Storm tracks in a moist shallow water atmosphere.

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Introduction

The goal of this project is to use a simple but nontrivial "toy model" of the atmosphere to study the behavior of mid-latitude storm tracks. Storm tracks are regions of relatively frequent atmospheric disturbances with a large spatial scale and short temporal duration (storms). Mid-latitude storm tracks occur in the Northern and Southern hemispheres and play an important role in the climate system. Storms redistribute energy, momentum, and moisture from the warm and humid equator towards the poles. Beyond their broader role in the energetics of the climate system, mid-latitude storms are a major source of extreme precipitation events in most of the non-equatorial world [3]. The climatic importance of mid-latitude storm tracks and the large human impact of their ensuing storm activity make understanding the behavior of mid-latitude storm tracks a worthwhile goal.

Climate models used to model storm track behavior vary in complexity from fairly constrained models on a plane to highly complex climate models taking into account everything from ice melt to atmospheric chemistry. This project aims to produce an idealized model of storm tracks operating on a spherical domain, taking into account the full effects of the Earth's rotation. Using shallow water approximations of the atmosphere, the model should reproduce storm track behavior. Using parametrizations of moist thermodynamics, the model should account for the effects of atmospheric moisture content on atmospheric fluid flow. Given a functional model, various moisture profiles are used to initialize the model and the ensuing atmospheric flows are compared to measure the response of mid-latitude storm tracks to moisture.

The Dry Model

The atmosphere is approximated as a shallow layer sitting atop the wide, spherical Earth. This allows for the shallow water approximation to be made. In shallow water, vertical accelerations of atmospheric parcels are considered necessarily smaller than horizontal (East-West and North-South) accelerations because of the shallowness of the fluid layer. Shallow water fluids conserve momentum and mass, allowing for partial differential equations of motion to be written. These PDEs lack analytical solutions but approximate numerical solutions can be obtained.

In this shallow water frame, storms are conceptualized as dynamical instabilities in the atmospheric fluid flow. Specifically, mid-latitude storms are baroclinic instabilities. The Charney-Stern-Pedlosky conditions perscribe the conditions met by fluid flows prone to baroclinic instability. A basic prerequisite is the presence of stratification in the fluid [4]. As a result, modeling mid-latitude storm tracks requires the use of a two-layer shallow water model. Each fluid layer is homogeneous (isentropic) and the two layers do not mix. Momentum and mass conservation equations are written for both layers, with the fluid motion being coupled by the gravitational impact of the upper layer on the lower layer.

Adding Moisture

The addition of moisture to this model requires approximating the thermodynamics of water and its movement in the atmosphere. The specific approximations described below are replicated from the 2012 work of Lambaerts et al [2]. Atmospheric water content is restricted to the lower layer of fluid. This mimics the real world abundance of water vapor in the lower troposphere and its relative sparsity higher in the atmosphere. Precipitation occurs when the local atmospheric moisture content (Q) exceeds some saturation value(Q_S). The Betts-Miller scheme is used, allowing excess moisture to rain out over some time scale τ (roughly an hour of simulation time in this model). When precipitation occurs, the phase change of water from vapor to liquid results in a latent heat release. This energy exchange is treated as a mass and momentum flux between the upper and lower layer. These approximations result in the equations of motion below.

$$\partial_t \vec{u_1} + (\vec{u_1} \cdot \nabla)\vec{u_1} + f\hat{k} \times \vec{u_1} = -g\nabla(h_1 + h_2) \tag{1}$$

$$\partial_t \vec{u_2} + (\vec{u_2} \cdot \nabla)\vec{u_2} + f\hat{k} \times \vec{u_2} = -g\nabla(h_1 + \alpha h_2) + \frac{\vec{u_1} - \vec{u_2}}{h_2}\beta P \tag{2}$$

$$\partial_t h_1 + \nabla \cdot (h_1 \vec{u_1}) = -\beta P \tag{3}$$

$$\partial_t h_2 + \nabla \cdot (h_2 \vec{u_2}) = \beta P \tag{4}$$

$$\partial_t Q + \nabla \cdot (Q \vec{u_1}) = -P \tag{5}$$

Lowercase h_1 and h_2 represent the thickness of the lower and upper layer respectively and $\vec{u} \cdot \hat{i}$ is the zonal (East-West) wind speed of a layer while $\vec{u_i} \cdot \hat{j}$ is its meridional (North-South) wind speed. α is a physical constant relating the relative density of the two fluid layers and β is a physical constant describing the latent heating associated with precipitation. Lowercase f is the coriolis parameter and g is the gravitational constant. Equations (1) and (2) are momentum conservation equations for the lower and upper layers respectively. Equations (3) and (4) are mass conservation equations for each layer and equation (5) imposes moisture conservation.

Running the Model

This model utilizes Dedalus, an open source python package that uses pseudo-spectral methods to solve PDEs [1]. The model is an initial value problem using one of Dedalus' Runge-Kutta solvers and its adaptive time-stepping capability. The model is initialized with a fluid flow that is stable but prone to baroclinic instability. Such a flow can be solved for by prescribing some desired wind profile (that satisfies the Charney-Stern-Pedlosky conditions) and using the IVP function in Dedalus to solve for other features of the flow. The specific flow chosen is shown in the figures below. A uniform initial moisture content Q_0 is set. A fixed perturbation is applied and the resulting fluid flow is numerically approximated by Dedalus.

Results and Next Steps

Running the model with initial relative humidities ($\frac{Q_0 - Q_s}{Q_s}$) ranging from 0 to 99.5%. Preliminary investigation of the fluid flow shows that very moist runs result in what appear to be stronger low pressure systems (storms) until some threshold after which the opposite phenomena is observed. This may be explained by a resolution of the flow's instability and local depletion of moisture in the perturbed region. Immediate next steps include quantifying the moisture depletion, instability strength and relative storm strength, as well as running the model with a stronger initial instability to better study this phenomena. Maintaining numerical stability while strengthening the physical instability may prove challenging. Further work includes better studying the energetics of the moist instability and evolving the equations of motion to better model the climate system as opposed to an IVP that results in storms that dissipate with time.

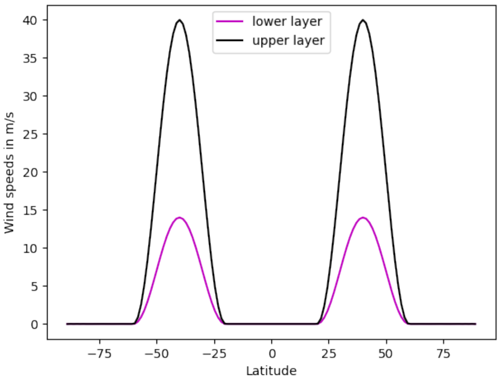

Figure 1: The initial zonal flow in the lower and upper layers as a function of latitude. The upper layer wind speed maxima in each hemisphere mimic the jet streams

Acknowledgments

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