Syllabus

TOPICS IN APPLIED PROBABILITY: THE MATHEMATICS OF FINANCIAL RISK MANAGEMENT

Instructor: Marco Avellaneda

Topics to be covered:

1. Probability and stochastic processes in financial modeling. Review of continuous-time finance and of the Black-Scholes-Merton theory of derivative asset pricing.
2. Liquidity constraints in dynamic risk-management: hedging with transaction costs and with limits on cash-market positions.
3. Stochastic volatility and its impact on pricing and hedging.
4. Implied Arrow-Debreu probabilities: the "inverse problem of Mathematical Finance".
5. The Uncertain Volatility Model: hedging exotic options and option portfolios using liquid options.

Approximate syllabus:

Lecture 1: Probabilities in Financial Mathematics. Modeling price fluctuations using local mean and variance of returns. Review of standard models: binomial random walk and Geometric Brownian Motion. (Source: class notes and J.Hull).

Lecture 2: Ito Calculus (introduction), Stochastic Integration, Ito processes. Introduction to continuous-time finance. (Source: class notes).

Lects. 3 & 4: The Black-Scholes option pricing theory as a result of cancelation of local risk. Sensitivities of the model (Delta, gamma, Rho, Vega). Extensions to other pricing problems, multifactor models, Arbitrage pricing theory and martingale measures (source: J. Hull (1992) Merton (1979) Harrison and Kreps (1980))

Lecture 5: Option pricing with transaction costs: lognormal model and binomial models (source: Leland (1985) Boyle and Vorst (1993))

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

Lecture 6: Hedging a derivatives warehouse in the presence of transaction costs (source: Hoggard, Whalley, Wilmott (1993), Avellaneda and Parás (1994), Bensaid Lesne Pages & Scheinkman (1992), Parás and Avellaneda (1995))

Lecture 7: Hedging under constraints on cash-market positions and application to the risk-management of digitals and barrier options (source: class notes)

Lecture 8: Stochastic volatility: the Hull & White approach and auto-regressive models (source: Hull & White (1989), Bollerslev, Engle and Nelson (1993).)

Lecture 9: Volatility smile and skew. Implied binomial trees and Arrow-Debreu probabilities. Synthetic structures (source: Dupire (1993), Rubistein (1994), Derman and Kani (1994))

Lecture 10: The "Uncertain Volatility Model": hedging under worst-case scenarios assuming a volatility range (Source: Avellaneda, Levy and Parás (1994), Garman (1976)).

Lecture 11: Application of UVM to volatility arbitrage and risk-management of exotic options (source: Avellaneda and Parás (1995)).

Lecture 12: Risk-management of financial portfolios. BIS recommendations for financial institutions for estimating market risk and J.P. Morgan's *RiskMetrics*. Towards a consistent theory for the risk-management of derivatives in financial portfolios (source: class notes, J.P. Morgan technical document).

Grades

There will be 3 homework assignments. These assignments will involve theoretical problems as well as some computer programming (MATLAB or C). Grades will be awarded according to quality of homework.