Information Theory for Neuroscience

Measuring the entropy of visual cortices

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Outline

1. Why entropy?
2. Algorithm
3. Discrete implementations
4. Continuous implementations
5. Neural data
6. Future work

Why Entropy?

What is entropy?

- Shannon entropy is a measure of “surprisal”
- Self entropy: or some random variable X following a distribution p

$$H(p,p) = -\sum_x p(x) \log p(x) \, .$$

- Cross entropy: for some random distribution p and an estimator distribution q

$$H(p,q) = -\sum_x p(x) \log q(x) \, .$$

Why Entropy?
Quantifying time reversal symmetry

- Kullback-Leibler divergence (KLD) - measure of distance from one probability distribution to a reference probability distribution.

$$D_{KL}(p \parallel q) = H(p, q) - H(p)$$

- Entropy production - the measure of the distance to time-reversal symmetry

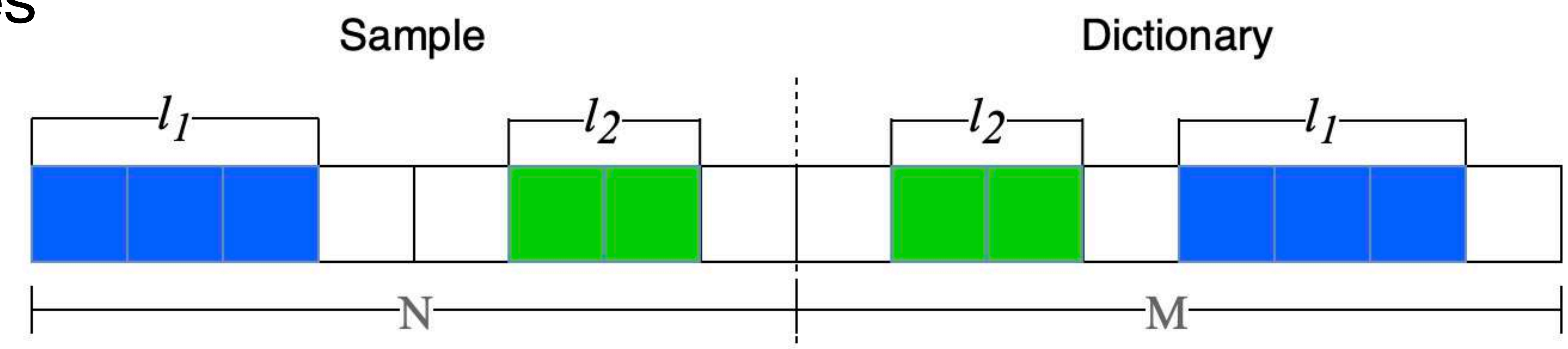
- For variable X' which is the flipped version of X and follows the distribution q,

$D_{KL}(p \parallel q) =$ Entropy production

Algorithm

Estimate the entropy when the distribution is unknown.

- Input series



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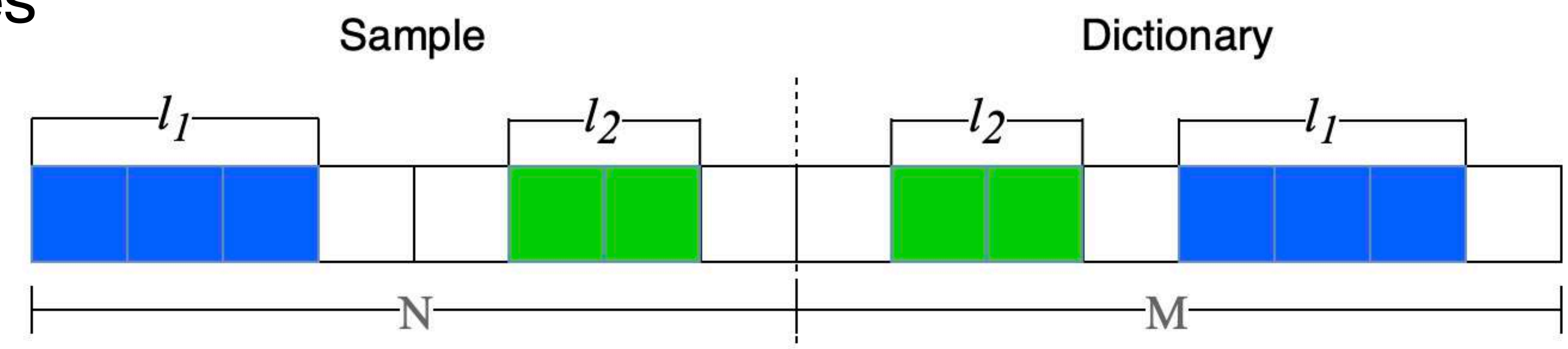

- Entropy estimator

$$H = \frac{log_2 M}{< l >}$$

Algorithm

Pattern matching estimator

- Data compression algorithm:

Lempel-Ziv 77 Factorization (LZ77)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
X =	a	b	c	a	b	a	b	a	b	a	b	a	b	a	b

$LZ^{77}(X)$ = [a,0], [b,0], [c,0], [0,2], [3,10]

Number of factors $C = 5$

Examples and derivations come from Stefano’s talk at Santa Fe, “The Other Side Of Entropy”

