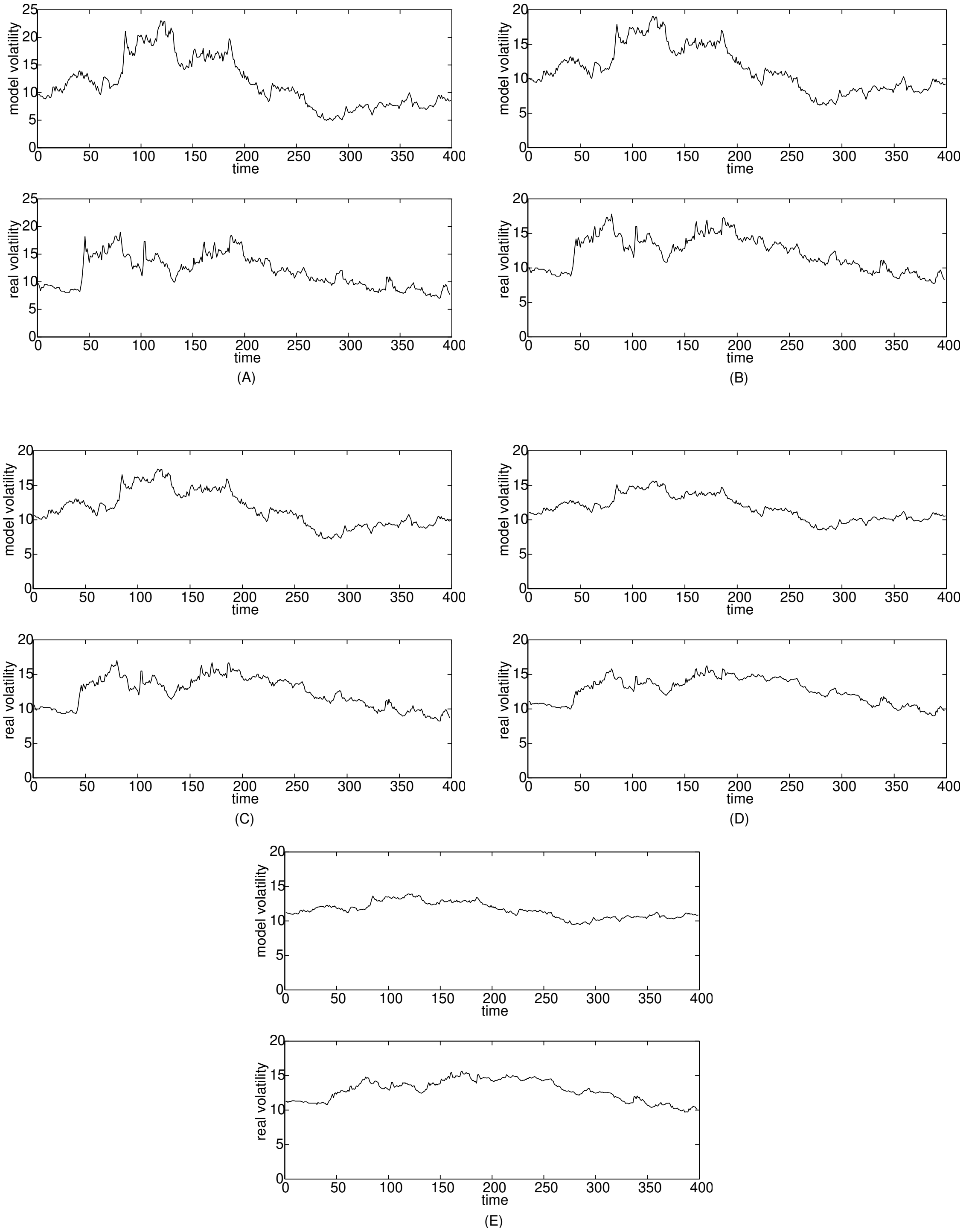

Figure 2: Simulated paths of E-ARCH Model (one realization) vs. Real Term Volatility for USD/JPY. A) 1 month, B) 2 month, C) 3 month, D) 6 month, E) 12 months. Notice the similar structure of paths.

2 Risk Factors Affecting the Volatility Term-Structure of FX options

2.1 Volatility Risk

By definition, Vega measures the sensitivity of an option's value to a parallel shift in the volatility term structure. In reality, however, the term volatility does not move in parallel fashion: it is well-known, for instance, that short-term volatility tends to be more volatile than long-term volatility.

A one-factor model of the term-structure effectively "decomposes" the overall volatility risk into (i) a systematic risk modeled by the one-factor model and (ii) an unsystematic risk, which is not accounted for explicitly, represented by the spreads between the realized term volatilities and the reference volatility predicted by the model. Consequently, if we used a statistical one-factor model for parallel shifts of the term structure of volatility, we would not be able to explain the relative changes or correlations between the prices of options with different expirations. Such an approach is too simplistic to be of practical use.

Empirical observation shows that each term volatility has a separate movement. This is why it is common practice in foreign-exchange markets to use "term Vegas" (i.e., 1-month Vega, 6-month Vega, etc.) for hedging the option book. However, this approach requires using large numbers of options and leads to the problem of hedging options with maturities which are not readily quoted in the over-the-counter market, such as a 75-day or a 47-day option.

The research conducted in this paper shows that the volatility term-structure can be decomposed essentially into three "principal components," or major sources of risk. This is due to the fact that the five maturities quoted on a day-to-day basis (1, 2, 3, 6 and 12 months) are highly correlated. This suggests the use of a multi-factor (three-factor) model to explain the fluctuations of the curve. This type of model offers the advantage of giving a better framework for hedging the book by hedging the exposure to each component rather than looking at individual expiration dates. It offers a solution to the aforementioned problems associated with "intensive" Vega-hedging, since fewer options will be involved if we hedge according to the principal components. We expect, in general, to hedge more than 95% of the risk in this way, in terms of a measure that will be made precise below.

The advantage of using a multi-factor model is that the underlying factors are not merely the quoted volatilities for standard maturities, ie., we take into account existing statistical correlations between volatilities. From the point of view of hedging, we are only concerned with the sensitivity of the portfolio to each factor.

