# Robust Control for Quantum Systems

## finding optimal control under noise

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## Introduction to quantum system

- simplest quantum system: **qubit**

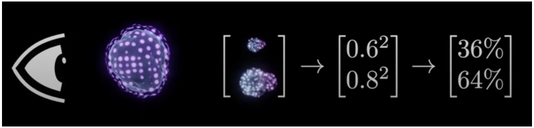
  classical bit is either 0 or 1, **qubit** can be both.
- state vector:

$$\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad |\psi|^2 = |\alpha|^2 + |\beta|^2 = 1, \quad \alpha, \beta \in \mathbb{C}.$$

- $\psi$ can have $> 2$ energy states.

examples:

$$\psi_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \psi_2 = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} \quad \psi_3 = \begin{pmatrix} \cos\theta \\ i \sin\theta \end{pmatrix}$$



## Introduction to quantum system

evolution of quantum state vector $\psi(t) \in \mathbb{C}^d$:

$$\text{Schrödinger's equation} \quad \dot{\psi}(t) = -iH(t; f(t))\psi(t).$$

- Hamiltonian $H$: $\mathbb{C}^{d\times d}$, Hermitian, $(t; f(t))$-dependent.
- $f(t)$: reflects our control.

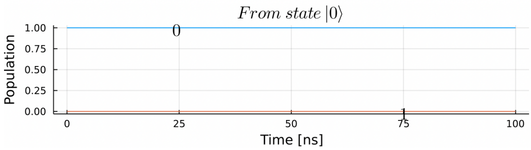
our Hamiltonian model:

$$H(t) = \omega \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + f(t) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

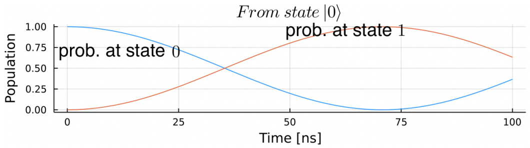
## Demonstration of "control":

$$\text{assume} \quad H(t) := f(t)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \text{start from} \quad \psi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

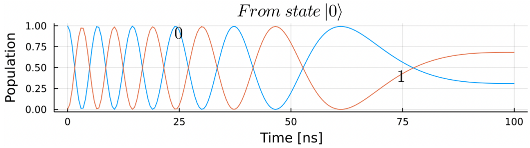
$f(t) \equiv 0$




$f(t) \equiv const$




$f(t) = c \cdot (100 - t)^2$



## Control problem

given:

$$\begin{cases} \dot{\psi} = -iH(t; f(t))\psi, \\ \psi(0) = \psi_0 := \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \end{cases}$$

we want:

$$\psi(T) = \psi_{tg} := \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

loss function:

$$L(f(t)) = 1 - \left|\langle \psi(T), \psi_{tg} \rangle\right|^2$$

Q: when does $L(f) = 0$? $\psi(T) = \psi_{tg}$!

## Noise attack

the optimization problem

$$\min_{f:[0,T]\to\mathbb{R}} L(f)$$

$$s.t. \quad \begin{cases} \dot{\psi} = -iH(t; f(t))\psi, \\ \psi(0) = \psi_0. \end{cases}$$

noise model:

$$\epsilon \sim \text{Unif}(-\delta, \delta), \qquad H(\epsilon) := H + \epsilon A$$

noise-aware loss function:

$$L_{NA} = \mathbb{E}^{\epsilon}[e^{L(f;\epsilon)}] = \frac{1}{2\delta}\int_{\epsilon} e^{L(f;\epsilon)} d\epsilon.$$

## Why is this challenging?

**updated optimization problem**

$$\min_{f:[0,T]\to\mathbb{R}} \mathbb{E}^{\epsilon}[e^{\mu L(f;\epsilon)}]$$

$$s.t. \quad \begin{cases} \dot{\psi} = -iH(t; f(t); \epsilon)\psi, \\ \psi(0) = \psi_0. \end{cases}$$

we use Juqbox, a Julia package!

