Homework # 3: Mathematics of Finance 1

Due in 3 weeks (end of term).

1. American options

A. Program a binomial tree for pricing derivative securities assuming that the underlying state variable X_n follows a multiplicative random walk, i.e.

$$X_{n+1} = X_n \cdot H_n \quad , \quad n = 1, 2, ..., N-1 \ ,$$

with

$$H_n = \begin{cases} U_n & \text{with prob. } P_U^n \\ D_n & \text{with prob. } P_D^n \ . \end{cases}$$

B. Apply the model to price and American option on S%P 500 with the following charateristics

- Type: American put
- Maturity: 90 days (on a 255 days/year basis)
- Strike price: 580
- Spot price : 600
- Interest rate: 5.90 %
- Dividend rate: 2.50 %
- Volatility : 15%

Comapre your result with that of a commercial pricing software. (You can use the following one available on Internet: http://www.intrepid.com/ robertl/option-pricer1.html)

C. Assume now that X_n represents the LIBOR interest rate. Price a **callable note** with the following charateristics:

- Maturity: 5 years
- Type: principal paid at the end 6-month coupons
- Coupon payment : 8% of principal
- Callable : after two years at par

Assume:

- LIBOR today : 5.50%
- LIBOR volatility : 10%
- LIBOR forward curve

1st yr	2nd yr	3rd year	4th yr	5th yr
5.50	5.75	6.00	6.00	6.50

2. Exotic Options

A. The value of an at-the-money European call option on index XYZ with maturity 90 days, assuming a volatility of 15%, interest rate of 6% and dividend rate of 2% is 3.46% of the current index price.

(a) Suppose that, instead, you want to market the option as a *deferred-premium* option in which no premium is collected upfront but the customer pays a premium at the expiration date if the option is in-the-money. What would be the fair value of the deferred premium?

(b) Suppose that you design the option as follows: 1% of the value of the index is paid upfront as an "installment" and a a deferred premium is to be paid at maturity if the option ends in the money. What is the fair value that should be paid then?

B. (a) Price a reverse-knockout option on Dollar-Mark with the following charateristics:

- Type: Dollar put/ Mark call
- Maturity: 90 days
- Strike: 1.40 DEM/USD
- Knockout: 1.20 DEM/USD

Assume:

- Spot: 1.35 DEM/USD
- Volatility: 13%
- US interest rate: 5.80%
- DEM interest rate : 4.00%

(b) (For the brave only...) Suppose that you sell the knockout as a "deferred premium option", in which the holder pays a premium only if the exchange rate is $\leq$ 1.40 DEM/USD at maturity. What is the fair value of the premium?