2.2 The Three-Factor Model

The Principal Component Analysis (PCA) approach for analyzing a time-series consists in studying the covariance matrix of successive shocks. If we view the term-structure of volatility as a 5-dimensional vector, $(\sigma^{(1)}, \sigma^{(2)}, \ldots, \sigma^{(5)})$, where the $\sigma^{(i)}$'s represent the 1, 2, 3, 6 and 12 month volatility, respectively, then we should analyze a 5×5 symmetric matrix of squares and cross-products of volatility changes. The approach that we take in this paper is to analyze the covariance of the differences of the logarithm of the implied volatilities $A = a_{ij}$, which are defined as

$$a_{ij} = \frac{1}{T-1} \sum_{t=1}^{T-1} (\log \sigma^{(i)}(t+1) - \log \sigma^{(i)}(t)) (\log \sigma^{(j)}(t+1) - \log \sigma^{(j)}(t))$$

where $1 \leq i, j \leq 5$ and t ranges over the number of days observed.[2] Other possible candidates for analyzing the principal components could be successive differences of the volatilities (instead of the logarithms), the logarithm of the data or simply the data itself (in the latter two cases, the sample mean of the term-structure enters the calculation as well). The reason for working with differenced data is the following: in conducting a unit root test on the implied volatility movements of FX rates for 13 currency pairs, we could not reject in any of the cases a unit root null-hypothesis. Hence, we concluded that the implied volatility curve does not behave like a stationary processes. Since, by convention, the PCA analysis leads to factors that have mean zero and constant variance, it is reasonable to select variables that are statistically stationary. The differenced data, by inspection or the unit-root test, is stationary. Moreover, to exclude the possibility of having negative volatilities, we chose to work with the differenced logarithms rather than the differences of the volatilities.

Let us denote by $Y_t^{(j)}$, the j-th differenced logarithm of implied volatility at time t and by $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_5$ the five normalized eigenvectors of the sample covariance matrix of $\{Y_t^{(j)}\}_{j=1}^{5}$. We can define the coordinates of the vectors $Y_t^{(1)}, \ldots Y_t^{(5)}$ in the orthonormal frame defined by the eigenvectors, *viz.*,

$$V_t^{(i)} = \sum_j v_{ij} Y_t^{(j)}$$

[2]The data used were daily closing prices posted electronically by brokers.

	V_1	V_2	V_3	V_4	V_5
1M	0.0636	0.3257	-0.0004	-0.6406	0.6925
2M	-0.5004	-0.6825	-0.0394	0.1533	0.5087
3M	0.7505	-0.1794	0.2818	0.4107	0.3956
6M	-0.0457	0.3897	-0.7288	0.4900	0.2737
12M	-0.4246	0.4941	0.6229	0.3967	0.1740
Eig-value	0.0083	0.0104	0.0045	0.0310	0.9459

Table 1: Eigenvectors and normalized Eigenvalues for USD/JPY

or

$$Y_t^{(j)} = \sum_i v_{ij} V_t^{(i)}.$$

In this formulation, the random variables $V^{(i)}$ are statistically uncorrelated linear combinations of the $Y^{(j)}$. This suggests the following *ansatz* for the term-structure of volatility

$$\sigma_t^{(j)} = \sigma_{t-1}^{(j)} exp(\sum_i v_{ij} V_{t-1}^{(i)}), \tag{1}$$

where the statistics of the processes $V_t^{(i)}$ $i = 1, \ldots, 5$ will be determined in the next section.

Numerical values for the components of $\{\vec{v}_i\}_{i=1}^5$ and their corresponding normalized eigenvalues are shown in Table 1 for USD/JPY,[3] using the period from January 1, 1995 to July 30, 1996 with daily observations. The eigenvalue normalization is made such that their total sums are equal to 1. Each normalized eigenvalue represents the importance of the corresponding component for explaining the variance of the curve. An important consequence of the PCA analysis is that, in all 13 cases, *the variability of the term-structure of volatility is explained to more than 95% by just 3 components or eigenvectors.*

Figure 3 exhibit the *factor sensitivities* for USD/JPY, USD/DEM and CAD/USD. The plotted curves represent the percentage change in term-vol for a one standard deviation shock in the the corresponding factor. Only the three factors with the largest eigenvalues are shown. We used cubic splines to generate a smooth curve interpolating between the five standard maturities.

Table 1 in Appendix shows that the first (largest eigenvalue) factor accounts for about 90% of the variance on average. We observe in Figure 3 that the percentage changes caused by this first factor are positive and relatively flat

[3]Table 1 in the Appendix shows results for all the 13 currency pairs.

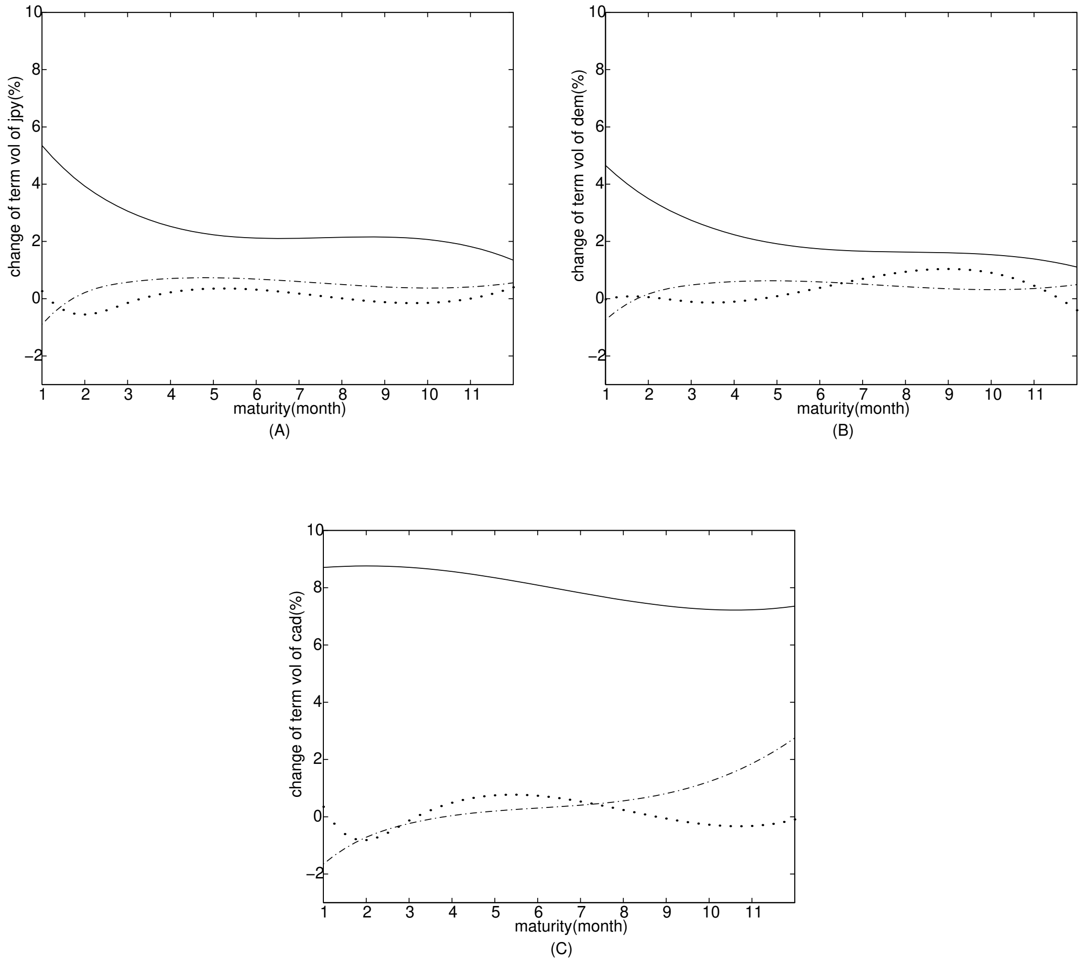

Figure 3: Factor Sensitivities for A) USD/JPY, B) USD/DEM and C) USD/CAD. Solid line is Factor 1, dot-dash line is Factor 2, and dotted line is Factor 3.

across all maturities. Thus, this first factor corresponds approximately to a parallel movement of the term-structure of volatility . Note however that the sensitivity curve of the first factor is downward-sloping for most currencies, which is consistent with the fact that the longer term volatility is less volatile than shorter term volatility.

