A Simple Dynamical Model Capturing the Key Features of the Central Pacific El Niño

Nan Chen[a,1] **and Andrew J. Majda**[b,1]

[a]Department of Mathematics, and Center for Atmosphere Ocean Science, Courant Institute of Mathematical Sciences, New York University, New York, NY, USA; [b]Department of Mathematics, and Center for Atmosphere Ocean Science, Courant Institute of Mathematical Sciences, New York University, New York, NY, USA, and Center for Prototype Climate Modeling, New York University Abu Dhabi, Saadiyat Island, Abu Dhabi, UAE

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Central Pacific (CP) El Niño has been frequently observed in recent decades. It is characterized by anomalous warm sea surface temperature (SST) confined to the central Pacific and has different teleconnections from the traditional El Niño. Here, a simple model is developed and shown to capture the key mechanisms of the CP El Niño. The starting model involves coupled atmosphere-ocean processes that are deterministic, linear and stable. Then systematic strategies are developed for incorporating several major mechanisms of the CP El Niño into the coupled system. First, simple nonlinear zonal advection with no ad hoc parameterization of the background SST gradient is introduced that creates coupled nonlinear advective modes of SST. Secondly, due to the recent multidecadal strengthening of the easterly trade wind, a stochastic parameterization of the wind bursts including a mean easterly trade wind anomaly is coupled to the simple atmosphere-ocean processes. Effective stochastic noise in the wind burst model facilitates the intermittent occurrence of the CP El Niño with realistic amplitude and duration. In addition to the anomalous warm SST in the central Pacific, other major features of the CP El Niño such as the rising branch of the Walker circulation being shifted to the central Pacific and the eastern Pacific cooling with a shallow thermocline are all captured by this simple coupled model. Importantly, the coupled model succeeds in simulating a series of CP El Niño that lasts for 5 years, which resembles the two CP El Niño episodes during 1990-1995 and 2002-2006.

nonlinear zonal advection | strengthening of the easterly trade wind | effective stochastic noise | Walker circulation

The El Niño-Southern Oscillation (ENSO) is the most prominent interannual climate variability on earth, affecting much of the tropics and subtropics. It consists of a cycle of anomalously warm El Niño conditions and cold La Niña conditions with considerable irregularity in amplitude, duration, temporal evolution and spatial structure. The well-known traditional El Niño involves unusual warming of sea surface temperature (SST) anomalies in the equatorial eastern Pacific ocean. The atmosphere response to such anomalous ocean warming is the reversal of the Walker circulation, where its rising branch is shifted from the Maritime Continent to the eastern Pacific and results in increased precipitation near the west coast of America [1].

In recent decades, a different type of El Niño has been frequently observed [2, 3], which is called the central Pacific (CP) El Niño (also known as El Niño Modoki [4], warm pool El Niño [5], date line El Niño [6] or S-Mode [7]). The CP El Niño is characterized by positive SST anomalies confined to the central Pacific, flanked by colder waters to both east and west where the corresponding thermocline becomes shallow. Such zonal SST gradients result in anomalous two-cell Walker circulation over the tropical Pacific, with a strong convection region in the central Pacific. See the illustrations in Figure 1. Associated with these distinct warming and cooling patterns, the teleconnections of the CP El Niño are quite different from those of the traditional El Niño with major societal impact [4, 8].

While the traditional El Niño is mainly associated with thermocline variation, the CP El Niño appears more related to zonal advection and atmospheric forcing [3, 9–11]. Composite analysis of reanalysis data shows that zonal advection has a significant contribution to the SST tendency [5] and is particularly important during the initiation phase of CP El Niño [12]. On the other hand, accompanying with the increasing occurrence of the CP El Niño since 1990's, a recent multidecadal acceleration of easterly trade winds is observed [13–15], which is linked with the strengthening of the Walker circulation [16–18]. Due to the atmosphere-ocean coupling, the enhanced atmospheric circulation then induces the intensification of the westward ocean current with an increased upwelling of cold water in the eastern Pacific ocean, which tends to prevent the full eastward extension of the anomalous warm SST during El Niño phases and leads to the occurrence of El Niño in the central Pacific ocean.

Despite the significant impact of the CP El Niño, many general circulation models fail to distinguish this new type of El Niño from the traditional one [19] and very few statistical or dynamical ENSO models are able to simulate the CP El Niño with realistic features. In the present article, a simple modeling framework is introduced and developed that captures the key mechanisms of the CP El Niño. The starting model involves

Significance Statement

Central Pacific (CP) El Niño has been frequently observed in recent decades. It is characterized by anomalous warm sea surface temperature (SST) confined to the central Pacific and has different teleconnections from the traditional El Niño with major societal impact. Here, a simple modeling framework is developed and shown to capture the key mechanisms of the CP El Niño. In addition to the SST, other major characteristics of the CP El Niño such as the rising branch of the Walker circulation being shifted to the central Pacific and the eastern Pacific cooling with a shallow thermocline are all captured by this simple coupled model. Key features of the model are nonlinear advection of SST and effective stochastic wind bursts.