Algorithm

Pattern matching algorithm

$H \leq$ Information required to specify the factor of a finite sequence

$C =$ Number of factors

$$H \leq \frac{C}{N} log_2 N + \frac{1}{N} \sum_{i=1}^{C} log_2 l_i$$

Length of compressed data

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Pattern matching estimator

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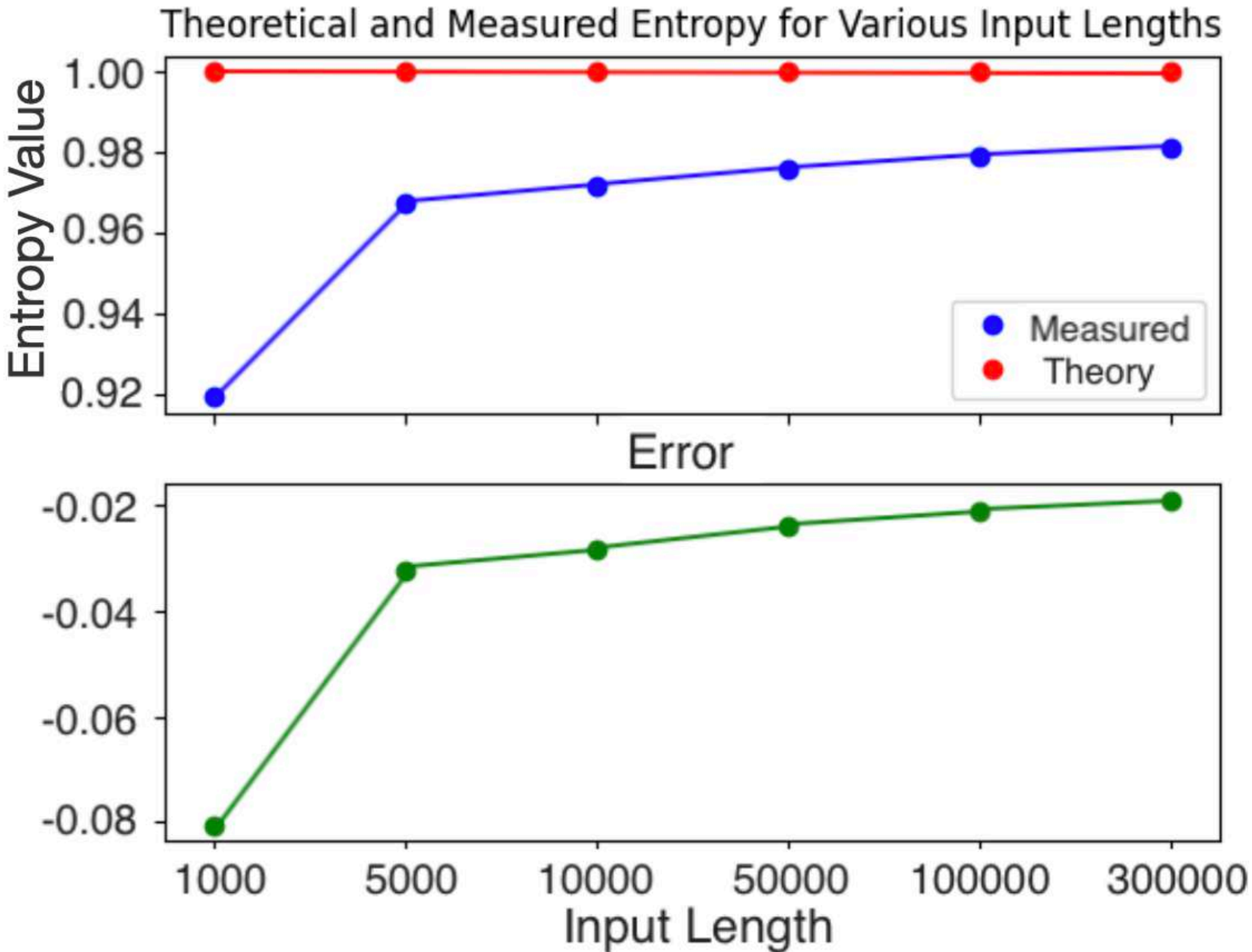
Discrete Implementations

Verification using Bernoulli series



As expected, the error of the estimator decreases as the input length grows.

Discrete Implementations

Introduce the Fast Fourier transform(FFT)

$$H = \frac{log_2 M}{< l >}$$

- Why Fast Fourier Transform?

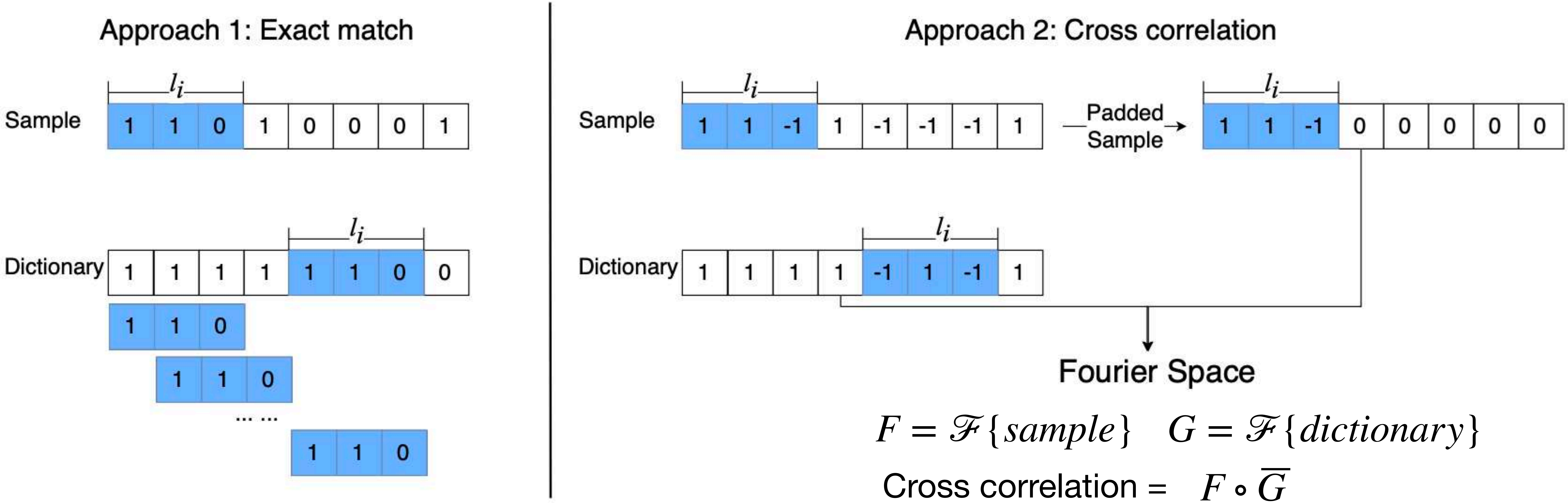
  It’s faster in higher dimensions.