The second factor, which explains about 5% of the variance on average, corresponds to the variation of the slope of the term-structure. It “lowers” the short-term volatilities and “raises” the long-term volatilities.

The third factor explains about 1% of the variability on average. We can view it as “twist component” of the term-structure curve: it tends to lower the term volatility for short and long maturities and raise it in the middle.

Term Vol	Total Variance Explained(%)	Factor 1(%)	Factor 2(%)	Factor 3(%)
1M	100.0	97.0	2.7	0.2
2M	99.2	97.0	0.3	1.9
3M	96.8	93.3	3.3	0.2
6M	97.1	86.1	9.0	1.9
1YR	91.8	72.9	12.4	6.4
Average	98.7	94.6	3.1	1.0

Table 2: Relative Importance of Factors

Table 2 exhibits the relative importance of the three factors of USD/JPY[4] in explaining the variation of each of the five volatilities corresponding to standard maturities.

We see that, on average, for all the currencies, the three factors account for more than 95% of the total variance. Although they explain the shorter term volatility better than the longer term volatility, the longer term volatility is much less volatile.

Based on this analysis and equation (1), we shall consider in the sequel the three-factor model for the volatility term-structure

$$\sigma_t^{(j)} = \sigma_{t-1}^{(j)} exp(\sum_1^3 v_{ij} V_{t-1}^{(i)}) \ , \tag{2}$$

where $V_t^{(i)}$, $i = 1, 2, 3$, represent the factors with the largest eigenvalues.

3 Three-factor Exponential ARCH model

A cursory inspection of the real volatility processes of JPY in Figure 2 shows that volatility of these processes is not constant across time. There are periods of unusually large volatility followed by periods of relatively low volatility of the term-structure. This clearly points to the inadequacy of naïve models in which the volatility of volatility is constant. Experiments with homoskedastic autoregressive (AR) statistics strongly support this, since the path fluctuations that result tend to be much more homogeneous across time than

[4] Table 2 in Appendix shows the results for all 13 currency pairs.

indicated by the data. More formal statistical testing (Lagrange Multiplier Test of Engle, 1984) points to a strong heteroskedastic effect. In order to construct an appropriate ARCH model, we first calculated the ACF (Autocorrelation Coefficient Function) of the differenced data, which shows no autocorrelation effect to any order. In fact, the ACF's resemble those of simulated white noise. Once we have an appropriate AR model, (in this case, no AR coefficients), the TR-square statistic of a regression of residuals for each $V^{(i)}$ is used, and it suggests that an ARCH(2) model is appropriate for most currency pairs. Therefore, we adopt the following model for each of the three factors:

$$\begin{aligned} V_t^{(i)} &= a^{(i)} + \epsilon_t^{(i)} \\ \epsilon_t^{(i)} &\sim N(0, h_t^{(i)}) \\ h_{t+1}^{(i)} &\equiv E_t \epsilon_{t+1}^{(i)2} = \alpha_0^{(i)} + \alpha_1^{(i)}(\epsilon_{t-1}^{(i)})^2 + \alpha_2^{(i)}(\epsilon_2^{(i)})^2 \ , \end{aligned} \tag{3}$$

where $i = 1, 2, 3$. E_t is expectation conditional on time t. The parameters a, α_0, α_1 and α_2 are then determined by the Maximum Likelihood method for each of the three factors. These values are shown in the Appendix.

Figure 2 shows the model vs. the real data for USD/JPY. We see that the model selects the correct range of motions of the term-structure and captures some of the finer details of the series.

We believe that the present Exponential ARCH(2) model is appropriate for simulating term-volatility movement because it captures two important features of the real processes: the unit root effect, or non-stationarity, and the heteroskedasticity of the process. These hypotheses are strongly supported by statistical testing.

One should not think of the 3-factor Exponential ARCH(2) model as a "local volatility model" in the sense of Hull and White. Thus, it cannot be used directly in a derivatives pricing model to simulate the dynamics of the spot volatility. Nevertheless, it gives a realistic version of how the prices of options with different maturities are correlated in the currency markets, and thus provides useful information for hedging a book with a spectrum of options.

To feed the market information directly into a derivatives pricing model, we would need to model the local volatility variations. To this effect, we could use the 3-factor E-ARCH to construct dynamics for the "forward" volatility processes, i.e., the 1 month-2 month, 2 month-3 month volatilities, etc. Another approach, which we explore below, is to determine "confidence bands" for the term-structure of volatilities, which could be used as inputs in the Uncertain Volatility Model (Avellaneda and Parás, 1996). The next section describes a methodology for computing confidence bands.

4 Application: Confidence Bands

Consider the model of equation (3) for $V^{(1)}, V^{(2)}$ and $V^{(3)}$. Let us make a change of variables expressing volatility term-structure in its original representation. Using the notations of the last section, we have,

$$\begin{aligned}\Delta \log \sigma_t^{(j)} &= \sum_{i=1}^{3} v_{ij} a^{(i)} + \sum_{i=1}^{3} v_{ij} \epsilon_t^{(i)} \\ &= A^{(j)} + \sum_{i=1}^{3} v_{ij} \epsilon_t^{(i)}, \quad (j = 1, \ldots, 5)\end{aligned}$$

Our goal is, for given confidence level, to find the upper and lower bounds for $\log \sigma_s^{(i)}$, $i = 1, \cdots, 5$, $s \in [0, t]$. To this end we

1. approximate the processes by their continuous time version.

2. assume that the drift terms $A^{(i)}$'s are small compared with diffusion coefficient $\sum_{i=1}^{3} v_{ij}^2$.

Both approximations are reasonable. Typically $A^{(i)}$ is of order 10^{-4}, while

$$\langle \Delta \log \sigma_t \rangle = \sum_i v_{ij}^2$$

is of order 10^{-2} ($\langle \cdot \rangle$ is the quadratic variation).

