YOUR SIGNATURE: ______________________________

(1) (Pre-calculus Review Set Problems 80 and 124.)

(a) Determine if each of the following statements is True or False. If it is true, explain why. If it is false, give a counterexample.

(i) If a and b are real numbers and $2a^5b^7 = 3ab$, then $2a^4b^6 = 3$. ____________.

(ii) When solving $x^2(x-2)^3 = (x-2)^3$, we get $x^2 = 1$, so the solutions are $x_1 = 1,\ x_2 = -1$. ____________.

(b) Simplify and write the following expression without negative exponents. Show your work.

$$\frac{6^{-1}r^{-3}r^2}{r^5}$$

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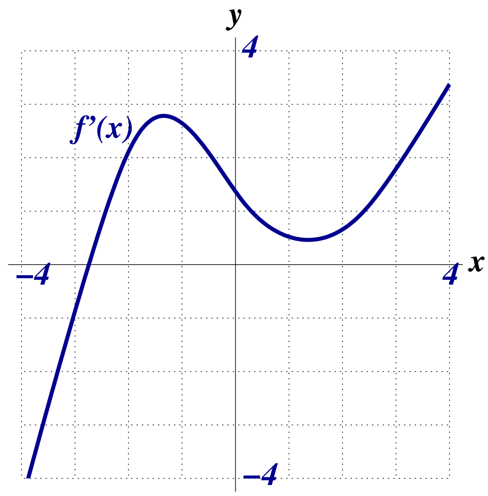
(2) Find the derivatives of the given functions. You must show your work. But you do not have to simplify your answers.

(a) $z(t) = \sqrt{t}\,(t^2 + t + 5)$

(b) $h(x) = \left(5x^2 - \dfrac{x^2}{\sqrt{x+1}}\right)^7$

(c) (6 points) Find dy/dx implicitly: $e^{x^2 y} = x + y$

(3) (a) (9 points) The graph below is the **derivative** of a function $f(x)$, $f'(x)$. Answer each of the following questions. You do not need to explain.



(i) $f(x)$ is increasing on the interval(s) ______________________

(If you are not sure about the coordinates of the end points, an estimate will do.)

(ii) $f(x)$ is concave up on the interval(s) ______________________.

(iii) The function $f(x)$ achieves its absolute maximum value on the interval $[-4, 4]$ at $x =$____________.

(b) (6 points) Write a parameterization for the line segment from $(2, -5)$ and $(-1, 3)$.

(c) (6 points) Consider the function: $f(x) = \begin{cases} |x-1|, & 0 \le x \le 2 \\ 3-x, & 2 < x \le 4 \end{cases}.$

(i) Sketch the graph of $f(x)$. Please label clearly the value of f at $x = 0, 1, 2, 3,$ and 4.

(ii) Find the average value of $f(x)$ on the interval $[0, 4]$.

(d) (6 points) Find the equation of the tangent line (in the form of $y = mx + b$) at the point $(2, 1)$ to the curve defined by the parametric equations

$$x = 2t, \quad y = t^3.$$

(4) Find each of the following limits when it exists, write DNE otherwise. Show your work.

(a) $\displaystyle\lim_{x\to\infty} \frac{13+2x-4x^4+3x^4}{2x-5x^2-5x^4}$

(b) $\displaystyle\lim_{x\to a^+} \frac{x-a}{\sqrt{x^2-a^2}}$, where $a>0$.

(c) $\displaystyle\lim_{x\to\infty} \frac{(\ln(x))^2}{x^2}$

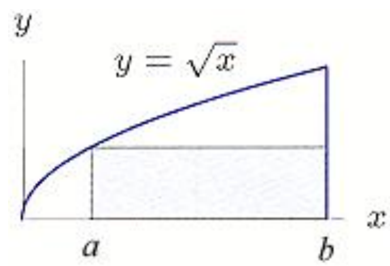
(5) Consider $f(x) = xe^{-2x^2}$. Answer each of the following questions. You must show all work.

(a) Find $f'(x)$ and all critical point(s) of $f(x)$.

(b) Determine the interval(s) where $f(x)$ is increasing and decreasing. State the x-coordinate(s) of the point(s) where f achieves it local maximum or/and local minimum.

(c) Find $f''(x)$ and all inflection point(s) of $f(x)$.

(6) The figure below shows the curve $y = \sqrt{x}$, and a rectangle with its upper-left corner on the curve, its sides parallel to the axes, its left end at $x = a$, and its right end at $x = b$. Let b be fixed as $b = 20$. Find the value of a such that the rectangle has the maximum possible area. What is that maximum possible area? Show your work and give exact values.



Useful formulas

- **The derivative of a function**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Some rules of differentiation**

$$\frac{d}{dx}(cf(x)) = cf'(x)$$
$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$
$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$
$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$
$\frac{d}{dx}(\sinh(x)) = \cosh(x)$	$\frac{d}{dx}(\cosh(x)) = \sinh(x)$	$\frac{d}{dx}(\tanh(x)) = \frac{1}{\cosh^2(x)}$

- **The linear approximation** of a function f at a is given by

$$f(x) \approx f(a) + f'(a)(x-a)$$

- **Derivative of the inverse function** If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function of f is differentiable at a and

$$\frac{d}{dx}\left(f^{-1}(a)\right) = \frac{1}{f'\left(f^{-1}(a)\right)}.$$

- **Parametric Equations for a straight line:** An object moving along a line through the point (x_0, y_0), with $dx/dt = a$ and $dy/dt = b$ has parametric equations
$$x = x_0 + at, \qquad y = y_0 + bt.$$
The slope of the line is $m = b/a$.

- The **instantaneous speed** of a moving object is defined to be
$$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}.$$
The quantity $v_x = dx/dt$ is the instantaneous velocity in the x-direction; $v_y = dy/dt$ is the instantaneous velocity in the y-direction. The velocity vector $\vec{v}$ is written $\vec{v} = v_x \vec{i} + v_y \vec{j}$.

- For parametric curves,
$$\text{Slope of curve} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}; \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}.$$

- **Fundamental Theorem of Calculus**: If f is continuous on the interval $[a, b]$ and $f(t) = F'(t)$, then
$$\int_a^b f(t)\, dt = F(b) - F(a).$$

- **The average value** of a function f on an interval $[a, b]$ is equal to $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\, dx$.

- **Comparison of Definite Integrals**: If f is continuous and $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $m(b-a) \leq \displaystyle\int_a^b f(x)\, dx \leq M(b-a)$.

- **Basic integration formulas:**

1. $\displaystyle\int x^n\ dx = \frac{x^{n+1}}{n+1} + C,\ (n \neq -1)$	2. $\displaystyle\int \frac{1}{x}\ dx = \ln\|x\| + C$
3. $\displaystyle\int e^x\ dx = e^x + C$	4. $\displaystyle\int a^x\ dx = \frac{a^x}{\ln(a)} + C$
5. $\displaystyle\int \sin(x)\ dx = -\cos(x) + C$	6 . $\displaystyle\int \cos(x)\ dx = \sin(x) + C$
7. $\displaystyle\int \sec^2(x)\ dx = \tan(x) + C$	8 . $\displaystyle\int \csc^2(x)\ dx = -\cot(x) + C$
9 . $\displaystyle\int \sec(x)\tan(x)\ dx = \sec(x) + C$	10. $\displaystyle\int \csc(x)\cot(x)\ dx = -\csc(x) + C$
11. $\displaystyle\int \frac{1}{x^2+1}\ dx = \tan^{-1}(x) + C$	12. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\ dx = \sin^{-1}(x) + C$