  A good practice before moving to the continuous case

Discrete Implementations

Find the longest length of best matches l_i

$$H = \frac{log_2 M}{< l >}$$



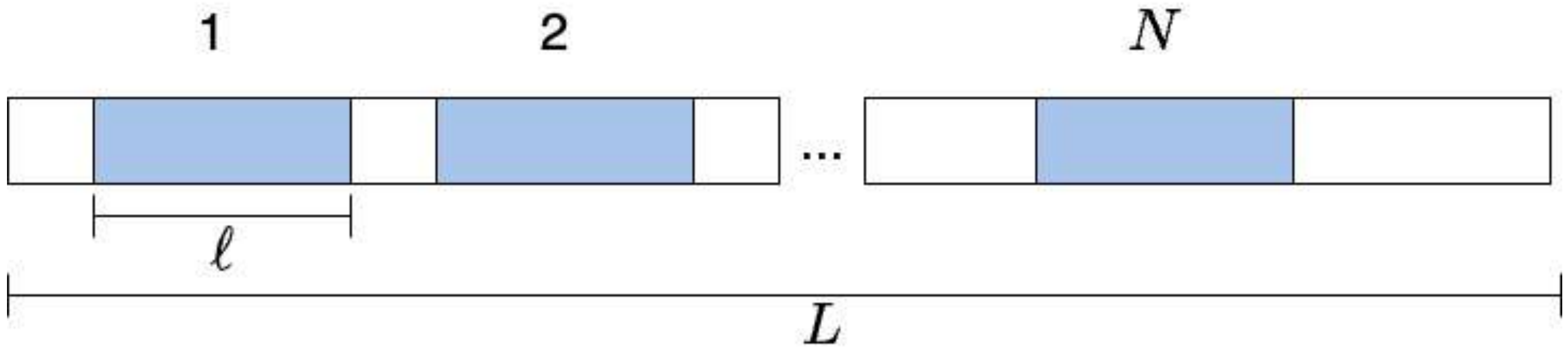
$$F = \mathscr{F}\{sample\} \quad G = \mathscr{F}\{dictionary\}$$