In principle, one could use Monte-Carlo to find the joint distribution of $\log \sigma_{min}^{(i)}$ and $\log \sigma_{max}^{(i)}$. This involves finding a two-dimensional histogram for each standard maturity, but this is time-consuming. Instead, we used Monte-Carlo simulation to find the distribution of the quadratic variation process of $\log \sigma_t^{(i)}$, ie.,

$$P(\langle \log \sigma_t \rangle \in dT)$$

which involves calculating only a one-dimensional histogram. Then, we used a time-change to transform, for each i, $\log \sigma_t^{(i)}$ to a Brownian Motion with drift.[5] We then used a formula for the Brownian first-passage time to compute

[5]Although in this case the drift is not constant, due to assumption 2 that the drift is small compared to typical T, this conditional distribution won't change much if we spread the drift out evenly during time $0 \to T$. When T is small, ie., the conditional motion is basically drift, the band is big enough that the probability won't be affected by the spreading out. Moreover, when T is small, constant drift is fairly a good approximation anyway.

the probability of the time-changed $\log \sigma_t$ exits a band. This is a well-known formula for the distribution of Brownian passage time with drift, often used to generate closed-form solutions for double-barrier options (Karatzas and Shreve, 1991).

Let us be more specific. First, we fix some notations. W_t is a Brownian Motion starting from x, with drift μ. T_0 and T_b are the exiting time of W_t from 0 and b, respectively. If $P^{(\mu)}[T_0 \wedge T_b \leq t]$ is the probability that the process $\{\mu s + W_s\}_{s=0}^{t}$ exits the band $[0, b]$, then

$$\begin{aligned}
P^{(\mu)}[T_0 \wedge T_b \leq t] \;=\; & \sum_{n=0}^{\infty} e^{\alpha_+}[1 - N(A_+) + e^{-2\alpha_+} N(A_-)] \\
- & \sum_{n=0}^{\infty} e^{\alpha_-}[1 - N(B_+) + e^{-2\alpha_-} N(B_-)] \\
+ & \sum_{n=0}^{\infty} e^{\alpha_-}[1 - N(A_+ + \frac{b}{\sqrt{t}}) + e^{-2\alpha_- + 2b\mu} N(A_- + \frac{b}{\sqrt{t}})] \\
- & \sum_{n=0}^{\infty} e^{\alpha_+ + b\mu}[1 - N(B_+ - \frac{b}{\sqrt{t}}) + e^{-2\alpha_+ - 2b\mu} N(B_- - \frac{b}{\sqrt{t}})]
\end{aligned}$$

where

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz$$

and

$$\begin{aligned}
\alpha_{\pm} &= -(2nb \pm x)\mu \\
A_{\pm} &= -\mu\sqrt{t} \pm \frac{2nb + x}{\sqrt{t}} \\
B_{\pm} &= -\mu\sqrt{t} \pm \frac{2nb - x}{\sqrt{t}} \;.
\end{aligned}$$

The terms in the series decay like $exp(-\alpha n^2)$, with $\alpha = \frac{2b^2}{t}$. Evaluation of the sum of the first two or three terms is sufficient in practice.

This calculation gives the conditional probability

$$P(\log \sigma_s \in [b_1, b_2], s \in [0, t] | \langle \log \sigma_t \rangle = T)$$

where again $\langle \cdot \rangle$ denotes the quardratic variation. Therefore,

$$\begin{aligned}
& P(\log \sigma_s \in [b_1, b_2], s \in [0, t]) \\
= & \int_0^{\infty} P(\log \sigma_s \in [b_1, b_2] | \langle \log \sigma_t \rangle = T) \cdot P(\langle \log \sigma_t \rangle \in dT)
\end{aligned}$$

For a given confidence level, say 95%, set

$$P(\log \sigma_s \in [b_1, b_2], s \in [0, t]) = 0.95.$$

There are many pairs of $[b_1, b_2]$ which satisfy the above equation. We simply choose the initial position x in such a way that the the probabilities for exiting the band on both sides are equal, namely

$$P_x[\tau_{b_1} \leq \tau_{b_2}] = P_x[\tau_{b_1} \geq \tau_{b_2}]$$

where τ_{b_i} is the exit time of the process $\log \sigma_s$ from boundary b_i. The subscript x denotes the initial condition. In practice, however, we approximate it by the following equation:

$$x = E\left\{ y \,|\, (P_y[T_{b_1} \leq T_{b_2} | \langle \log \sigma_t \rangle] = P_y[T_{b_1} \geq T_{b_2} | \langle \log \sigma_t \rangle]) \right\}$$

Table 3 shows the 95%-confidence bands for 11 currency pairs[6] over different time periods T.[7] We find that the bands for the term-structure of volatility form a "cone": short maturities have wider confidence intervals than long maturities for any given time-window t. The bands were calculating using a specific initial condition: we used the first three days in the dataset as the initial condition for the ARCH(2) processes. Note that this is a *conditional* term volatility band, i.e., different initial conditions give rise to different bands, which depend on the positions of the five term volatilities over the past three days.

5 Conclusion

The analysis of the implied volatilities of currency options for 13 currency pairs shows that the movements of the term-structure are explained to more than 95% with a three-factor model. These factors are derived by a Principal Component Analysis of the sample covariance matrix of the changes in the log-differences of the implied volatilites of the first five standard maturities.

A heteroskedastic model for the evolution of the three factors driving the volatility curve was derived. We found that an ARCH(2) model was consistent with the data for each currency pair.

Finally, this model was used to calculate confidence bands for the term-structure over different periods of time. In all cases, these bands are "cone-like", in the sense that the confidence intervals become narrower as the option's expiration date increases.

[6]CAD and DEMESP are more suitable for jump-diffusion processes, here we exclude these two cases.

[7]The Table 3 shows time periods over 15, 30, 60, 90 and 120 days.

