Ordinary Differential Equations, Fall 2012
Homework 5 (50pts)
Intro to Systems of ODEs

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1 [10 points] Trivial System

Find a solution of the system of ODEs

$$\begin{aligned} y_1' &= -y_1 \\ y_2' &= y_1 + y_2 \end{aligned}$$

that satisfies the initial condition $\boldsymbol{y}(0) = (2, 1)$. *Hint: Solve the first equation and substitute the solution in the second equation. The initial condition means $y_1(0) = 2$ and $y_2(0) = 1$.*

2 [5 points] General Form

Write the system of ODEs

$$\begin{aligned} y_1' &= y_1 - t y_2 + e^t \\ y_2' &= t^2 y_1 - y_3 \\ y_3' &= y_1 + y_2 - y_3 + 2e^{-t} \end{aligned}$$

in the form $\boldsymbol{y}'(t) = \boldsymbol{A}(t)\boldsymbol{y}(t) + \boldsymbol{g}(t)$, i.e., identify the matrix $\boldsymbol{A}$ and the vector $\boldsymbol{g}$.

3 [15pts] Relationship to higher-order ODEs

Write the scalar third-order linear equation for the unknown function $x(t)$,

$$x''' + a_1(t)x'' + a_2(t)x' + a_3(t)x = b(t)$$

as a system of the form $\boldsymbol{y}'(t) = \boldsymbol{A}(t)\boldsymbol{y}(t) + \boldsymbol{g}(t)$.

4 [10pts] Linear Algebra

4.1 [2.5pts] Matrices and Vectors

Show that the vectors

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

are linearly independent and explain your method.

4.2 [7.5pts] Matrices and Vectors

Show that the vectors of functions

$$\boldsymbol{v}_1 = \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$$

are linearly independent for $-\infty < t < \infty$.

5 [10pts] Fundamental matrix

Show that

$$\boldsymbol{\Phi} = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$$

is a fundamental matrix for the system $\boldsymbol{y}' = \boldsymbol{A}\boldsymbol{y}$, where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$