Cross correlation = $F \circ \overline{G}$

Continuous Implementations

Generating hard rods

- Non-overlapping constant-length rods

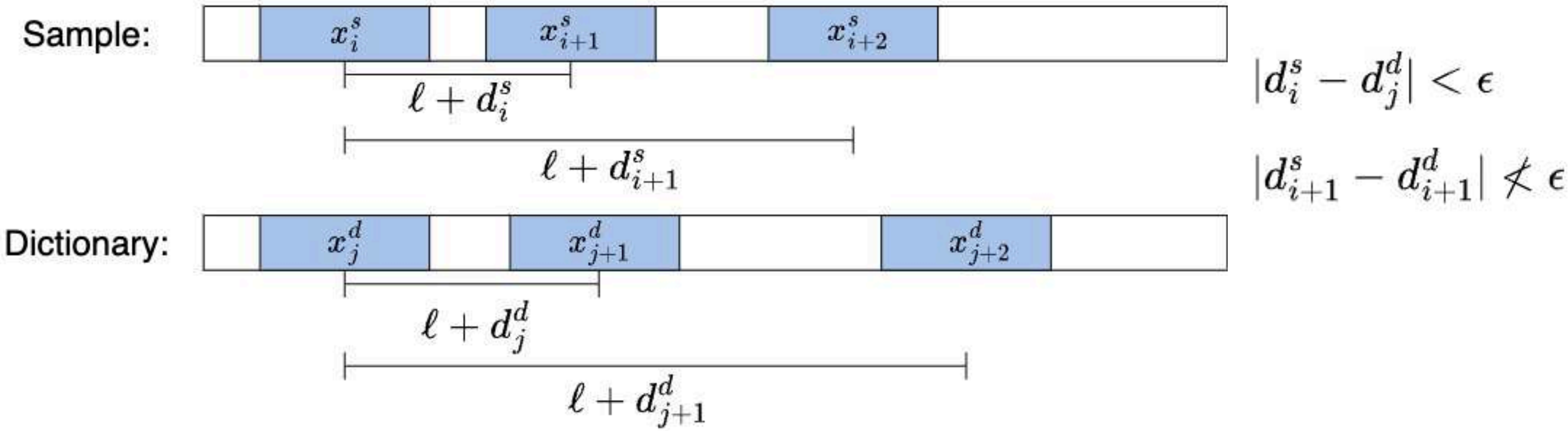


- Analytic entropy is known
- Discretization does not produce a good estimate

Continuous Implementations

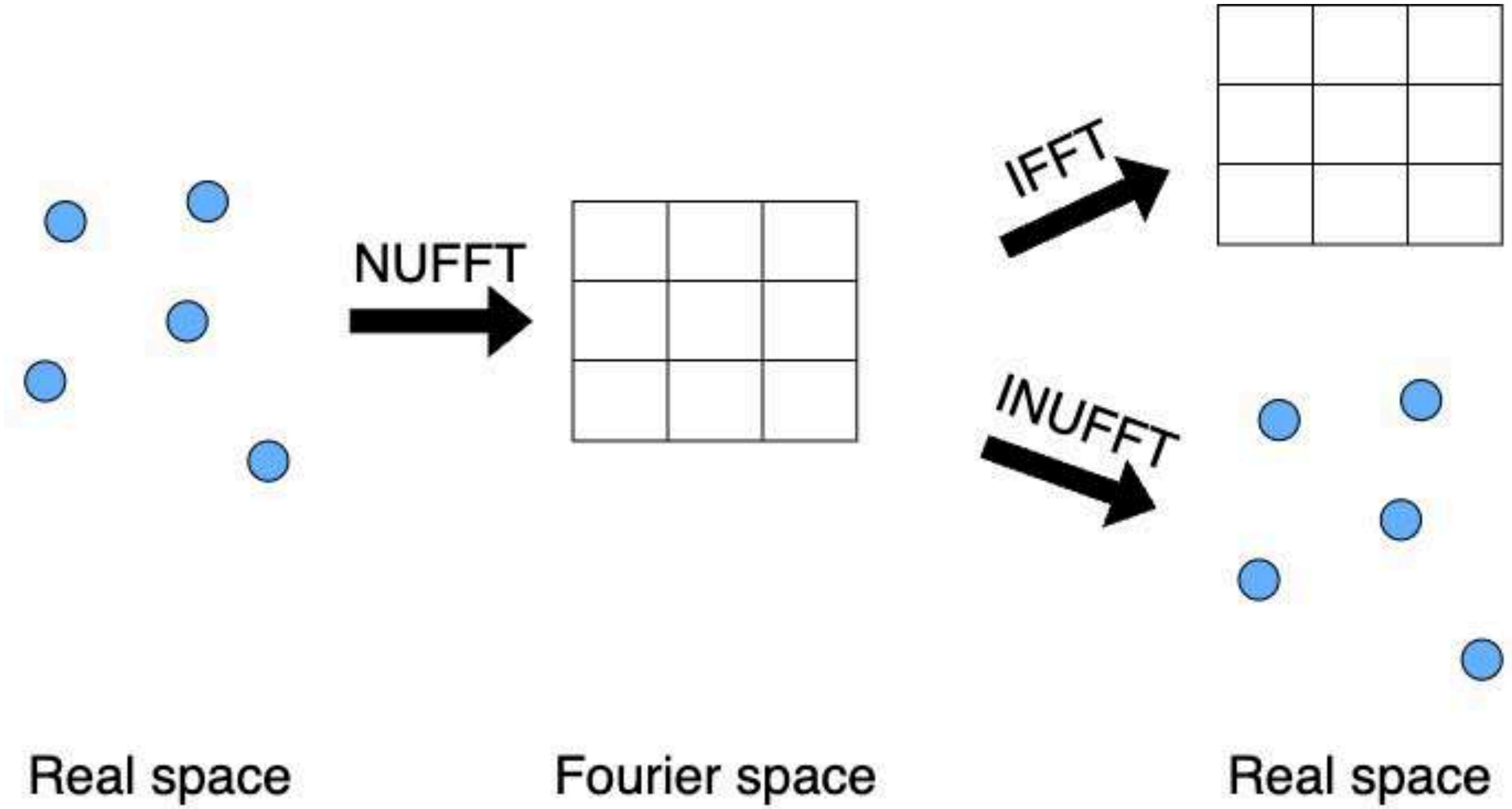
Real-space

- Used hard rods encoded by center location
- Match distances between rod centers with some error ϵ



Continuous Implementations
Fourier-space

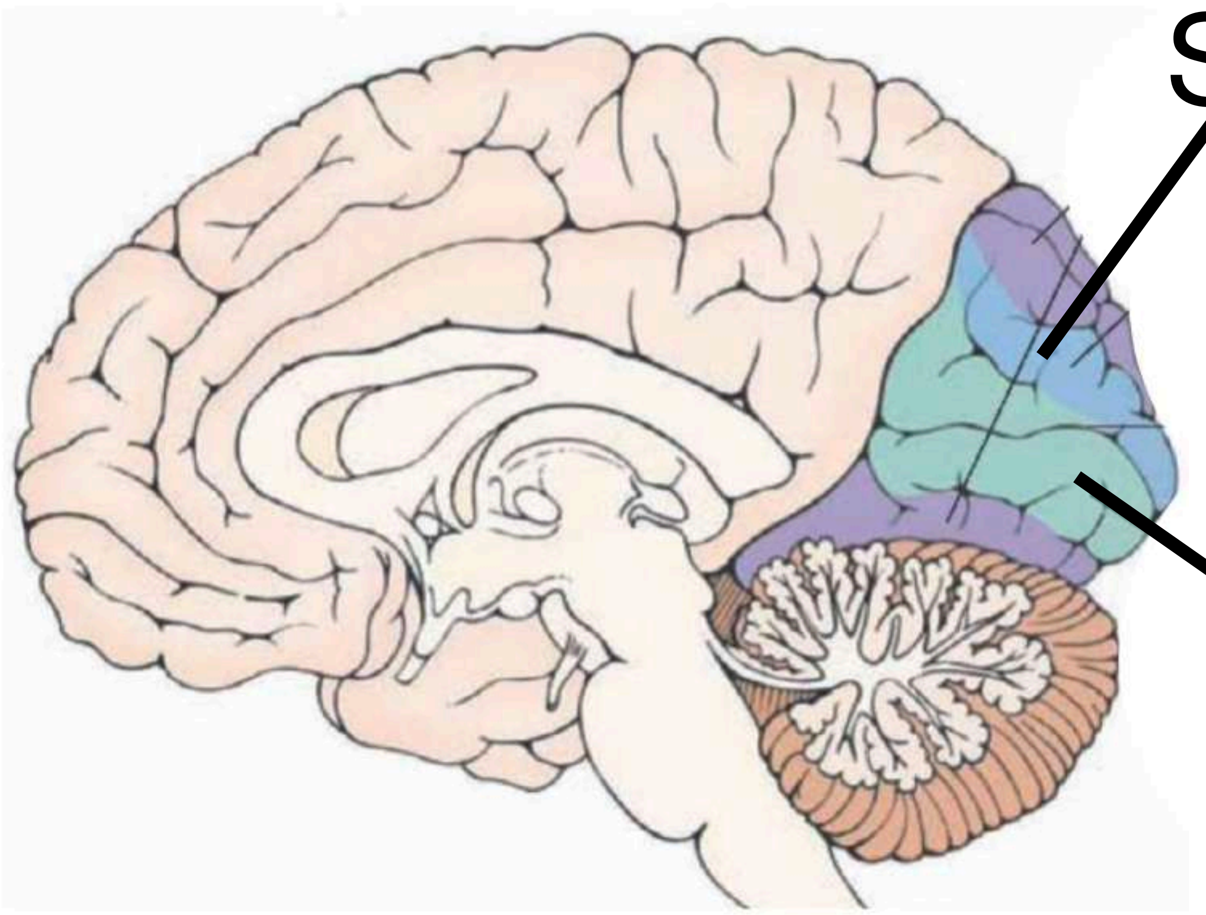
- Use non-uniform FFT (NUFFT)