	T		1M	2M	3M	6M	12M
USD-AUD	15	$\overline{\sigma}$	9.9781	9.4833	9.0959	8.9046	13.5821
		$\underline{\sigma}$	6.0989	6.7484	7.0336	7.3649	4.9966
	30	$\overline{\sigma}$	10.9664	10.1199	9.5547	9.2323	9.2005
		$\underline{\sigma}$	5.5504	6.3236	6.6938	7.1008	7.4829
	60	$\overline{\sigma}$	12.5266	11.0881	10.2405	9.7172	9.5792
		$\underline{\sigma}$	4.8612	5.7710	6.2416	6.7414	7.1829
	90	$\overline{\sigma}$	13.8944	11.9040	10.8097	10.1157	9.8876
		$\underline{\sigma}$	4.3845	5.3750	5.9094	6.4709	6.9547
	120	$\overline{\sigma}$	15.1229	12.6170	11.2958	10.4492	10.1433
		$\underline{\sigma}$	4.0299	5.0709	5.6516	6.2597	6.7754
USD-CHF	15	$\overline{\sigma}$	18.6214	16.6631	16.0034	14.4959	13.8153
		$\underline{\sigma}$	5.3403	6.4607	7.3952	8.6314	9.7241
	30	$\overline{\sigma}$	24.7961	20.7054	19.1007	16.3118	14.9680
		$\underline{\sigma}$	4.0007	5.1903	6.1854	7.6594	8.9649
	60	$\overline{\sigma}$	34.2818	26.4360	23.3269	18.6939	16.4289
		$\underline{\sigma}$	2.8726	4.0410	5.0389	6.6597	8.1488
	90	$\overline{\sigma}$	46.4759	33.2947	28.1467	21.1912	17.8945
		$\underline{\sigma}$	2.1064	3.1936	4.1588	5.8571	7.4653
	120	$\overline{\sigma}$	56.1850	38.4087	31.6422	22.9463	18.8931
		$\underline{\sigma}$	1.7328	2.7561	3.6851	5.3941	7.0571
USD-DEM	15	$\overline{\sigma}$	15.5809	15.4993	15.3015	14.4975	13.6618
		$\underline{\sigma}$	4.3031	4.5380	4.7065	5.0866	5.4010
	30	$\overline{\sigma}$	19.0959	18.8207	18.4282	17.1042	15.8199
		$\underline{\sigma}$	3.4960	3.7194	3.8893	4.2926	4.6481
	60	$\overline{\sigma}$	28.3647	27.4421	26.4692	23.5806	21.0344
		$\underline{\sigma}$	2.3459	2.5418	2.6979	3.1034	3.4865
	90	$\overline{\sigma}$	36.6813	35.0758	33.4948	29.0573	25.3024
		$\underline{\sigma}$	1.7990	1.9704	2.1122	2.4972	2.8791
	120	$\overline{\sigma}$	50.8531	47.9054	45.1775	37.9016	32.0228
		$\underline{\sigma}$	1.2953	1.4400	1.5630	1.9111	2.2713

Table 3: Implied Vol 95% Band. T is the time period for which different bands are valid. *overlineσ* and *underlineσ* are the upper bound and lower bound respectively.

DEM-CHF	15	$\overline{\sigma}$	6.0702	5.6669	5.8365	5.0656	5.1074
		$\underline{\sigma}$	2.3736	2.6795	2.7385	3.1564	3.4521
	30	$\overline{\sigma}$	7.1700	6.4581	6.7021	5.5293	5.4984
		$\underline{\sigma}$	2.0066	2.3488	2.3830	2.8902	3.2053
	60	$\overline{\sigma}$	9.1467	7.8282	8.1736	6.2656	6.1022
		$\underline{\sigma}$	1.5682	1.9334	1.9509	2.5479	2.8859
	90	$\overline{\sigma}$	11.0733	9.1044	9.5463	6.9102	6.6207
		$\underline{\sigma}$	1.2915	1.6588	1.6678	2.3079	2.6578
	120	$\overline{\sigma}$	12.9096	10.2714	10.8350	7.4873	7.0821
		$\underline{\sigma}$	1.1045	1.4672	1.4672	2.1278	2.4827
DEM-FRF	15	$\overline{\sigma}$	4.8144	4.6203	4.7438	5.0590	5.1818
		$\underline{\sigma}$	0.8281	1.0439	1.3136	1.7752	1.9728
	30	$\overline{\sigma}$	6.6910	6.0860	6.0197	6.1571	6.2193
		$\underline{\sigma}$	0.5944	0.7907	1.0331	1.4563	1.6415
	60	$\overline{\sigma}$	10.6467	8.9752	8.4334	8.1206	8.0486
		$\underline{\sigma}$	0.3719	0.5338	0.7345	1.1008	1.2652
	90	$\overline{\sigma}$	14.9690	11.9164	10.7845	9.9504	9.7342
		$\underline{\sigma}$	0.2633	0.4003	0.5721	0.8957	1.0435
	120	$\overline{\sigma}$	20.8403	15.7589	13.7252	12.1167	11.6729
		$\underline{\sigma}$	0.1883	0.3013	0.4477	0.7334	0.8679
DEM-ITL	15	$\overline{\sigma}$	14.5603	13.0722	12.0480	11.0815	10.4499
		$\underline{\sigma}$	4.3889	4.8904	5.3072	5.7698	6.1192
	30	$\overline{\sigma}$	18.1806	15.6822	14.0276	12.5067	11.5391
		$\underline{\sigma}$	3.5107	4.0730	4.5550	5.1082	5.5376
	60	$\overline{\sigma}$	24.9891	20.3243	17.4252	14.8850	13.3318
		$\underline{\sigma}$	2.5483	3.1376	3.6618	4.2855	4.7862
	90	$\overline{\sigma}$	32.0328	24.8852	20.6415	17.0511	14.9160
		$\underline{\sigma}$	1.9834	2.5584	3.0870	3.7354	4.2718
	120	$\overline{\sigma}$	39.1518	29.3086	23.6716	19.0319	16.3451
		$\underline{\sigma}$	1.6191	2.1688	2.6882	3.3417	3.8929
DEM-JPY	15	$\overline{\sigma}$	11.0145	11.1917	11.4694	11.7281	11.2358
		$\underline{\sigma}$	5.1023	6.1512	7.0585	7.8577	8.8987
	30	$\overline{\sigma}$	12.7203	12.5137	12.5610	12.6586	11.7450
		$\underline{\sigma}$	4.4149	5.4984	6.4425	7.2799	8.5118
	60	$\overline{\sigma}$	15.6032	14.6631	14.2883	14.1115	12.5038
		$\underline{\sigma}$	3.5942	4.6875	5.6591	6.5298	7.9931
	90	$\overline{\sigma}$	18.1937	16.5169	15.7446	15.3323	13.1124
		$\underline{\sigma}$	3.0781	4.1570	5.1315	6.0095	7.6201
	120	$\overline{\sigma}$	20.8027	18.3292	17.1362	16.4517	13.6622
		$\underline{\sigma}$	2.6883	3.7420	4.7109	5.6002	7.3114