A.J.M. designed research. N.C. and A.J.M performed research. N.C. and A.J.M. wrote the paper.

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[1]To whom correspondence should be addressed. E-mail: chennan@cims.nyu.edu; jonjon@cims.nyu.edu

coupled ocean-atmosphere processes that are deterministic, linear and stable. Then systematic strategies are developed for incorporating several major causes of the CP El Niño into the coupled system. First, simple nonlinear zonal advection with no ad hoc parameterization of the background SST gradient is introduced that creates a coupled nonlinear advective mode of SST. Second, due to the recent multidecadal strengthening of the easterly trade wind, a mean easterly trade wind anomaly is included in the parameterization of the wind that couples to the atmosphere-ocean processes. The combined effect of the nonlinear zonal advection and the enhanced easterly trade wind enables the coupled model to generate regular patterns that are associated with the CP El Niño. Then a hierarchy of effective stochastic noise models [20] is incorporated into the parameterization of the wind bursts that facilitates the intermittent occurrence of the CP El Niño with realistic amplitude and duration.

The remainder of this article is organized as follows. After introducing the coupled model, both the regular patterns associated with the CP El Niño due to the deterministic nonlinear advection and the role of the effective stochastic wind bursts are studied. Details of model derivations, mathematical background of the effective stochastic wind bursts and more supporting information of the results are included in the *SI Appendix*.

Basic coupled model

ENSO model. The ENSO model considered in this article consists of a non-dissipative atmosphere coupled to a simple shallow-water ocean and SST budget [20]. This reads:
Interannual atmosphere model

$$\begin{aligned}&-yv-\partial_x\theta=0\\&yu-\partial_y\theta=0\\&-(\partial_x u+\partial_y v)=E_q/(1-\overline{Q}),\end{aligned}\qquad [1]$$

Interannual ocean model

$$\begin{aligned}&\partial_\tau U-c_1YV+c_1\partial_x H=c_1\tau_x\\&YU+\partial_Y H=0\\&\partial_\tau H+c_1(\partial_x U+\partial_Y V)=0,\end{aligned}\qquad [2]$$

Interannual SST model

$$\partial_\tau T+\underline{\mu\partial_x(UT)}=-c_1\zeta E_q+c_1\eta H,\qquad [3]$$

with

$$\begin{aligned}E_q&=\alpha_q T\\ \tau_x&=\gamma(u+u_p).\end{aligned}\qquad [4]$$

In the above model, x is zonal direction and τ is interannual time, while y and Y are meridional direction in the atmosphere and ocean, respectively. The u, v are zonal and meridional winds, θ is potential temperature, U, V, are zonal and meridional currents, H is thermocline depth, T is SST, E_q is latent heating, and τ_x is zonal wind stress. All variables are anomalies from an equilibrium state, and are non-dimensional. The coefficient c_1 is a non-dimensional ratio of time scales, which is of order $O(1)$. The term u_p in Eq. (4) is a stochastic wind burst perturbation described in the next section. The atmosphere extends over the entire equatorial belt $0 \le x \le L_A$ with periodic boundary conditions $u(0, y, \tau) = u(L_A, y, \tau)$, while the Pacific ocean extends over $0 \le x \le L_O$ with reflection

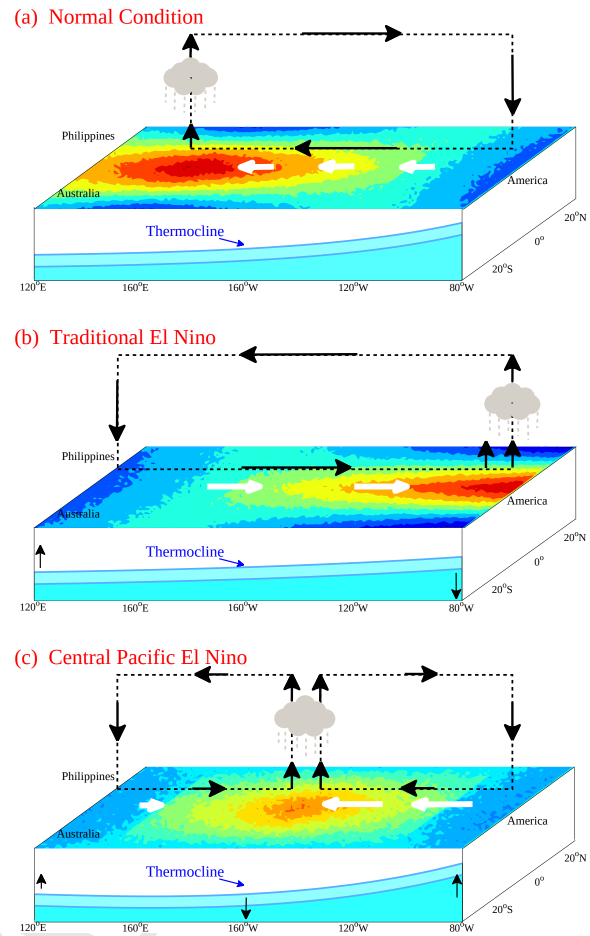

Fig. 1. Illustrations of (a) normal condition, (b) traditional El Niño and (c) CP El Niño. The black arrows with the dashed box show the Walker circulation and the white arrows show the direction of ocean current. Positive and negative SST anomalies are displayed in blue and red colors, respectively.

boundary conditions for the ocean model and zero normal derivative at the boundaries for the SST model.