Neural Data

Neuron activity in the visual cortex

- Spike train of neurons: recording of neuron activations.
- Virtual cortex



Neural Data

Neuron activity in the visual cortex

- Spike train of neurons: recording of neuron activations.
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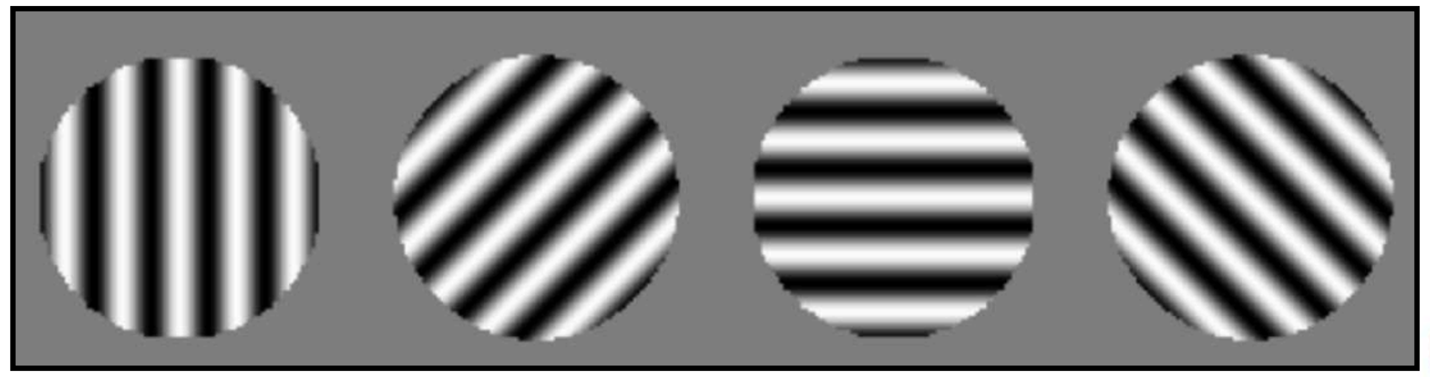
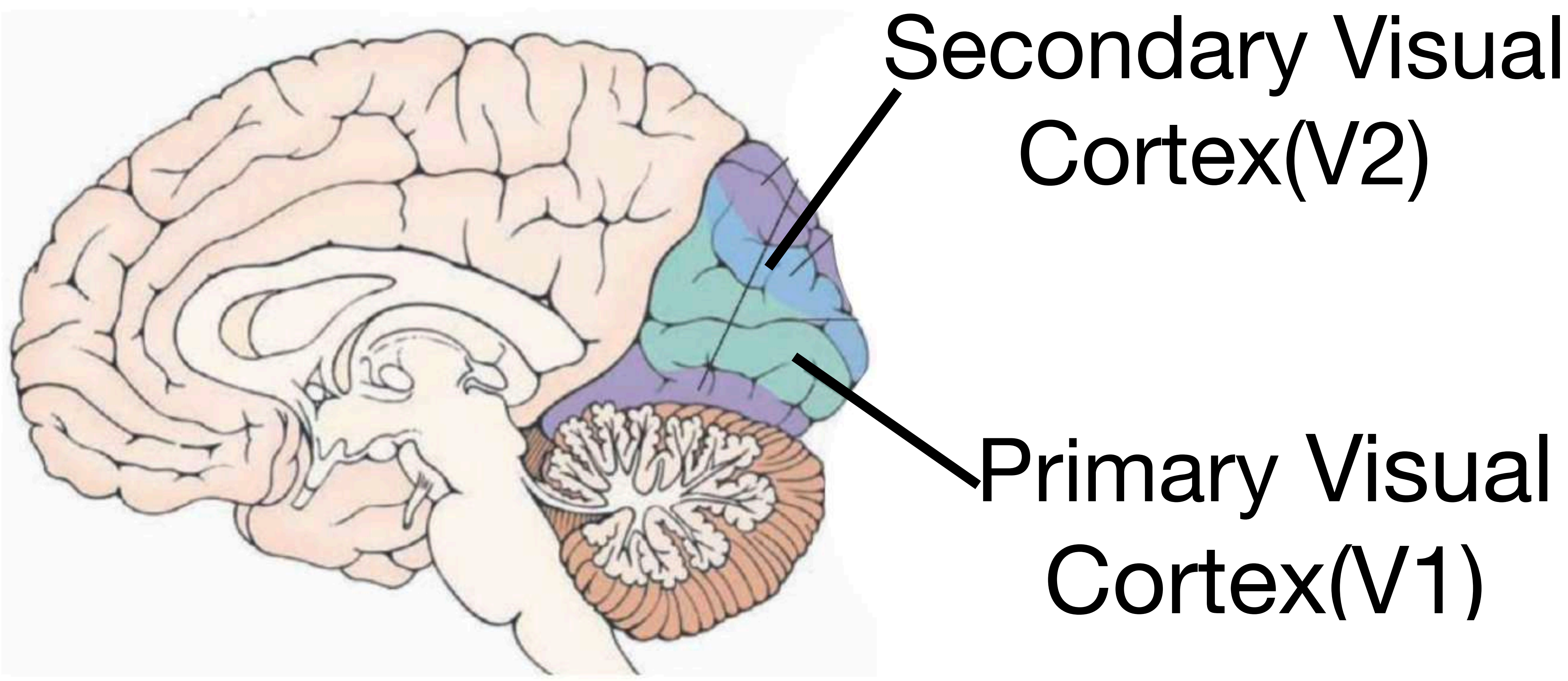
- Experiment Setup



Oriented gratings

Macaques



Neural Data

Quantities measured

- Hypothesis

  Neural code is more time reversible in different brain areas, revealing the computing happening in the visual cortex.