- $f$ in infinite-dim space
- need $\partial_f L$ to update $f$
- ODE constraint
- integral $\int_\epsilon$ in loss

- approximate $f$ by finite basis
- get gradient by adjoint method
- quadrature/Monte-Carlo sampling

## Test problem: 1-dim uniform noise

### Hamiltonian

$$H(\epsilon) = \omega A + f(t)B + \epsilon C$$

where

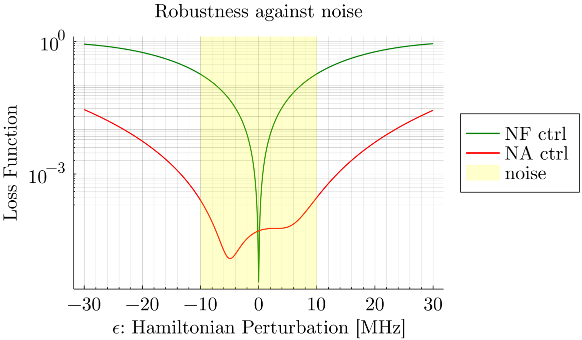
- $\omega A$ : natural evolution,
- $f(t)B$ : human control,
- $\epsilon C$ : uniform noise,
- $A, B, C$ are fixed, Hermitian matrices $\in \mathbb{C}^{3\times 3}$.

### our goal

start from $\psi(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, arrive at $\psi(T) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

## Comparison between NF and NA methods

- noise-free method is only accurate for $\epsilon \approx 0$;
- noise-aware method behaves robustly under large noise;
- good generalization ability.



## Test problem: high-dim noise

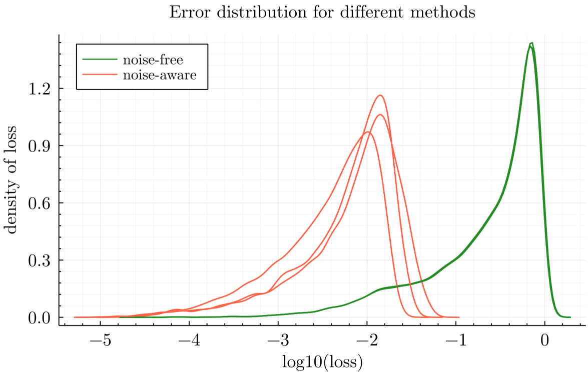
- noise model:

$$\begin{pmatrix} \epsilon_1 & \epsilon_4 & \\ \epsilon_4 & \epsilon_2 & \epsilon_5 \\ & \epsilon_5 & \epsilon_3 \end{pmatrix}$$

$$\epsilon_i \sim Unif(-\delta_i, \delta_i).$$

- curse of dimensionality:

–MC method.



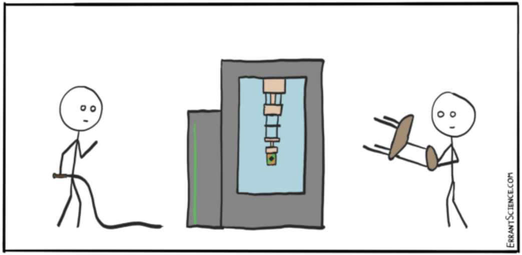
## Contribution and future work

contribution:

- implement risk-aware loss functions
- test on 1-dim noise
- test on higher-dim noise

future work:

- N-qubit system
- non-uniform noise
- new loss function
- ...




Two physicists trying to control a qubit in the lab

## References

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  *Quantum Computation and Quantum Information: 10th Anniversary Edition*.
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  Optimal control of closed quantum systems via b-splines with carrier waves, 2021.

## References

other risk aware loss functions we experimented on:

- risk-neutral:
  $$F(\alpha) := \mathbb{E}^{\epsilon}[L(\alpha, \epsilon)]$$
- risk-sensitive:
  $$F(\alpha) := \mathbb{E}^{\epsilon}[e^{L(f;\epsilon)}]$$
- mean-variance:
  $$F(\alpha) := \mathbb{E}^{\epsilon}[L(\alpha, \epsilon)] + \frac{\theta}{2} \cdot \mathrm{Var}[L(\alpha, \epsilon)]$$
- conditional value-at-risk:
  $$\mathrm{CVaR}_{\beta}[X] := \inf_{t \in \mathbb{R}}\{t + (1-\beta)^{-1}\mathbb{E}[(X-t)_{+}]\}$$