The above model retains a few essential processes that model the ENSO dynamics in a simple fashion. Latent heating E_q that is proportional to SST T is depleted from the ocean and forces an atmospheric circulation. The resulting zonal wind stress τ_x in return forces an ocean circulation that can have feedback on the SST through thermocline depth anomalies H. This thermocline feedback is maximal in the eastern Pacific, as shown by the profile of η in Figure 2.

The model introduces unique theoretical elements such as a non-dissipative atmosphere consistent with the skeleton model for the MJO in the tropics [21, 22], valid here on the interannual timescale and suitable to describe the dynamics of the Walker circulation [23–25]. In addition, the meridional axis y and Y are different in the atmosphere and ocean as they each scale to a suitable Rossby radius. This allows for a systematic meridional decomposition and truncation of the flow into the well known parabolic cylinder functions, which keeps the system low-dimensional [26]. For instance, here model solutions Eq. (1) are projected and truncated to the first parabolic cylinder function of the atmosphere [21], while Eq. (2)–Eq. (3) are projected and truncated to the first parabolic cylinder function of the ocean [27].

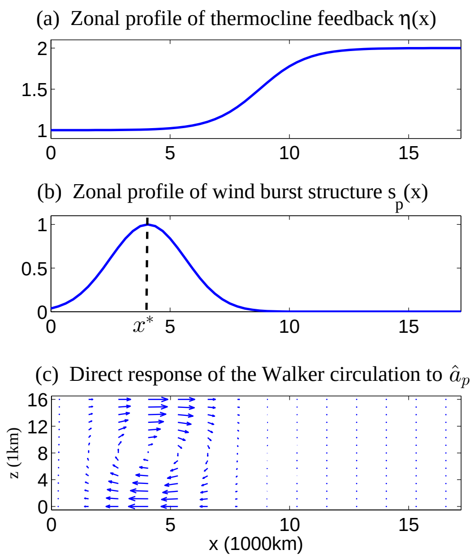

Fig. 2. Profiles of (a) the thermocline coefficient $\eta(x)$, (b) the zonal structure of the wind burst $s_p(x)$ with its peak at x^*, and (c) Direct response of the Walker circulation to a mean easterly trade wind anomaly $\hat{a}_p$.

The coupled system Eq. (1)–Eq. (4) without the nonlinear zonal advection in Eq. (3) was systematically studied in [20]. It succeeds in recovering the traditional El Niño and capturing the ENSO statistics in the eastern Pacific as in nature. Note that if the stochastic wind burst u_p is further removed, the resulting coupled system is linear, deterministic and stable. The *SI Appendix* of [20] provides detailed derivations of the model from an asymptotic expansion as well as the low-order meridional truncation [28].

The observational significance of the zonal advection has been shown for the CP El Niño [5, 12]. Yet, unlike the previous works [29, 30] where the advection is mostly linear and requires ad hoc parameterization of the background SST gradient, a simple nonlinear advection is adopted in Eq. (3) that contributes significantly to the SST tendency. Such nonlinear advection provides the mechanism of transporting anomalous warm water to the central Pacific region by the westward ocean zonal current. Importantly, when stochasticity is included in the wind burst u_p, this nonlinear zonal advection involves the contribution from both mean and fluctuation, the latter of which is usually ignored in the previous works.

Stochastic wind burst model. Stochastic parameterization of the wind bursts with speed u_p are added to the model that represent both the recent multidecadal strengthening of the easterly trade and several important ENSO triggers such as westerly wind bursts, easterly wind bursts, as well as the convective envelope of the MJO. This reads:

$$u_p = a_p(\tau)s_p(x)\phi_0(y), \qquad [5]$$

with amplitude $a_p(\tau)$ and fixed zonal spatial structure $s_p(x)$ shown in Figure 2. Here, $\phi_0(y)$ has a Gaussian profile centered as the equator and it equals to the first parabolic cylinder function of the atmosphere (See *SI Appendix*). Both the wind bursts perturbations [31] and the strengthening of the trade wind [13, 14] are localized over the western equatorial Pacific according to the observations and for simplicity they share the same zonal extent.

The evolution of wind burst amplitude a_p reads:

$$\frac{da_p}{d\tau} = -d_p(a_p - \hat{a}_p(T_W)) + \sigma_p(T_W)\dot{W}(\tau), \qquad [6]$$

where d_p is noise dissipation and $\dot{W}(\tau)$ is a white noise source. The amplitude of the wind burst noise source σ_p can either be a constant or depends on T_W, which is the average of SST anomalies in the western half of the equatorial Pacific ($0 \le x \le L_O/2$). The term $\hat{a}_p < 0$ represents the mean strengthening of the easterly trade wind. Corresponding to a nonzero constant easterly trade wind $\hat{a}_p < 0$, the direct response of the Walker circulation at the equatorial Pacific ocean is shown in Panel (c) of Figure 2, which is computed based on first baroclinic mode structure in the vertical direction (See *SI Appendix*).