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  Neural code is more time reversible in different brain areas, revealing the computing happening in the visual cortex.

- Entropy: $H(p) = -\sum_x p(x)\log p(x)\,.$
- Cross Entropy: $H(p, q) = -\sum_x p(x)\log q(x)\,.$
- Kullback-Leibler divergence (KLD):

$$D_{KL}(p \parallel q) = H(p, q) - H(p)\,.$$

  - p - distribution of spike train data
  - q - distribution of reversed spike train data

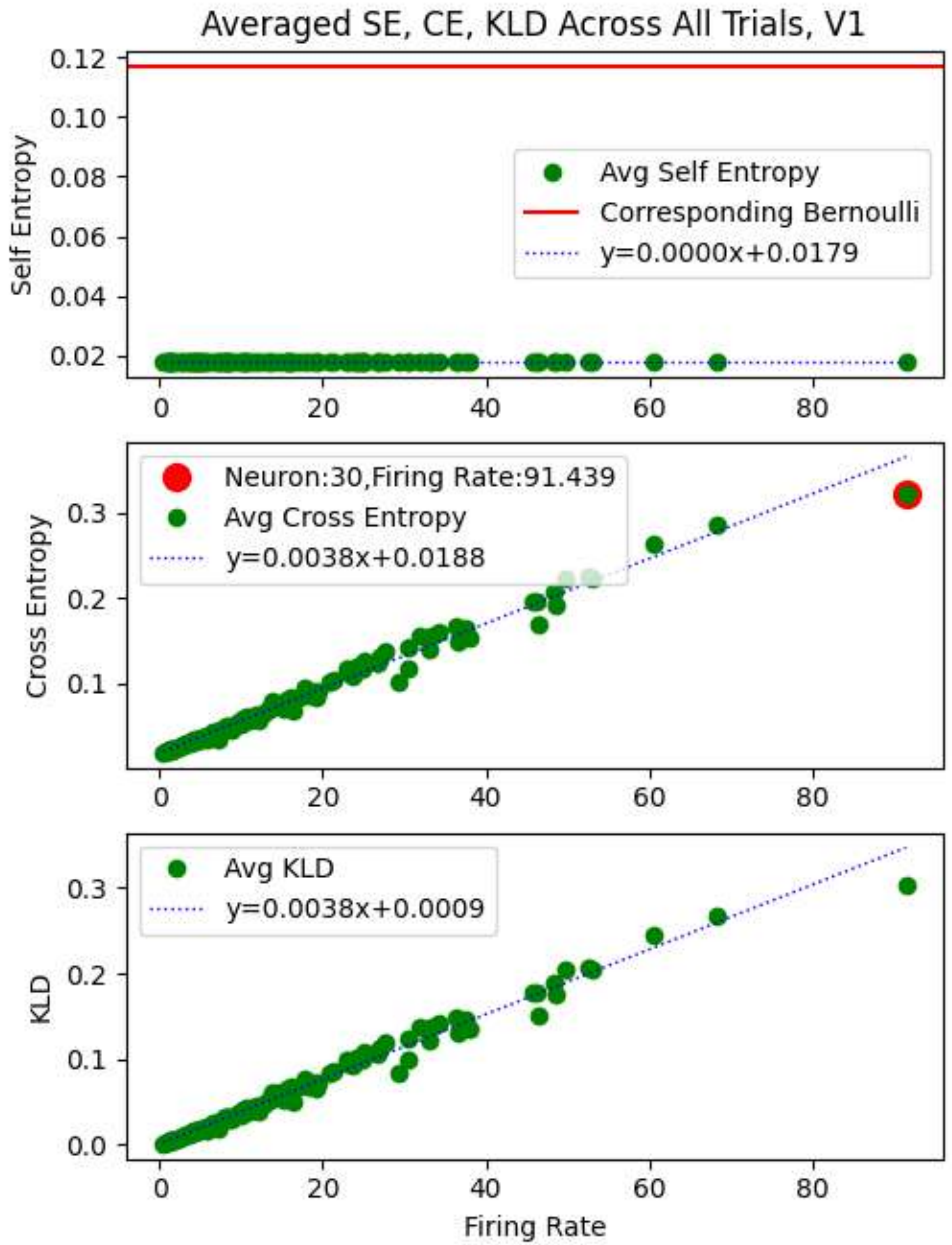
A measurement of time-reversal symmetry

Neural Data

Avg KLD vs. Firing Rate in V1

- Firing rate (Hz): $v = \frac{n_{sp}}{T}$
- In V1

  The KLD has a positive linear relationship with the firing rate of neurons.

