Topics in Probability: Risk Management in Finance

Homework # 2

1. The "engine". Construct a trinomial tree for pricing currency option portfolios. The tree will allow for the following inputs

1. Implied volatilities data for at-the-money options.
2. Term deposit rates in domestic currency.
3. Term deposit rates in foreign currency.

For later use, build the tree in such a way that the spot volatility at any node can vary from 0 to 20% by adjusting the probabilities.

Check the accuracy of your tree by pricing an at-the-money 180-day Dollar call/Mark put with strike 1.4850. Use a volatility of 13.1%, a U.S. rate of 5.80 % and a Mark rate of 3.30%. (Volatility is annualized using 365 business days and deposits are continuously compounded.)

2. Pricing odd-date options. Brokers in the interbank market quote prices of at-the-money options with standard maturities (1m, 2m, 3m, 6m, 1yr). The available data usually consists of term deposits in the two currencies and the implied volatility curve $\sigma_i(T_1)$, $\sigma_i(T_2)$, $...\sigma_i(T_N)$. This exercise explores the pricing of "odd-date" options (i.e. options that do not fall on these dates.) In most models, the "model spot volatility" is constructed by interpolating linearly the variance, i.e.,

$$\sigma_f^2(T_1, T_2) = \frac{\sigma_i^2(T_2)\, T_2 - \sigma_i^2(T_1)\, T_1}{T_2 - T_1} .$$

Thus, $\sigma_f(T_1, T_2)$ is the **forward volatility** for the period.

A. Use the following Dollar/Yen data to price a 100 day option which is (a) at the money and (b) 10% out of the money.

Spot: 104.50 yen per dollar

USD rate: 5.50%

JPY rate: 0.70%

Implied volatility: 30 days 11.0%, 60 days 11.5%, 180 days 12%

B. Using the above model, calculate the amount (hedge ratio) of 60-day ATM options that can be purchased by selling the 100-day ATM using the model value.

C. Introduce bands of $\pm 2\%$ volatility on each side of the forward vols. In other words, assume that

$$\sigma_f - 0.02 \le \sigma(t) \le \sigma_f + 0.02$$

where σ_f is the forward volatility for the period. Price the 100-day option in two ways:

i. using the **uncertain volatility model** (UVM) with the above bands. Calculate both the worst-case buy-price and sell-price.

ii. Compute the buy/sell value of the 100-day option by pricing the 60-day / 100-day calendar spread with the uncertain volatility model and adding/subtracting the price of the 60-day option.

iii. Let λ be the hedge-ratio for the 60-day option computed in part A. Price the calendar spread consisting of λ 60-day options (long) and one 100-day option (short) using the UVM (sell price). Then add the cost of the λ options. Do the same thing for the buy price.

3. Transaction costs. Interest rates in country XYZ are 40% a year and you think the stock of its telephone company, "TEL-TEL", has a volatility of 50%. Bid-offer spreads on TEL-TEL are typically 2% of the price of the stock.

(i) Calculate the implied volatility at which you would write an option on TEL-TEL if you expect to delta-hedge with adjustments of the hedge made approximately every 3 days. What would be your bid on volatility (at what volatility would you buy options if you expect to Delta-hedge)?

(ii) Assuming that TEL-TEL is trading at 100, estimate the cost of delta-hedging a call-spread consisting of short 1 call with strike 90 and maturity 30 days and long 1 call with strike 110.