Agent Based Trading Model of Heterogeneous and Changing Beliefs*

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Abstract

I construct an agent based model of a stock market in which investors trade based on heterogeneous and changing beliefs. In this model, each agent trades based on the theory of dynamic investment in Merton (1969), with his own belief on the return of the stock. The baseline model implies that optimistic investors buy stocks from pessimistic investors. Then, I extend the model such that the beliefs of agents are time-varying: each agent adjusts his belief differently with the arrival of news information. By simulating the extended model, I find that trade volume increases with higher heterogeneity of agents' response to news information. I also find that return volatility decreases with more positive or more negative average level of response to news information.

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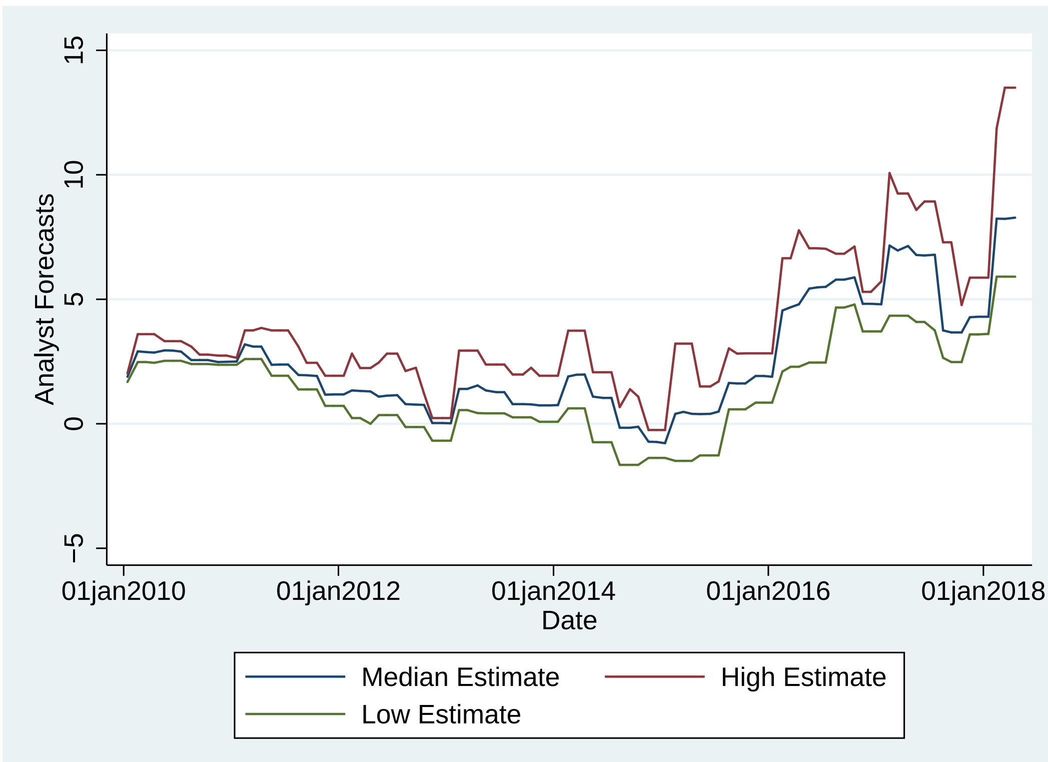
1 Introduction

Stock prices are determined by trading, which arises from differences in opinions among investors. If all investors had the same expectations on stock returns, no trades would occur, because everyone would seek to trade in the same direction. Thus, it makes sense to study the stock price as a representation of heterogeneous beliefs on stock returns materialized through trading.

Figure 1: Time Series of Analyst Forecasts of Amazon's EPS



It is also reasonable to think that these beliefs on stock returns change over time, with the influx of new information. More specifically, Figure 1 illustrates the analysts Amazon's earnings per share (EPS) forecast from 2010. Figure 1 reveals that the opinions are time-varying and heterogeneous across analysts. This may suggest that investors make different conclusions from the same set of information. This motivates modeling the heterogeneity of beliefs as an accumulation of the idiosyncratic response to the identical set of randomly arriving news information.

There is a large body of literature[1] that documents the empirical evidence on the effect of heterogeneous beliefs on asset price dynamics. This may involve an investigation of the relationship between belief dispersion and the mean returns of stocks. Goetzmann and Massa (2005) and Yu (2011) find that dispersion in beliefs is negatively correlated with future returns. On the other hand, Avramov et al. (2009) show that such negative relation is limited to worst-rated firms, while Doukas et al. (2006) find that the relationship is positive.