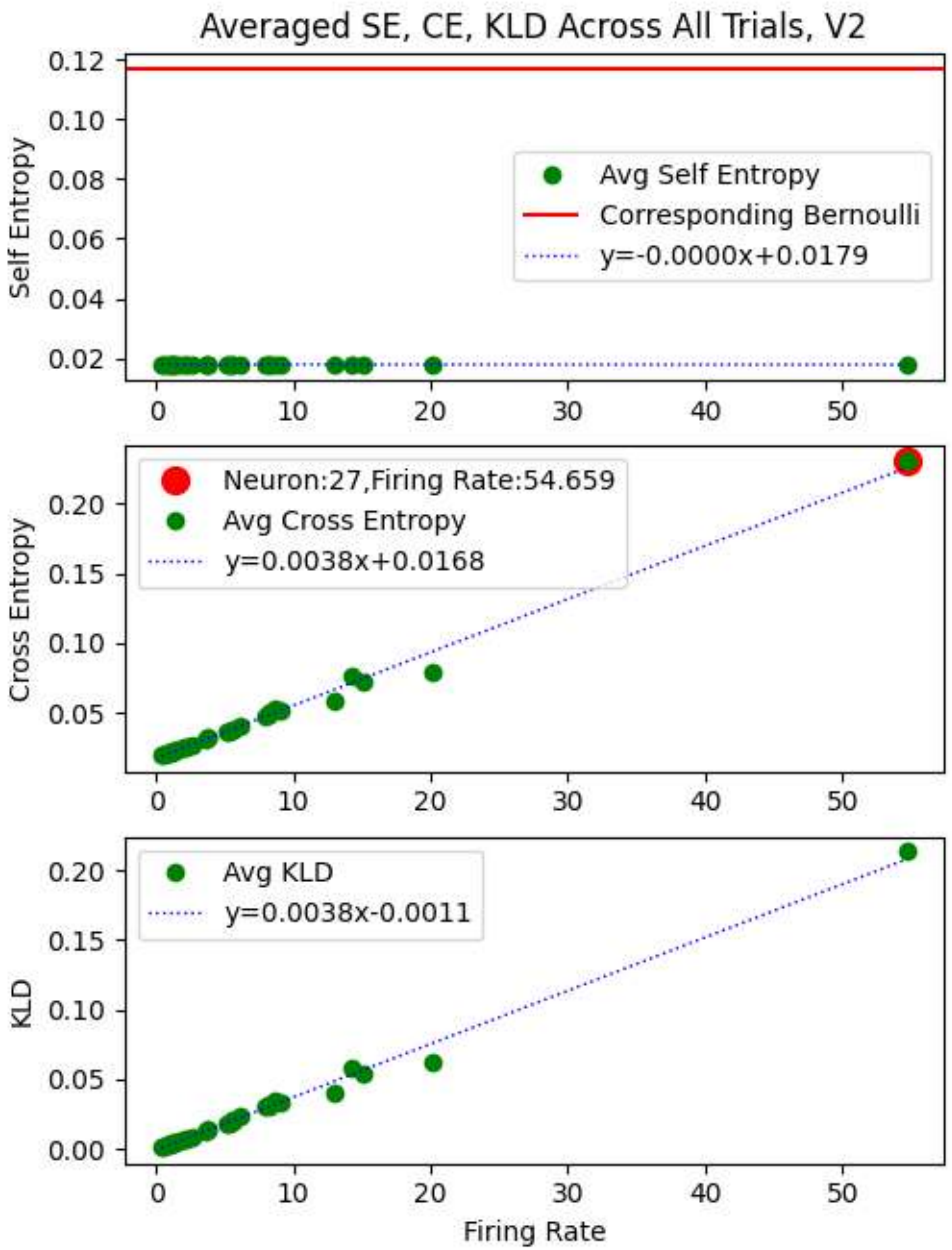


Neural Data

Avg KLD vs. Firing Rate in V2

- Firing rate (Hz): $v = \frac{n_{sp}}{T}$
- The same observation holds in V2:

  The KLD has a positive linear relationship with the firing rate of neurons.



Future work

- Neural data:
  - Apply algorithm to visual cortices of other animals
- Refine continuous implementation in real and Fourier space
- Extend to higher dimensions

Acknowledgments

People

Shivang Rawat

Mathias Casiulis

Stefano Martiniani

Institutions

NYU

NYU COURANT

SIMONS FOUNDATION

Why Entropy (Backup)

Cross-entropy

$$H(p, q) = -\sum_x p(x) \log q(x) .$$

- Measure of bits needed when the distribution is assumed to be q but is actually p
- Useful in machine learning context

Algorithm (Backup)

Pattern matching algorithm

- LZ77 is proved to asymptotically converge to the entropy

$H \leq$ Information required to specify the factor of a finite sequence

$$H \leq \frac{C}{N} logN + \frac{1}{N} \sum_{i=1}^{C} logl_i$$

$$= \frac{C}{N} logN + \frac{C}{N} < logl >$$

$$\leq \frac{C}{N} logN + \frac{C}{N} log < l >$$

$$H \leq \frac{logN}{< l >} + \frac{log < l >}{< l >}$$

$$H \leq \frac{logN}{< l >}$$

Pattern matching estimator

Examples and derivations come from Stefano's talk at Santa Fe, "The Other Side Of Entropy"

Phase Correlation (Backup)

Non-binary input

- Cross correlation is valid only for binary input
- Phase correlation:

$$F = \mathscr{F}\{sample\} \quad G = \mathscr{F}\{dictionary\}$$

$$\text{Phase correlation} = \frac{F \circ \overline{G}}{|F \circ \overline{G}|}$$

Algorithm (Backup)

Pattern matching estimator

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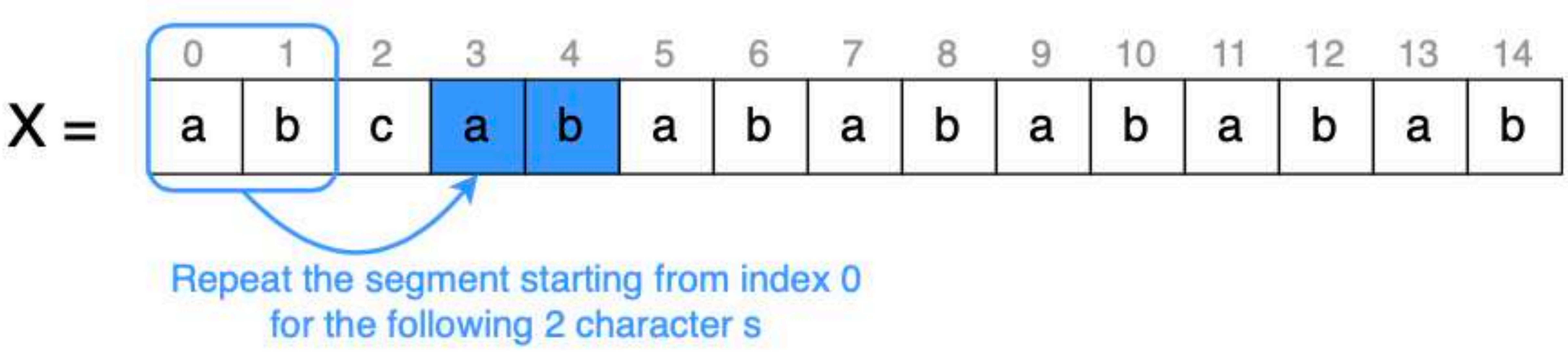