Table 3: (continued)Implied Vol 95% Band

USD-FRF	15	$\overline{\sigma}$	15.5483	14.4455	13.6527	12.8797	12.2370
		$\underline{\sigma}$	4.6258	5.7136	6.5904	7.7481	8.8269
	30	$\overline{\sigma}$	19.2527	17.0004	15.5144	14.0962	12.9726
		$\underline{\sigma}$	3.7254	4.8451	5.7888	7.0693	8.3187
	60	$\overline{\sigma}$	26.9794	21.9913	19.0005	16.2384	14.2082
		$\underline{\sigma}$	2.6430	3.7293	4.7084	6.1184	7.5806
	90	$\overline{\sigma}$	34.2630	26.4156	21.9281	17.9700	15.1665
		$\underline{\sigma}$	2.0693	3.0918	4.0642	5.5128	7.0883
	120	$\overline{\sigma}$	41.9166	30.8467	24.7406	19.5792	16.0226
		$\underline{\sigma}$	1.6825	2.6370	3.5894	5.0456	6.6975
USD-GBP	15	$\overline{\sigma}$	12.2662	11.7865	11.4443	11.2836	11.4482
		$\underline{\sigma}$	4.6945	5.6929	6.5996	8.1575	9.0802
	30	$\overline{\sigma}$	14.7347	13.5512	12.7100	12.0034	11.9589
		$\underline{\sigma}$	3.8993	4.9435	5.9327	7.6605	8.6863
	60	$\overline{\sigma}$	18.9455	16.4122	14.6803	13.0765	12.7119
		$\underline{\sigma}$	3.0191	4.0686	5.1197	7.0178	8.1602
	90	$\overline{\sigma}$	22.9590	19.0154	16.3953	13.9706	13.3411
		$\underline{\sigma}$	2.4801	3.5004	4.5693	6.5556	7.7644
	120	$\overline{\sigma}$	27.1407	21.5973	18.0421	14.7839	13.8886
		$\underline{\sigma}$	2.0888	3.0721	4.1389	6.1827	7.4478
GBP-DEM	15	$\overline{\sigma}$	8.8646	8.5452	8.0783	7.5907	7.5108
		$\underline{\sigma}$	3.4126	3.8027	4.3093	4.7422	5.1174
	30	$\overline{\sigma}$	10.6893	10.0164	9.1372	8.3295	8.1053
		$\underline{\sigma}$	2.8302	3.2445	3.8100	4.3213	4.7417
	60	$\overline{\sigma}$	13.8544	12.4780	10.8378	9.4682	8.9995
		$\underline{\sigma}$	2.1837	2.6051	3.2124	3.8009	4.2698
	90	$\overline{\sigma}$	17.0060	14.8488	12.4031	10.4760	9.7741
		$\underline{\sigma}$	1.7792	2.1897	2.8072	3.4348	3.9306
	120	$\overline{\sigma}$	20.1387	17.1396	13.8636	11.3890	10.4642
		$\underline{\sigma}$	1.5025	1.8975	2.5117	3.1589	3.6708
USD-JPY	15	$\overline{\sigma}$	14.5472	13.4890	13.0636	12.8907	12.7556
		$\underline{\sigma}$	5.3224	6.4092	7.3488	8.5502	9.6576
	30	$\overline{\sigma}$	17.5806	15.5250	14.5580	13.9374	13.4512
		$\underline{\sigma}$	4.4036	5.5670	6.5926	7.9064	9.1568
	60	$\overline{\sigma}$	22.7916	18.8156	16.8861	15.5093	14.4728
		$\underline{\sigma}$	3.3960	4.5907	5.6805	7.1021	8.5081
	90	$\overline{\sigma}$	28.1021	21.9741	19.0323	16.9034	15.3508
		$\underline{\sigma}$	2.7536	3.9286	5.0372	6.5136	8.0193
	120	$\overline{\sigma}$	33.5397	25.0451	21.0559	18.1734	16.1270
		$\underline{\sigma}$	2.3066	3.4449	4.5506	6.0559	7.6312

Table 3: (continued)Implied Vol 95% Band

References

[1] Avellaneda, M., and A. Parás (1996). "Managing the volatility risk of portfolios of derivative securities: The Lagrangian Uncertain Volatility model." *Applied Mathematical Finance 3*, 21-52.

[2] Derman, E., and I. Kani (1994). "Riding on the Smile." *Risk* 7 (February): 32-39.

[3] Dupire, B. (1994). "Pricing with a Smile." *Risk* (January) 7:18-20.

[4] Enders, W. (1995). *Applied Econometric Time Series* John Wiley & Sons, Inc.

[5] Engle, Robert F. (1984), "Wald, Likelihood Ratio, and Lagrange Multiplier Tests in Econometrics " in *Handhook of Econometrics, Vol. II,* (eds. Z.Griliches and M.D. Intriligator), North-Holland, Amsterdam.

[6] Noh, J., and R.F. Engle, and Alex Kane (1994). "Forecasting Volatility and Option Prices of The S& P 500 Index". *The Journal of Derivatives* (Fall)

[7] Engle, Robert, Joseph Mezrich (1995). " Grappling With GARCH ". *Risk* (September) 8:112-117.

[8] Engle, R., and J.V. Rosenberg (1995). "Garch Gamma." *The Journal of Derivatives 2*.47-59.

[9] Hull, J., and A. White (1987) "The Pricing of Options on Asset with Stochastic Volatilities." *Journal of Finance* (June):281-300.

[10] Judge,G.G., W.E. Griffiths, R.C. Hill, H. Lutkepohl and T.C.Lee (1988) *Introduction to the Theory and Practice of Econometrics*, 2nd ed. New York: John Wiley.

[11] Karatzas, I., and S. Shreve (1991) Brownian Motion and Stochastic Calculus. Springer-Verlag.

[12] Litterman, R., and J. Scheinkman (1991) "Common Factors Affecting Bond Returns." *The Journal of Fixed Income* (June): 55-61.

[13] Rubinstein, M. (1994). "Implied Binomial Trees." *Journal of Finance* 49:771-818.

A Appendix: Tables

List of tables:

Table 1: **Eigenvectors and Normalized Eigenvalues**

	V_1	V_2	V_3	V_4	V_5
1M	0.1142	0.4142	0.0460	-0.5382	0.7236
2M	-0.4633	-0.7075	-0.1437	0.0342	0.5128
3M	0.7490	-0.2424	0.1727	0.4704	0.3594
6M	-0.1624	0.4320	-0.6588	0.5485	0.2282
12M	-0.4300	0.2871	0.7165	0.4324	0.1796
USD-AUD	0.0188	0.0321	0.0083	0.0769	0.8640
1M	-0.0694	0.3003	-0.6623	-0.4984	0.4669
2M	0.4755	-0.7004	0.1263	-0.2163	0.4696
3M	-0.7708	-0.1139	0.4121	-0.0714	0.4670
6M	0.4172	0.6321	0.4795	0.0918	0.4337
12M	-0.0301	-0.0817	-0.3817	0.8314	0.3943
USD-CAD	0.0017	0.0037	0.0137	0.0299	0.9510
1M	0.0759	-0.0207	0.3928	-0.6209	0.6738
2M	-0.5324	0.1068	-0.6602	0.0715	0.5139
3M	0.7051	-0.3374	-0.2571	0.3867	0.4164
6M	0.0458	0.7409	0.3507	0.5007	0.2745
12M	-0.4600	-0.5705	0.4698	0.4574	0.1819
USD-CHF	0.0105	0.0061	0.0181	0.0389	0.9264
1M	-0.0475	-0.0420	0.3554	-0.6285	0.6889
2M	-0.2964	0.0972	-0.7846	0.1394	0.5174
3M	0.7962	-0.1924	0.0149	0.4055	0.4055
6M	-0.2304	0.6707	0.4284	0.4973	0.2576
12M	-0.4722	-0.7085	0.2728	0.4169	0.1639
USD-DEM	0.0092	0.0067	0.0164	0.0286	0.9392
1M	0.4129	-0.1051	0.0190	-0.6588	0.6198
2M	-0.8486	-0.1054	-0.0935	-0.0395	0.5084
3M	0.1731	0.7438	-0.0096	0.4367	0.4754
6M	0.1311	-0.4650	0.6982	0.4448	0.2853
12M	0.2495	-0.4565	-0.7095	0.4194	0.2239
DEM-CHF	0.0289	0.0511	0.0118	0.0718	0.8365
1M	-0.1898	0.0173	-0.4174	-0.6681	0.5858
2M	0.6912	0.0364	0.5089	-0.0687	0.5072
3M	-0.6426	-0.3050	0.4953	0.2457	0.4340
6M	-0.1057	0.7812	-0.1920	0.4722	0.3445
12M	0.2492	-0.5432	-0.5335	0.5153	0.3044
DEM-FRF	0.0147	0.0092	0.0231	0.0663	0.8867

