Exercises: One-period arbitrage pricing

1. Consider a hypothetical country where the government has declared a "currency band" policy, in which the exchange rate between the domestic currency, denoted by XYZ, and the U.S. dollar is guaranteed to fluctuate in a prescribed band, namely

$$USD\ 0.95 \ \leq\ XYZ \quad \leq\ USD\ 1.05\ ,$$

for at least one year. Suppose also that the governement has issued 1-year notes denominated in the domestic currency which pay a simply compounded annualized rate of 30 %. Assuming that the coresponding interest rate for U.S. deposits is 6%, show that this market is not arbitrage-free in the "pure" sense. Describe the situation in terms of the Arrow-Debreu model. Propose some realistic scenarios that could make this pure arbitrage dissapear in practice.

2. (i) Show that the set of all probability measures on a finite state-space of M elements can be represented as a convex subset $\mathbf{P}$ of the Euclidean space $\mathbf{R}^M$. Given a security s defined by its price and its cash-flows, verify that the set of measures which are risk-neutral for this security corresponds to the intersection of the set $\mathbf{P}$ with a hyper-plane in $\mathbf{R}^M$. Similarly, show that the set of admissible risk-neutral measures for a securities market with N securities corresponds to the intersection of $\mathbf{P}$ with N hyper-planes.

(ii) Apply this analysis to the trinomial model of Chapter 1§6 – assuming that $S =$ \$100, $U = 1.10$, $M = 1.00$, $D = .80$, $R = .05$ and that a call option with strike \$105 is trading at a premium of $C =$ \$3.80. Show that if, instead, $C =$ \$1.00, there is an arbitrage opportunity.

3. According to *The Economist* (Sept. 7 - Sept 14, 1996), Ladbroke, a London betmaker, gave the following odds regarding the upcoming U.S. presidential election: Clinton 1-6, Dole 7-2, Perot 1-50. [For instance, Ladbroke pays one pound for every 6 pounds bet on Clinton if he wins.]. Calculate the corresponding risk-neutral probabilities for the victory of each candidate implied by these odds assuming that one of them will necessarily win.