

NEW YORK UNIVERSITY
COURANT INSTITUTE SCHOOL OF OF MATHEMATICS, COMPUTING, AND DATA SCIENCE
DEPARTMENT OF MATHEMATICS

Summer Undergraduate Research Experience (SURE) Program 2026

SURE Participant	Project Title	Faculty Mentor
Alex Hassett	Eigenvalue Exclusion for Sample Covariance Matrices with Independent Columns and Martingale-Difference Coordinates	Dr. Simona Diaconu
Yuliang Liu	Accelerating Centroid Molecular Dynamics via Generative Gibbs Sampling	Dr. Jonathan Weare
Dinh Khang Nguyen	Hirzebruch Signature Theorem and Milnor's Construction of an Exotic 7-Sphere	Dr. Sylvain Cappell
James Pagán	Complex Manifolds via Condensed Mathematics	Dr. Sylvain Cappell
Mark Shapiro	Defining the Higher K-Groups	Dr. Sylvain Cappell
Yihan (Ian) Wang	Faithfulness of the Burau Representation for B_4	Dr. Liming Pang
Guangxun Zhang	Zero-Shot SDE Recovery Through Conditional Velocity Drift Matching	Dr. Wenzheng Shi

Eigenvalue Exclusion for Sample Covariance Matrices with Independent Columns and Martingale-Difference Coordinates

SURE Participant: Alex Hassett

Faculty Mentor: Dr. Simona Diaconu

Sample covariance matrices estimate the mutual variance among multivariate data coordinates, with their eigenvalues forming the foundation of principal component analysis. While global spectral laws characterize the macro-level eigenvalue distribution, they fail to prevent isolated outliers from escaping the theoretical support. We address this strong spectral containment problem in the regime where data dimension and sample size scale proportionally.

We examine sample covariance matrices generated via deterministic linear maps of independent random vectors whose entries constitute martingale-difference sequences with unit conditional variance. The corresponding population covariance matrices maintain uniformly bounded operator norms. Assuming high-order moment constraints, quantitative support criteria, and structural stability, we establish that fixed compact intervals on the positive real axis—strictly separated from the deterministic comparison measure supports—contain no eigenvalues almost surely for large sample sizes. Building on the framework of Bai and Silverstein, our proof leverages quadratic form moment estimates for dependent variables to bound Stieltjes transforms near the real axis, yielding summable probability bounds for spectrum-free intervals. We present results under both uniform moment constraints and prescribed higher-order polynomial growth bounds.

Accelerating Centroid Molecular Dynamics via Generative Gibbs Sampling

SURE Participant: Yuliang Liu

Faculty Mentor: Dr. Jonathan Weare

Path-integral molecular dynamics (PIMD) captures nuclear quantum effects by representing each quantum particle as a cyclic ring polymer of P classical replicas, or beads, connected by harmonic springs, and samples that ring by integrating a fictitious Hamiltonian, which is expensive and decorrelates slowly. Generative Gibbs sampling for path integrals (GG-PI) accelerates this by drawing each bead from its distribution conditioned on all the others, using a generative model trained once for the purpose. That conditional distribution factorizes into the Boltzmann weight of the physical potential at the bead and a Gaussian centred on the midpoint of its two neighbours, and because alternate beads are conditionally independent, all odd or even beads can be drawn in parallel.

Centroid molecular dynamics (CMD) builds on PIMD to approximate quantum dynamics properties by evolving the ring polymer's centroid on the potential of mean force obtained by integrating out the non-centroid normal modes at fixed centroid. We extend GG-PI to CMD while keeping the original GG-PI structure intact. Our approach is to replace the exact centroid constraint with a harmonic restraint of stiffness K , which modifies the bead's conditional distribution in GG-PI by shifting its Gaussian off the neighbour midpoint toward satisfying the constraint and narrowing it, both through a dimensionless parameter μ proportional to K . Conditioning a single generative model on μ allows sampling across restraint strengths and reaching the exact-constraint limit $K \rightarrow \infty$; by extrapolation.

Validated against reference PIMD for liquid water, the sampler reproduces the ring radius of gyration to 1.7%, the mean force to 1.6%, and the librational and bending bands of the vibrational spectrum to within 2%, at an

effective sample size per second eight times PIMD's, and needs a third of the force evaluations per picosecond of trajectory that traditional adiabatic CMD requires.