CP El Niño model, the deterministic nonlinear advective modes

Consider an intensification of the easterly trade wind with a constant amplitude $a_p \equiv \hat{a}_p < 0$, where the stochastic effect σ_p is set to be zero in Eq. (6). Without the randomness in the wind burst model, the nonlinear advection becomes deterministic. Note that the nondimensional value $a_p = -0.25$ adopted below is roughly -0.94m/s at its maximum of the equator x^*, which is comparable to the observational record [13, 14].

The solution of the coupled model illustrates three different spatial-temporal structures depending on the strength of both the easterly trade wind a_p and the nonlinear zonal advection μ. See the phase diagram in Panel (a) of Figure 3.

In regime I, the steady state solution has constant values at each longitude. Particularly, with a suitably strong a_p, even without the nonlinear advection, the anomalous warm SST is shifted to the central-eastern Pacific region (Panel (b)). The corresponding ocean zonal current is westward and the rising branch of the Walker circulation is shifted to the central-eastern Pacific region (See *SI Appendix*).

With a nonzero zonal advection μ and a suitably strong easterly trade wind a_p, all the atmosphere, ocean and SST anomaly fields become time-periodic, and the period is much longer than 2 years (Regime II and Panel (c)). In each period, the positive SST anomaly develops from central-eastern Pacific and evolves slowly towards the western Pacific where it arrives at the maximum value. The corresponding ocean zonal current is westward at the anomalous warm SST phase and the rising branch of the Walker circulation is shifted towards the west accompanying with the warm water (See *SI Appendix*). It is worth noting that, except at the end of each period, the eastern Pacific remains cool from the ocean upwelling, which is one of the features of the CP El Niño.

When both μ and a_p are sufficiently large, the steady state solution shows regular oscillation patterns with period around 1.6 years (Regime III and Panel (d)). Within each period, warm water is transported westward and the maximum of anomalous warm SST is at the central Pacific.

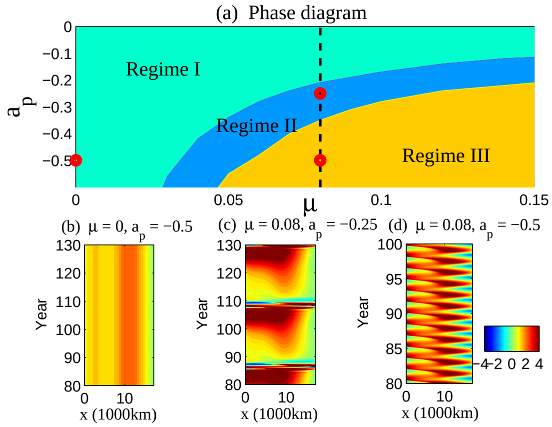

Fig. 3. (a) Phase diagram of a_p and μ, where $a_p \equiv \hat{a}_p$ is a negative constant, representing a mean easterly trade wind. Here $d_p = 3.4$ is fixed. Panel (b)-(d) show the three typical spatial-temporal SST patterns at the equator associated with each regime, corresponding to the three red dots in Panel (a). The corresponding atmospheric wind, ocean zonal current and thermocline feedback are included in *SI Appendix*. Note that the range of the y-axis in Panel (d) is different from that in Panel (b) and (c).

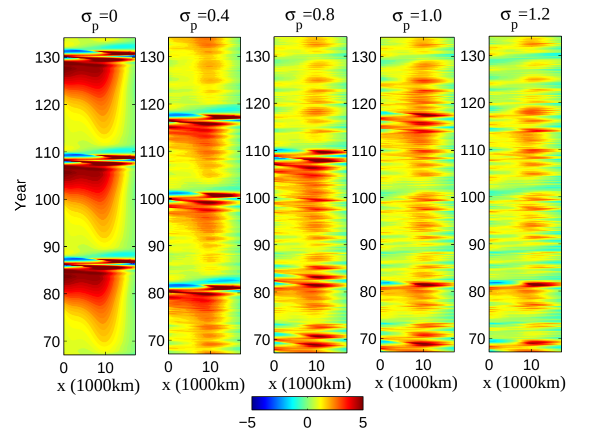

Fig. 4. Hovmoller diagrams of SST field at the equator with different levels of additive noise. Here $a_p = -0.25$ and $d_p = 3.4$ are fixed. The seeds of random number generators in different columns are kept to be the same.

The effective stochastic wind burst and the occurrence of CP El Niño

The deterministic nonlinear advection with an intensified easterly trade wind is able to generate regular patterns that are associated with the CP El Niño. However, the irregularity of nature and the intermittent occurrence of CP El Niño are not captured. Effective stochastic wind bursts help generate a more realistic CP El Niño.