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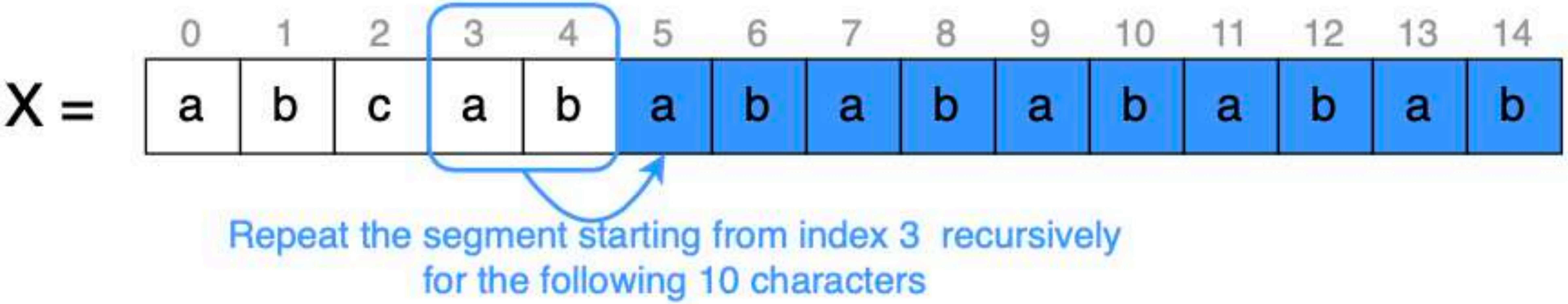

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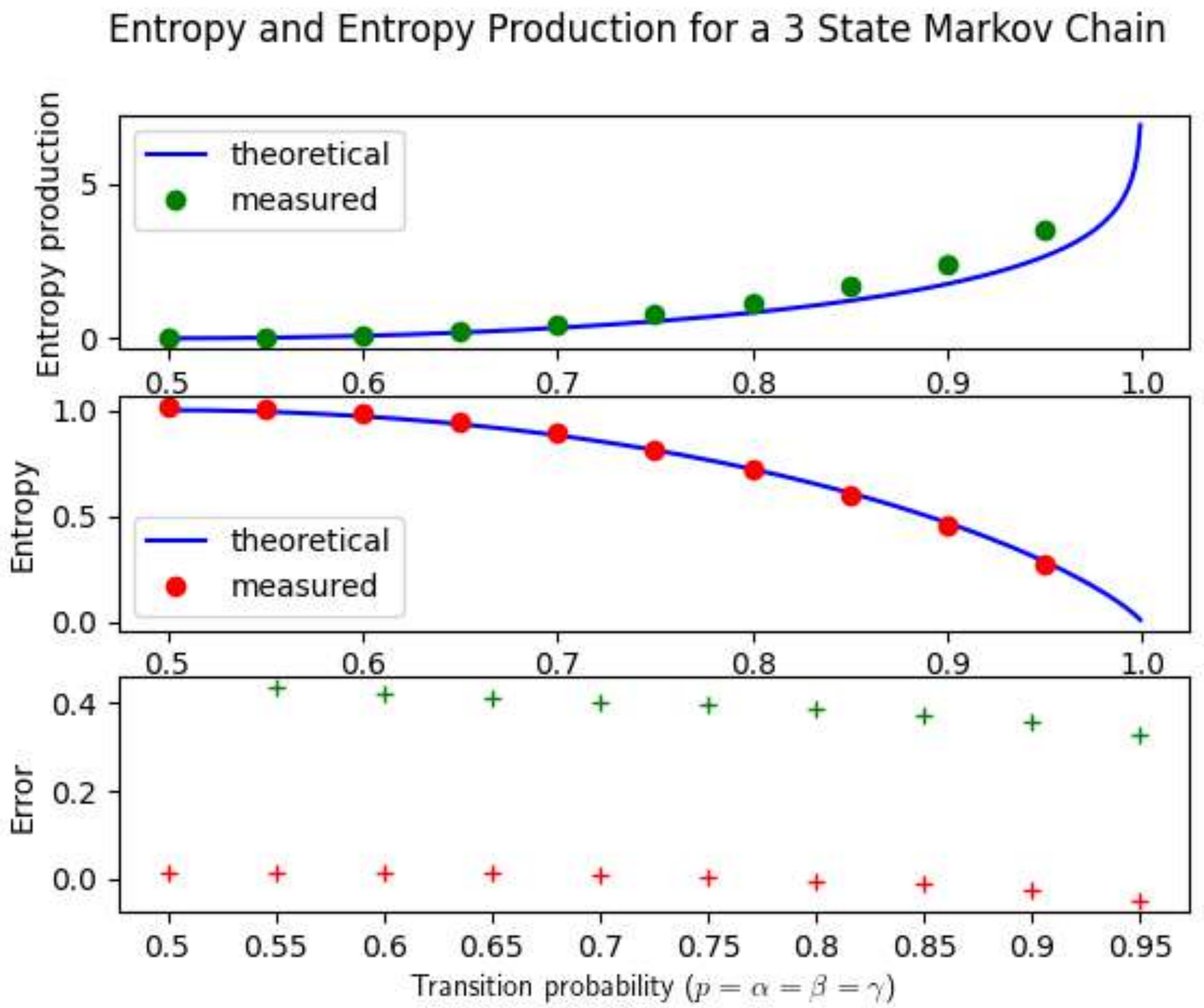
Discrete Implementations (Backup)

Verification using 3-state Markov chain



As expected, the error of the estimator increases as the bias grows.