Moist Rayleigh-Bénard Convection: Investigation of Different Convective Regimes

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1 Introduction

Rayleigh-Bénard convection is a fascinating phenomenon that occurs when a fluid is heated from below, leading to the generation of fluid motion and the formation of intricate flow patterns. It has widespread occurrences in various engineering, industrial, and geophysical systems [1]. Its moist variant, Moist Rayleigh-Bénard convection, further characterizes the energy variation due to the phase change of water in the system, and thus more accurately represents the atmospheric convection. In this project, we focused on the Moist Rayleigh-Bénard convection and investigated the different convective regimes by simulating these models with different set-ups.

2 Rayleigh-Bénard Convection Model

The Rayleigh-Bénard Convection has been widely studied, and the system can be described through the set of following three equations: 1) the Navier-Stokes equation modified by the Boussinesq approximation; 2) the equation of continuity; and 3) the equation for the convection and diffusion of heat

$$\begin{aligned} \rho\frac{D\vec{u}}{Dt} = \rho_0(\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}) &= -\alpha\rho_0(T-T_0)g - \nabla P + \nu\nabla^2\vec{u} \\ \nabla\cdot\vec{u} &= 0 \\ \frac{DT}{Dt} = \frac{\partial T}{\partial t} + (\vec{u}\cdot\nabla)T &= \kappa\nabla^2 T \end{aligned} \tag{1}$$

By rescaling and substitutions we can obtain our standard RBC equations:

$$\begin{aligned} \frac{D\vec{u}}{Dt} &= B\hat{k} - \nabla P + \nu\nabla^2\vec{u} \\ \nabla\cdot\vec{u} &= 0 \\ \frac{DB}{Dt} &= \kappa\nabla^2 B \end{aligned} \tag{2}$$

3 Moist Rayleigh-Bénard Convection Model

However, the model above is only for the "dry case," in which water vapor in the atmosphere is not considered. The reason why water vapor is so important is that "of all atmospheric gases, only water is present in all three phases within the Earth's atmosphere. And the phase transitions of water are associated with the conversion of latent energy and sensible (thermal) energy. Thus, if we take water into account, the system becomes more complex as phase change happens.

To provide a more applicable model and avoid directly characterizing the highly non-linear process of phase transitions. We use two variables D and M to represent the "dry buoyancy" and "moist buoyancy" of the air, which corresponds to the buoyancy of the unsaturated air and saturated air respectively. The idea is that we use the moist buoyancy for the buoyancy if the air is moister than the saturation line, and we use the adjusted dry buoyancy if it is dryer than the saturation line. The full derivation of the model can be found in [2]. We are then able to describe the non-linear phase-change process through the two linear variables and a piece-wise linear buoyancy:

$$\begin{aligned}
\frac{d\vec{u}}{dt} &= B\hat{k} - \nabla P + \nu\nabla^2\vec{u} \\
\nabla \cdot \vec{u} &= 0 \\
\frac{dD}{dt} &= \kappa\nabla^2 D \\
\frac{dM}{dt} &= \kappa\nabla^2 M \\
B(M, D, z) &= \max(M, D - N_s^2 z)
\end{aligned} \tag{3}$$

4 MRBC Simulations

For the simulation, we used the PDE solver package Dedalus, which is a flexible framework for numerical simulations using spectral methods (A general sparse tau method) [3]. Due to the nature of spectral methods, it is excellent for complex problems on simple domains. Dedalus also provides MPI parallelization for efficient solutions. Both of the features are well-suited for our problems.

Due to the limitation of time, we focused mainly on the 2D simulations during the summer. We did a few 3D simulations for general insights, but we mainly explored the convective regimes in 2D. First, we want to see how the pattern of 2D convection changes with aspect ratio (A) and Rayleigh number (RA), so we did a 2D MRBC Parameter Space of A and RA.

4.1 2D MRBC Parameter Space

As we want to explore convection and cloud patterns, we have to find a way to quantify the "convection" and "clouds" in our model. There are many different ways to quantify convection, and one of the simplest ones is to use the spatial integration of kinetic energy over the domain, which corresponds to the motion of the fluid. And for the "clouds," we will define it as the extra buoyancy $B - D + N_s^2 z$ [1]. This quantity is proportional to the extra heat released due to condensation, thus being able to represent the amount of condensed water. Given these definitions, we are able to investigate the 2D convection pattern.

