Ordinary Differential Equations, Fall 2012
Homework 9: Using computers to solve ODEs
75 pts + 25 pts extra credit = 100 pts

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Due Thursday **Dec. 13th 2012**

1 [45 points] Undamped Pendulum

Using a computer program of your choice, numerically solve the undamped pendulum equation

$$\theta''(t) = -\omega^2 \sin\theta(t),$$

for $\omega = 1$ over the interval for $1 \leq t \leq 50$ for several sets of initial conditions:

1. [20pts] $\theta(0) = \pi$ and $\theta'(0) = -0.001$. Plot the solution in the phase plane. From the plot, can you tell whether $\theta = \pi$ a stable or unstable point?
2. [25pts] $\theta(0) = \pi/4$ and $\theta'(0) = -1$. Plot the solution in the phase plane, and also plot our analytical solution for the trajectory from class to compare. Comment on your observations.

2 [55 points] Chaos in the Lorenz System

Consider the nonlinear system of ODEs (Lorenz equations)

$$\begin{aligned} x' &= \sigma\,(y - x) \\ y' &= x(\rho - z) - y \\ z' &= xy - \beta z, \end{aligned}$$

where σ, ρ and β are parameters. Write a Matlab code to solve these equations numerically, using a built-in higher-order method.

1. [20pts] Solve the equations for $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$ for $x(0) = 1$, $y(0) = 1$, and $z(0) = 1$ and plot the solution in the $x - y$ plane over the time interval $0 \leq t \leq 100$.
   [5pts Extra Credit] Also try to plot the solution in three-dimensions, in the $x - y - z$ coordinate system.
2. [10pts] Now repeat the same but change $x(0) = 1.00001$ and plot the trajectory with a different color on the same plot.
3. [20pts Extra Credit] Plot the Euclidean distance

$$d(t) = \sqrt{\left(x_1(t) - x_2(t)\right)^2 + \left(y_1(t) - y_2(t)\right)^2 + \left(z_1(t) - z_2(t)\right)^2}$$

between the two numerical solutions from part 1 (denoted with subscript 1) and part 2 (subscript 2) as a function of time. Comment on what you observe.
Hint: If you are using Matlab, look at the Matlab help page for the function "deval" and look at the example at the end.