Additive noise. First, additive noise is adopted in the stochastic wind burst model Eq. (6), where the mean easterly trade wind intensification $\hat{a}_p = -0.25$ is fixed and the wind burst noise σ_p is a constant that has no dependence on T_W.

The SST field shown in Figure 4 becomes more irregular with a gradual increase of the stochastic noise amplitude σ_p. When σ_p is around $\sigma_p = 1.0$, most of the anomalous warm SST is located in the central Pacific region and each single event resembles the SST pattern associated with the CP El Niño. It is shown in the *SI Appendix* that both the nonlinear advection and the easterly mean trade wind are the necessary ingredients in generating CP El Niño. It is also shown there that the CP El Niño disappears if the amplitude σ_p of the stochastic noise is too large.

In Figure 5, the Hovmoller diagrams for different fields are shown with $\hat{a}_p = -0.25$ and $\sigma_p = 1.0$. Most events with anomalous positive SST in the central Pacific represent CP El Niño. At these phases, the surface westerly wind and easterly wind converge in the central Pacific region, where the rising branch of the Walker circulation is formed (See Figure 7 in the next section). The westward zonal ocean current in the central Pacific region serves to transport the warm water to the central Pacific via the nonlinear zonal advection. The associated thermocline becomes deeper in the central Pacific but is shallower in the eastern Pacific, resulting in an upwelling of cold water. All these features are consistent with the observational record during the CP El Niño years.

To understand the role of the zonal nonlinear advection, the budget of SST tendency $dT/d\tau$ in Eq. (3) is studied. The positive flux divergence $-\mu\partial_x(UT)$ in the central-western Pacific region indicates its dominant role in transporting anomalous warm water to the central and western Pacific. On the other hand, the combined effect of the latent heat $-c_1\zeta E_q$ and thermocline feedback $c_1\eta H$ leads to the increase of the anomalous warm SST only in the eastern Pacific region. In fact, even the positive component of thermocline feedback itself does not extend as much to the central-western Pacific region as the flux divergence (not shown). These results are consistent with the observational findings that the CP El Niño appears more related to zonal advection than thermocline feedback [3, 9–11].

A two-state Markov jump model. To obtain the occurrence of the CP El Niño with realistic duration and amplitude, a two-state Markov jump process [20, 32] is adopted to model the stochastic wind burst Eq. (6). Here, both σ_p and $\hat{a}_p$ switch between one quiescent phase (State 0) and one active phase (State 1),

$$\begin{aligned} &\text{State 0:} \quad && \sigma_p = 0.2, \quad && \text{and} \quad && \hat{a}_p = 0, \\ &\text{State 1:} \quad && \sigma_p = 1.0, \quad && \text{and} \quad && \hat{a}_p = -0.25. \end{aligned}$$

The transition rates between the two states are functions of T_W, where a larger (smaller) T_W corresponds to a higher probability of transition from State 0 (1) to State 1 (0). This is because wind burst activity is usually favored by warmer SST in the western Pacific, and conversely [31, 33, 34]. The mathematical formulae of the transition probability and the profiles of switching rates are shown in the *SI Appendix*.

In Figure 6, Hovmoller diagrams of different fields for a 30-year period are shown. The stochastic switching process in the wind burst model leads to the intermittent occurrence of the CP El Niño with realistic amplitude and duration. Both a 1-year CP El Niño event ($t = 342$) and a series of CP El Niño events that lasts for 5 years (from $t = 329$ to $t = 334$) are simulated. The latter is particularly important since it resembles the two CP El Niño episodes as observed during 1990-1995 and 2002-2006, which have much longer durations than the traditional El Niño.

Similar to Figure 5, detailed analysis shows that the flux divergence is the main contributor to the occurrence of the central Pacific El Niño, where the westward zonal ocean current transports the anomalous warm water to the central Pacific