1M	0.1694	-0.0293	-0.4132	-0.6475	0.6169
2M	-0.6962	0.1148	0.4819	0.0002	0.5196
3M	0.6906	0.1368	0.5042	0.2598	0.4273
6M	-0.0535	-0.7700	-0.2453	0.4878	0.3258
12M	-0.0830	0.6118	-0.5317	0.5248	0.2465
DEM-ITL	0.0138	0.0091	0.0201	0.0651	0.8918
1M	0.2524	0.0513	-0.5393	-0.4509	0.6630
2M	-0.6978	0.3048	0.3807	-0.0423	0.5230
3M	0.2860	-0.6892	0.5105	0.0863	0.4185
6M	-0.1191	-0.0939	-0.4304	0.8454	0.2775
12M	0.5945	0.6486	0.3440	0.2696	0.1867
DEM-JPY	0.0139	0.0105	0.0255	0.0580	0.8921
1M	0.7092	-0.0628	-0.3035	0.0210	0.6328
2M	0.0301	-0.0089	0.4937	0.8515	0.1739
3M	0.0172	-0.0205	0.7702	-0.5221	0.3654
6M	-0.4656	0.7045	-0.1877	0.0305	0.5007
12M	-0.5282	-0.7065	-0.1888	0.0329	0.4302
USD-DEMESP	0.0102	0.0006	0.1170	0.3123	0.5600
1M	0.0276	-0.0619	0.4520	-0.5713	0.6817
2M	-0.4377	0.0285	-0.7365	0.0075	0.5149
3M	0.8053	0.1097	-0.1839	0.3708	0.4100
6M	-0.3771	0.4926	0.4367	0.5938	0.2680
12M	-0.1298	-0.8606	0.1696	0.4284	0.1737
USD-FRF	0.0089	0.0100	0.0168	0.0331	0.9312
1M	-0.6982	0.0842	-0.0409	0.0291	0.7092
2M	0.6039	0.5603	-0.1952	-0.1075	0.5212
3M	0.3022	-0.4638	0.7241	-0.1018	0.3986
6M	0.2249	-0.4441	-0.3961	0.7392	0.2210
12M	0.0766	-0.5163	-0.5282	-0.6563	0.1332
USD-GBP	0.0548	0.0495	0.0310	0.0182	0.8465
1M	0.1346	0.5494	0.0637	-0.5264	0.6316
2M	-0.5690	-0.6063	0.0036	-0.1297	0.5402
3M	0.7627	-0.4038	0.0520	0.2803	0.4171
6M	-0.1523	0.2882	-0.7277	0.5268	0.2943
12M	-0.2306	0.2906	0.6809	0.5916	0.2207
GBP-DEM	0.0166	0.0232	0.0095	0.0535	0.8973
1M	0.0636	0.3257	-0.0004	-0.6406	0.6925
2M	-0.5004	-0.6825	-0.0394	0.1533	0.5087
3M	0.7505	-0.1794	0.2818	0.4107	0.3956
6M	-0.0457	0.3897	-0.7288	0.4900	0.2737
12M	-0.4246	0.4941	0.6229	0.3967	0.1740
USD-JPY	0.0083	0.0104	0.0045	0.0310	0.9459

Table 1: (continued) Eigenvectors and Normalized Eigenvalues

Table 2: **Relative Importance of Factors**

	Term Volatility	Total Variance Explained(%)	Factor 1(%)	Factor 2(%)	Factor 3(%)
	1M	100.0	94.2	4.6	1.1
	2M	98.3	91.8	0.0	6.5
	3M	92.4	79.0	12.0	1.3
	6M	94.8	57.5	29.6	7.7
	1YR	85.3	53.0	27.3	5.0
AUD	Average	97.3	86.4	7.7	3.2
	1M	99.8	93.8	3.4	2.7
	2M	99.0	98.2	0.7	0.1
	3M	99.5	98.3	0.1	1.1
	6M	99.0	97.2	0.1	1.7
	1YR	100.0	86.7	12.1	1.2
CAD	Average	99.5	95.1	3.0	1.4
	1M	100.0	95.9	3.4	0.6
	2M	98.8	95.6	0.1	3.1
	3M	96.6	92.6	3.4	0.7
	6M	96.1	82.0	11.4	2.6
	1YR	91.1	65.3	17.3	8.5
CHF	Average	98.3	92.6	3.9	1.8
	1M	100.0	97.1	2.5	0.5
	2M	99.7	95.6	0.2	3.8
	3M	96.3	93.5	2.8	0.0
	6M	95.4	82.1	9.3	4.0
	1YR	85.4	68.5	13.5	3.3
DEM	Average	98.4	93.9	2.9	1.6
	1M	98.6	89.8	8.7	0.2
	2M	91.2	90.9	0.1	0.2
	3M	99.6	81.5	5.9	12.2
	6M	93.7	68.4	14.3	11.1
	1YR	89.4	57.5	17.3	14.6
DEM-CHF	Average	95.9	83.7	7.2	5.1