Hirzebruch Signature Theorem and Milnor's Construction of an Exotic 7-Sphere

SURE Participant: Dinh Khang Nguyen

Faculty Mentor: Dr. Sylvain Cappell

The Hirzebruch signature theorem expresses the signature of the intersection form on $H^{2k}(M; \mathbb{R})$, for M a smooth, closed, oriented $4k$ -manifold, as a rational linear combination of the Pontryagin numbers of M . We develop the background needed to apply this theorem to the classification of smooth structures on spheres. Following Milnor, we construct a parallelizable $4k$ -manifold W with boundary ($k \geq 2$) by plumbing disk bundles over S^{2k} , and show that ∂W is a homology $(4k - 1)$ -sphere. Since ∂W is simply connected, the Hurewicz and Whitehead theorems give a homotopy equivalence $\partial W \simeq S^{4k-1}$ as $4k - 1 \geq 7$, Smale's work on the generalized Poincaré conjecture in dimensions ≥ 5 then shows that ∂W is homeomorphic to S^{4k-1} . We show, however, that ∂W is not diffeomorphic to S^{4k-1} . Because a diffeomorphism would allow W to be capped off with a disk, producing a closed smooth manifold whose k -th Pontryagin number is determined by the signature theorem but fails to be an integer. This exhibits smooth structures on S^{4k-1} that are not diffeomorphic to the standard one.

Complex Manifolds via Condensed Mathematics

SURE Participant: James Pagán

Faculty Mentor: Dr. Sylvain Cappell

The holomorphic functions defined on each open subset of a complex manifold are recorded by its structure sheaf. In this project, we construct the structure sheaf of a complex manifold using condensed mathematics, a recent theory of Clausen and Scholze. Condensed mathematics proposes that instead of understanding a space as a set with a topology, one should understand how totally disconnected spaces map into it.

This approach enlarges the categories of 'nice' topological Abelian groups and topological vector spaces, leading to categories with extremely well-behaved algebraic properties. Using condensed mathematics, Clausen and Scholze developed a unified framework for analytic geometry across all base fields by gluing analytic rings, analogous to how algebro-geometric spaces arise from gluing ordinary rings. We treat the relevant condensed mathematics from scratch, making it accessible to nonspecialists in algebraic geometry.

Defining the Higher K-Groups

SURE Participant: Mark Shapiro

Faculty Mentor: Dr. Sylvain Cappell

Since their independent introductions, the first few algebraic K-groups proved quite useful in disparate fields of math, for example, in geometric topology (in manifold classification), geometric analysis on manifolds, and number theory. Defining these groups generally thus became a focus of research efforts.

We outline the first steps made in higher algebraic K-theory: Daniel Quillen's solutions to the problem of defining the higher K-groups. Starting from the classical K-groups, we touch on Quillen's use of the 'plus construction' to define the higher K-groups before detailing a generalized definition via his novel 'Q

construction.' These methods stem from the realization that starting from some object one can construct a 'K-theory space,' the homotopy groups of which are the K-groups of the object.

Faithfulness of the Burau Representation for B_4

SURE Participant: Yihan (Ian) Wang

Faculty Mentor: Dr. Liming Pang

The reduced Burau representation ($\bar{\rho}_4 : B_4 \rightarrow GL_3(\mathbb{Z}[t^{\pm 1}])$) connects the algebraic structure of the four-strand braid group with the geometry of punctured disks. This project examines the recent proposed proof of its faithfulness by Bharathram, Birman, and Brendle.

We analyze the structure of their geometric argument, with particular attention to the reduction to the Brunnian subgroup and the use of point-pushing maps and parity conditions to control cancellation in intersection polynomials. We compare these methods with earlier geometric criteria for faithfulness and identify the key steps specific to the four-strand case. We further examine consequences for the Jones braid representation and revisit Korzun, Lanina, and Sleptsov's work on closed four-braids and the Jones unknot problem. This comparison clarifies which proposed constructions are excluded by faithfulness and which questions concerning unknot detection remain unresolved.

Zero-Shot SDE Recovery Through Conditional Velocity Drift Matching

SURE Participant: Guangxun Zhang

Faculty Mentor: Dr. Wenzheng Shi

We study recovery of drift and diffusion functions in stochastic differential equations (SDEs) from discretely observed sample paths. Existing methods based on local increments or predefined function libraries often struggle with nonlinear and time-dependent dynamics. We introduce Conditional Velocity Drift Matching (CVDM), a two-stage neural framework.

First, CVDM jointly learns drift and diffusion using Euler–Maruyama transition likelihood. It then freezes the diffusion and refines the drift by matching multi-step conditional displacement velocities between observed and simulated trajectories, capturing longer-horizon dynamics obscured by one-step noise. Across 60 SDEs of varying difficulty, CVDM outperforms Kramers–Moyal estimation, PyDaddy, sparse SINDy, and Tripura–Chakraborty sparse Bayesian regression, with the largest gains on nonlinear, out-of-dictionary, and time-dependent systems.