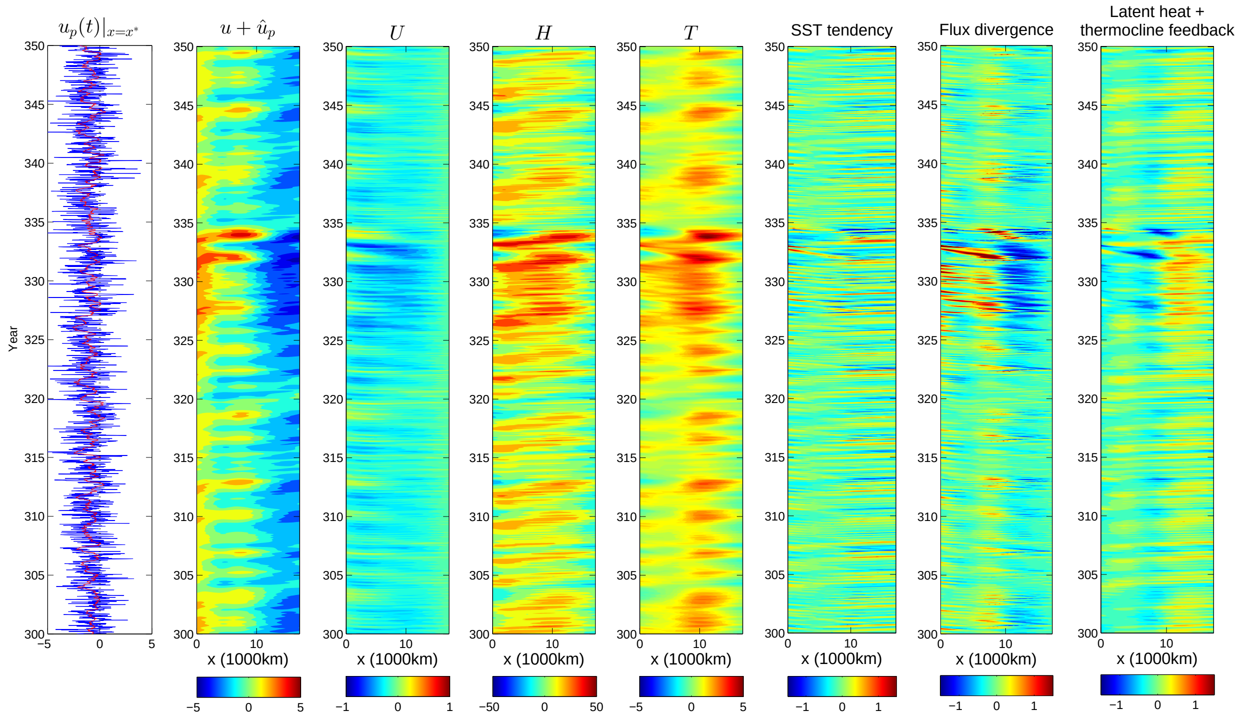

Fig. 5. Solutions of the coupled system with an additive noise in the wind burst model Eq. (6), where $\hat{a}_p = -0.25, \sigma_p = 1.0$ and $d_p = 3.4$. Different columns show the wind burst $u_p(t)$ at the peak x^* of its zonal profile with its 120-day moving average (red), Hovmoller diagrams of the atmospheric wind $u + \hat{u}_p$, the ocean zonal current U, the thermocline depth H, the SST field T, the SST tendency dT/dt, the flux divergence $-\mu\partial_x(UT)$ and the combined effect of the latent heat $-c_1\zeta E_q$ and thermocline feedback $c_1\eta H$ in Eq. (3). Here $\hat{u}_p$ is defined as $\hat{u}_p = \hat{a}_p s_p(x)\phi_0(y)$. All variables shown are at the equator.