	Term Volatility	Total Variance Explained(%)	Factor 1(%)	Factor 2(%)	Factor 3(%)
	1M	99.8	89.9	8.7	1.2
	2M	97.1	94.5	0.1	2.5
	3M	96.2	91.0	2.2	3.1
	6M	95.4	83.1	11.7	0.7
	1YR	96.7	74.7	16.0	6.0
DEM-FRF	Average	97.6	88.7	6.6	2.3
	1M	99.9	91.6	7.4	0.9
	2M	97.3	95.5	0.0	1.9
	3M	96.2	90.9	2.5	2.9
	6M	95.4	81.0	13.3	1.0
	1YR	95.7	66.6	22.1	7.0
DEM-ITL	Average	97.7	89.2	6.5	2.0
	1M	99.8	95.1	2.9	1.8
	2M	97.0	95.5	0.0	1.4
	3M	96.4	92.2	0.2	3.9
	6M	99.8	59.6	36.0	4.1
	1YR	80.4	65.2	8.8	6.3
DEM-JPY	Average	97.6	89.2	5.8	2.5
	1M	97.9	93.3	0.1	4.5
	2M	100.0	6.2	83.3	10.5
	3M	100.0	32.6	37.1	30.3
	6M	98.3	95.3	0.2	2.8
	1YR	97.2	93.1	0.3	3.8
DEMESP	Average	98.9	56.0	31.2	11.7
	1M	100.0	96.8	2.4	0.8
	2M	99.3	95.8	0.0	3.5
	3M	96.5	93.4	2.7	0.3
	6M	95.7	78.3	13.7	3.8
	1YR	82.1	66.6	14.4	1.1
FRF	Average	98.1	93.1	3.3	1.7

Table 2: (continued) Relative Importance of Factors (continued)

	Term Volatility	Total Variance Explained(%)	Factor 1(%)	Factor 2(%)	Factor 3(%)
	1M	100.0	94.0	0.1	5.9
	2M	99.5	86.2	5.8	7.5
	3M	90.1	80.7	6.4	3.0
	6M	78.5	60.2	14.2	4.0
	1YR	63.4	33.4	29.3	0.7
GBP	Average	95.1	84.7	5.0	5.5
	1M	99.9	94.2	3.9	1.8
	2M	98.1	94.7	0.3	3.1
	3M	94.4	89.8	2.4	2.2
	6M	94.6	77.8	14.9	1.9
	1YR	92.4	62.7	26.9	2.8
GBP-DEM	Average	97.4	89.7	5.3	2.3
	1M	100.0	97.0	2.7	0.2
	2M	99.2	97.0	0.3	1.9
	3M	96.8	93.3	3.3	0.2
	6M	97.1	86.1	9.0	1.9
	1YR	91.8	72.9	12.4	6.4
JPY	Average	98.7	94.6	3.1	1.0

Table 2: (continued) Relative Importance of Factors (continued)

Table 3: **Estmated ARCH(2) Parameters and Errors**

	α_0	α_1	α_2	a
	0.0000	0.2013	0.0018	-0.0001
	(0.0000)	(0.0028)	(0.0008)	(0.0000)
	0.0001	0.2587	0.0011	-0.0010
	(0.0000)	(0.0021)	(0.0002)	(0.0000)
	0.0011	0.1473	0.0006	-0.0002
AUD	(0.0000)	(0.0018)	(0.0003)	(0.0000)
	0.0000	0.3556	0.1798	-0.0002
	(0.0000)	(0.0044)	(0.0035)	(0.0000)
	0.0002	0.0609	0.0547	-0.0005
	(0.0000)	(0.0025)	(0.0027)	(0.0000)
	0.0020	0.7520	0.0727	-0.0055
CHF	(0.0000)	(0.0090)	(0.0026)	(0.0001)
	0.0000	0.5276	0.1496	-0.0004
	(0.0000)	(0.0054)	(0.0026)	(0.0000)
	0.0001	0.0315	0.0973	-0.0004
	(0.0000)	(0.0024)	(0.0027)	(0.0000)
	0.0023	0.8640	0.0515	-0.0074
DEM	(0.0000)	(0.0103)	(0.0020)	(0.0001)
	0.0002	0.4792	0.0007	0.0002
	(0.0000)	(0.0051)	(0.0011)	(0.0000)
	0.0003	0.2658	0.0005	0.0005
	(0.0000)	(0.0042)	(0.0025)	(0.0000)
	0.0037	0.3706	0.0007	-0.0035
DEM-CHF	(0.0000)	(0.0051)	(0.0015)	(0.0002)
	0.0004	0.1937	0.0005	-0.0003
	(0.0000)	(0.0050)	(0.0000)	(0.0000)
	0.0013	0.0852	0.0005	-0.0003
	(0.0000)	(0.0028)	(0.0018)	(0.0001)
	0.0121	0.4344	0.0504	-0.0069
DEM-FRF	(0.0001)	(0.0064)	(0.0027)	(0.0003)

	α_0	α_1	α_2	a
	0.0001	0.4247	0.0009	0.0005
	(0.0000)	(0.0045)	(0.0025)	(0.0000)
	0.0005	0.2409	0.0006	-0.0003
	(0.0000)	(0.0031)	(0.0033)	(0.0001)
	0.0067	0.2269	0.1043	-0.0031
DEM-ITL	(0.0000)	(0.0036)	(0.0034)	(0.0002)
	0.0001	0.1336	0.0000	-0.0001
	(0.0000)	(0.0030)	(0.0054)	(0.0000)
	0.0002	0.1584	0.0009	0.0004
	(0.0000)	(0.0031)	(0.0009)	(0.0000)
	0.0029	0.1724	0.0431	-0.0016
DEM-JPY	(0.0000)	(0.0035)	(0.0022)	(0.0001)
	0.0001	0.4138	0.1652	-0.0003
	(0.0000)	(0.0041)	(0.0034)	(0.0000)
	0.0002	0.0138	0.0508	-0.0006
	(0.0000)	(0.0027)	(0.0022)	(0.0000)
	0.0036	0.1601	0.4218	-0.0068
FRF	(0.0000)	(0.0040)	(0.0085)	(0.0002)
	0.0002	0.2189	0.0009	-0.0003
	(0.0000)	(0.0036)	(0.0018)	(0.0000)
	0.0002	0.2121	0.0009	0.0008
	(0.0000)	(0.0029)	(0.0020)	(0.0000)
	0.0035	0.2532	0.0535	-0.0057
GBP	(0.0000)	(0.0042)	(0.0021)	(0.0002)
	0.0001	0.1965	0.2524	-0.0002
	(0.0000)	(0.0032)	(0.0036)	(0.0000)
	0.0003	0.1020	0.0000	-0.0002
	(0.0000)	(0.0043)	(0.0016)	(0.0000)
	0.0061	0.0636	0.0007	0.0001
GBP-DEM	(0.0000)	(0.0018)	(0.0017)	(0.0002)
	0.0001	0.2095	0.0006	0.0001
	(0.0000)	(0.0031)	(0.0026)	(0.0000)
	0.0001	0.0314	0.0973	-0.0004
	(0.0000)	(0.0033)	(0.0036)	(0.0000)
	0.0036	0.2808	0.1029	-0.0007
JPY	(0.0000)	(0.0042)	(0.0023)	(0.0002)

Table 3: (continued) Estmated ARCH(2) Parameters and Errors