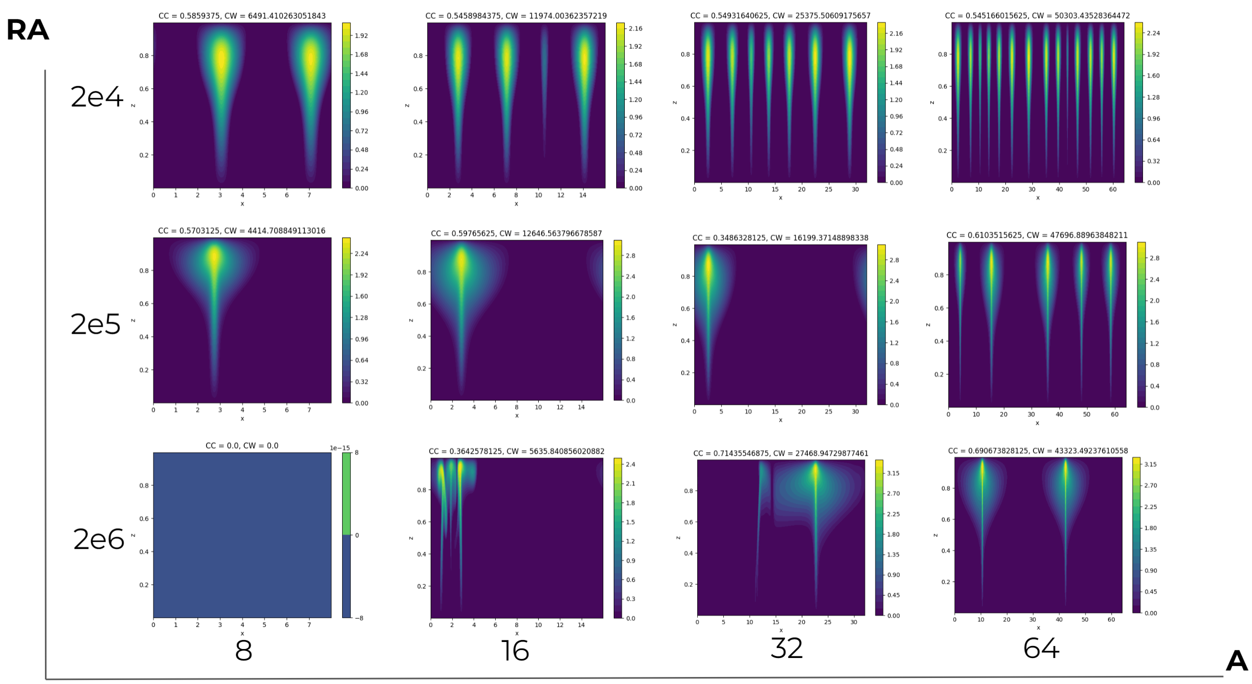

Figure 1: Parameter Space (A, RA): Final Frame of Clouds

The layout of the 2D MRBC parameter space of A and RA is shown in Figure 1. The contour plots show the extra buoyancy (clouds) at the last ticks of the simulations over the domain. Here we observe different cloud patterns: some of them have reached the equilibrium state while some haven't. The subfigures show the characteristic updrafts of MRBC. They also show the amount of condensed water and the number of cloud clusters in different settings, which typically decreases with RA and increases with A. Before analyzing the cloud patterns, we will examine the convection intensity pattern over time first.

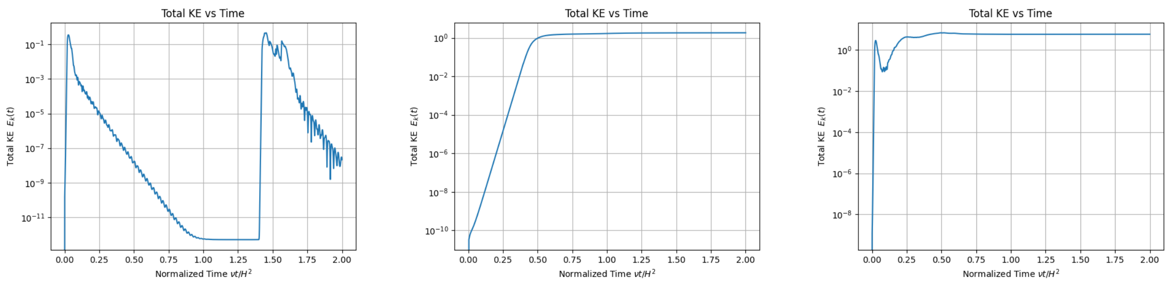

Figure 2: The convection intensity (total KE) vs time. Left (A=8, RA=2e6); middle (A=32, RA=2e4); right (A=64, RA=2e6)

Figure 2 shows the integrated kinetic energy over the domain versus time. There are three typical different patterns of kinetic energy evolution. In the left case, we have the aspect ratio being small (8) and the Rayleigh number being large ($2 \cdot 10^6$), and the kinetic energy demonstrates a "discharge-recharge" behavior, where it has outbursts followed by recessions. This behavior suggests the system cannot sustain a stable convection pattern and the convection disappear until the system is "recharged" and ready for the next convective outburst. In the middle case, the Rayleigh number is small and the Aspect ratio is large enough, so the system quickly reaches a stable convection pattern, corresponding to the unchanging kinetic energy after some time. On the right, both the aspect ratio and the Rayleigh number are large, and the system will give an initial outburst, followed by a short recession, and reach and stable convective pattern again.

4.2 More Simulations and Results

Much more simulations with different settings and extra factors [4, 5] have been done, and some yields very interesting results. If interested, please contact zn2021@nyu.edu

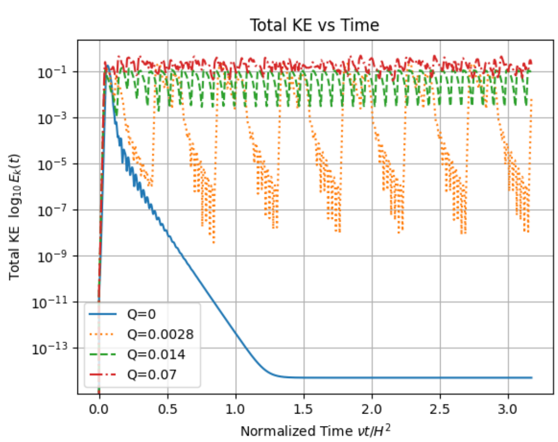

Figure 3: Impacts of four different levels of Radiation on total KE

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