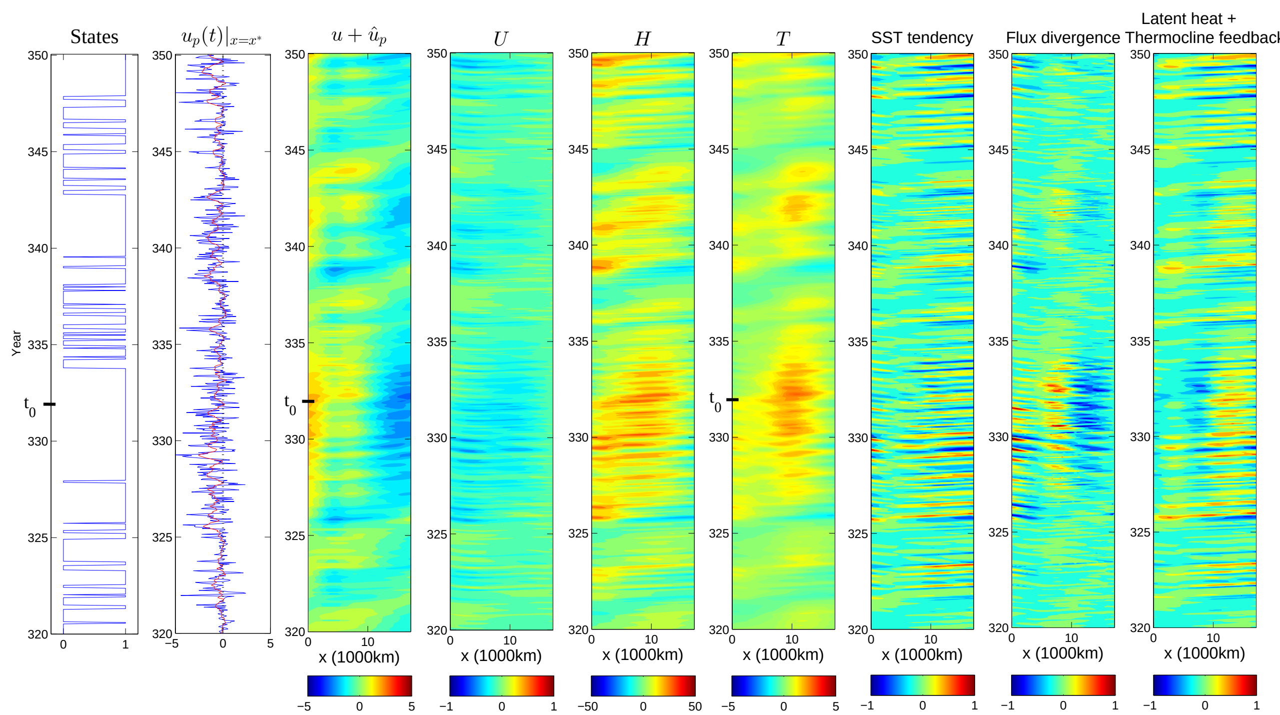

Fig. 6. Solutions of the coupled system with a two-state Markov jump process in the wind burst model Eq. (6). Different columns show the indicators of switching between the two states, the wind burst $u_p(t)$ at the peak x^* of its zonal profile with its 120-day moving average (red), Hovmoller diagrams of the atmospheric wind $u + \hat{u}_p$, the ocean zonal current U, the thermocline depth H, the SST field T, the SST tendency dT/dt, the flux divergence $-\mu\partial_x(UT)$ and the combined contribution due to the latent heat $-c_1\zeta E_q$ and thermocline feedback $c_1\eta H$ in Eq. (3). Here $\hat{u}_p$ is defined as $\hat{u}_p = \hat{a}_p(\tau)s_p(x)\phi_0(y)$. All variables shown are at the equator.

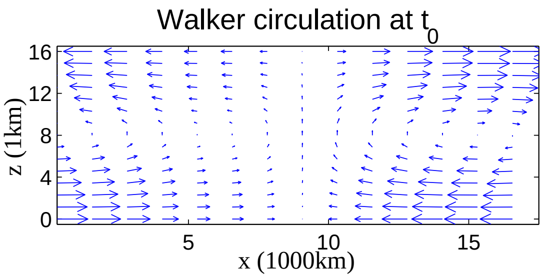

Fig. 7. The Walker circulation as a function of x over the equatorial Pacific and height z at time t_0 shown in Figure 6.

region and leads to the eastern Pacific cooling with a shallow thermocline depth.

The Walker circulation at the CP El Niño phase t_0 is shown in Figure 7. It is important to note that the rising branch is located at the central Pacific, where the atmosphere surface wind $u + \hat{u}_p$ is westerly in the western Pacific and easterly in the eastern Pacific. These features in the Walker circulation are consistent with observations during CP El Niño years [5] and are distinct from those associated with the traditional El Niño where the rising branch is located at the eastern Pacific.

Conclusion and discussion

A simple dynamical model is developed here that captures the key mechanism of the CP El Niño. Systematic strategies are developed for incorporating several major mechanisms of the CP El Niño into simple coupled atmosphere-ocean processes which are otherwise deterministic, linear and stable. First, a simple nonlinear zonal advection with no ad hoc parameterization of the background SST gradient is introduced that contributes to the SST tendency through a coupled nonlinear advective mode. Secondly, due to the recent multidecadal strengthening of the easterly trade wind, a stochastic parameterization of the wind bursts including a mean easterly trade wind anomaly is coupled to the simple atmosphere-ocean processes.

The deterministic nonlinear advection model involving an easterly trade wind anomaly shows regular patterns that are associated with the CP El Niño. The irregularity of nature is recovered by involving stochastic noise in the wind burst model. To capture the intermittent occurrence of the CP El Niño with realistic amplitude and duration, effective stochastic noise that accounts for its dependence on the strength of the western Pacific warm pool through a two-state Markov jump process [20, 32] is incorporated into the wind burst model. In addition to the anomalous warm SST in the central Pacific, other major features of the CP El Niño such as the rising branch of the Walker circulation being shifted to the central Pacific and the eastern Pacific cooling with a shallow thermocline are all captured by this simple coupled model. Importantly, the coupled model succeeds in simulating a series of CP El Niño that lasts for 5 years, which resembles the two CP El Niño episodes during 1990-1995 and 2002-2006.

It is worthwhile mentioning the possibility of developing a simple stochastic model for ENSO that involves both the CP El Niño and the traditional El Niño and La Niña. This probably can be achieved by combining the simple dynamical model for the CP El Niño developed here with the one studied in [20] that generates the traditional El Niño and super El Niño events with westerly wind bursts.

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1. Clarke AJ (2008) *An introduction to the dynamics of El Nino & the outhern Oscillation.* (Academic press).
2. Lee T, McPhaden MJ (2010) Increasing intensity of El Niño in the central-equatorial Pacific. *Geophysical Research Letters* 37(14).
3. Kao HY, Yu JY (2009) Contrasting eastern-Pacific and central-Pacific types of ENSO. *Journal of Climate* 22(3):615–632.
4. Ashok K, Behera SK, Rao SA, Weng H, Yamagata T (2007) El Niño Modoki and its possible teleconnection. *Journal of Geophysical Research: Oceans* 112(C11).
5. Kug JS, Jin FF, An SI (2009) Two types of El Niño events: cold tongue El Niño and warm pool El Niño. *Journal of Climate* 22(6):1499–1515.
6. Larkin NK, Harrison D (2005) On the definition of El Niño and associated seasonal average US weather anomalies. *Geophysical Research Letters* 32(13).
7. Guilyardi E (2006) El Nino–mean state–seasonal cycle interactions in a multi-model ensemble. *Climate Dynamics* 26(4):329–348.
8. Weng H, Behera SK, Yamagata T (2009) Anomalous winter climate conditions in the Pacific rim during recent El Niño Modoki and El Niño events. *Climate Dynamics* 32(5):663–674.
9. Yu JY, Kao HY (2007) Decadal changes of ENSO persistence barrier in SST and ocean heat content indices: 1958–2001. *Journal of Geophysical Research: Atmospheres* 112(D13).
10. Yeh SW et al. (2009) El Niño in a changing climate. *Nature* 461(7263):511–514.
11. Yeh SW, Kug JS, An SI (2014) Recent progress on two types of El Niño: observations, dynamics, and future changes. *Asia-Pacific Journal of Atmospheric Sciences* 50(1):69–81.
12. Su J, Li T, Zhang R (2014) The initiation and developing mechanisms of central Pacific El Niños. *Journal of Climate* 27(12):4473–4485.
13. England MH et al. (2014) Recent intensification of wind-driven circulation in the Pacific and the ongoing warming hiatus. *Nature Climate Change* 4(3):222–227.
14. Sohn B, Yeh SW, Schmetz J, Song HJ (2013) Observational evidences of Walker circulation change over the last 30 years contrasting with GCM results. *Climate Dynamics* 40(7-8):1721–1732.
15. Merrifield MA, Maltrud ME (2011) Regional sea level trends due to a Pacific trade wind intensification. *Geophysical Research Letters* 38(21).
16. L'Heureux ML, Lee S, Lyon B (2013) Recent multidecadal strengthening of the Walker circulation across the tropical Pacific. *Nature Climate Change* 3(6):571–576.
17. Choi JW, Kim IG, Kim JY, Park CH (2016) The recent strengthening of Walker circulation. *SOLA* 12(0):96–99.
18. McGregor S et al. (2014) Recent Walker circulation strengthening and Pacific cooling amplified by Atlantic warming. *Nature Climate Change* 4(10):888–892.
19. Ham YG, Kug JS (2012) How well do current climate models simulate two types of El Nino? *Climate dynamics* 39(1-2):383–398.
20. Thual S, Majda AJ, Chen N, Stechmann SN (2016) A simple stochastic model for El Niño with westerly wind bursts. *Proc. Natl. Acad. Sci.* Accepted.
21. Majda AJ, Stechmann SN (2009) The skeleton of tropical intraseasonal oscillations. *Proc. Natl. Acad. Sci.* 106:8417–8422.
22. Majda AJ, Stechmann SN (2011) Nonlinear dynamics and regional variations in the mjo skeleton. *Journal of the Atmospheric Sciences* 68(12):3053–3071.
23. Majda A, Klein R (2003) Systematic multiscale models for the Tropics. *J. Atmos. Sci.* 60:393–408.
24. Stechmann S, Ogrosky H (2014) The walker circulation, diabatic heating, and outgoing longwave radiation. *Geophys. Res. Lett.* 41:9097–9105.
25. Stechmann SN, Majda AJ (2015) Identifying the skeleton of the Madden-Julian oscillation in observational data. *Mon. Wea. Rev.* 143:395–416.
26. Thual S, Thual O, Dewitte B (2012) Absolute or convective instability in the equatorial Pacific and implications for ENSO. *Q. J. Meteorol. Soc.* 138.
27. Clarke AJ (2008) *An Introduction to the Dynamics of El Nino & the Southern Oscillation.* (Academic Press).
28. Majda A (2003) *Introduction to PDEs and Waves for the Atmosphere and Ocean.* (American Mathematical Soc.) Vol. 9.
29. Jin FF, An SI (1999) Thermocline and zonal advective feedbacks within the equatorial ocean recharge oscillator model for ENSO. *Geophysical research letters* 26(19):2989–2992.
30. Dewitte B, Yeh SW, Thual S (2013) Reinterpreting the thermocline feedback in the western-central equatorial Pacific and its relationship with the ENSO modulation. *Climate dynamics* 41(3-4):819–830.
31. Tziperman E, Yu. L (2007) Quantifying the dependence of westerly wind bursts on the large-scale tropical Pacific SST. *J. Climate* 20:2760–2768.
32. Majda AJ, Harlim J (2012) *Filtering Complex Turbulent Systems.* (Cambridge University Press).
33. Vecchi G, Harrison D (2000) Tropical Pacific sea surface temperature anomalies, El Nino, and equatorial westerly wind events. *J. Climate* 13:1814–1830.
34. Hendon H, Wheeler M, Zhang C (2007) Seasonal Dependence of the MJO-ENSO Relationship. *J. Climate* 20:543.