To study how interactions among investors with heterogeneous and changing beliefs affect stock prices, I apply an Agent Based Model (ABM) approach. The ABM approach offers several advantages that representative agent models do not. Firstly, an ABM allows its agents to have heterogeneous beliefs, not just heterogeneous utilities. Secondly, agents in an ABM follow a simple behavioral rule, instead of optimizing with the full knowledge of the state of the world. Thirdly, the key dynamics in an ABM are generated endogenously from the interaction among the agents. In relation to ABM and economics, Westerhoff and Franke (2012) illustrate the usefulness of ABM in designing economic policies with their example of technical traders, fundamental traders, and a central authority model to study the impact of simple intervention strategies on asset price dynamics. Lengnick (2011) constructs an ABM for business cycles in an economy and illustrates that an ABM can reproduce many stylized facts without the strict assumption of rationality. Lengnick also finds that the aggregate behavior generated by this ABM is not equal to the results of a microeconomic optimization by the representative agent. A more empirical application was conducted by Baptista et al. (2016), who developed an ABM for the UK housing market to examine the effect of macroprudential policies on key housing market indicators. Their results imply that a larger buy-to-let sector might amplify house price cycles and lead to higher price volatility. These papers show that the ABM can be economically useful in reproducing stylized facts without strong assumptions on agents or equilibriums; the ABM also generates non-standard

[1] Diether et al. (2002), Chen et al. (2002), Park (2005), Berkman et al. (2009), Qu et al. (2003) and others

aggregate behavior that is markedly different from that of a representative agent.

In this paper, I extend Goodman (2016) model, such that agents trade based on heterogeneous and changing beliefs. In the baseline model, each agent trades based on his beliefs and the theory of dynamic investment in Merton (1969), in which an agent maximizes his power utility by holding a fixed proportion of his wealth in risky assets. This baseline model implies that optimistic investors buy stocks from pessimistic investors. I then extend the model by having the agents adjust their beliefs in a manner that differs from randomly arriving news information. By simulating the extended model, I find that the trade volume increases with the higher heterogeneity of the agents' response to the news information. I also find that the return volatility decreases with a more positive or negative average response level to news information.

The rest of this paper is organized as follows. Section 2 presents the baseline and extended models. Section 3 reports the simulation results on return volatility and trade volume. Section 4 discusses a potential extension to the model with an extrapolative component. Section 5 concludes.

2 Model

To define the trading behavior of the agents, I use the result from Merton (1969). In this model, an agent maximizes power utility $V(T)$ by allocating his wealth $W(t)$ between a riskless asset and a risky asset, the value of which evolves with mean μ and volatility σ.

$$\max_{x} E[V(W(T))]$$

subject to

$$dW = rW(t)dt + (\mu - r)X(t)dt + \sigma X(t)d\omega_t$$

The solution to this problem is to keep a fixed proportion of his wealth in risky assets.

This proportion π is a function of the risky asset's excess return $\mu - r$, return volatility σ, and the agent's risk aversion parameter γ.

$$X(t) = \pi W(t), \quad \text{where} \quad \pi = \frac{\mu - r}{(1-\gamma)\sigma^2}$$

This proportion π determines the agents' trading behavior: each agent will buy or sell stocks to maintain that proportion of wealth in stocks. For instance, if the price of the risky asset goes up while π does not change, the agent will sell the risky asset to maintain the proportion of his wealth in the risky asset.

In the baseline model, each agent trades based on the Merton proportion and there are n such agents in the market. An important assumption of this model is that each agent has his own belief: agent k has beliefs μ_k and σ_k and risk aversion parameter γ_k. I further assume that the total number of stocks in the market does not change and that there is only one risky asset (stock) and one riskless asset (cash).

2.1 Model Implications

From the assumptions of the model, the following identities hold:

1. The wealth of each agent is the sum of her cash and stock value:

$$W_k(t) = B_k(t) + X_k(t) = B_k(t) + N_k(t)S(t)$$

   Then, the wealth of the economy can be expressed as:

$$\sum_{k=1}^{n} W_k(t) = \sum_{k=1}^{n} (B_k(t) + N_k(t)S(t))$$

2. In an equilibrium, each agent has the optimal ratio of risky assets.

$$W_k(t)\pi_k(t) = X_k(t) = N_k(t)S(t)$$

The wealth of the economy at equilibrium can thus be expressed as:

$$\sum_{k=1}^{n} W_k(t) = \sum_{k=1}^{n} \frac{1}{\pi_k(t)} N_k(t)S(t)$$

3. Total number of stocks remains invariant:

$$\sum_{k=1}^{n} N_k(t_1) = \sum_{k=1}^{n} N_k(t_2)$$

Let N denote the total number of stocks in the market ($N = \sum_{k=1}^{n} N_k(t)$)

4. Total cash in the economy remains invariant:

$$\sum_{k=1}^{n} B_k(t_1) = \sum_{k=1}^{n} B_k(t_2)$$

5. From 3 and 4, I find that the wealth of the economy is increased only by the rise in stock price:

$$\sum_{k=1}^{n} W_k(t_2) - \sum_{k=1}^{n} W_k(t_1) = N \times (S(t_2) - S(t_1))$$

Consider a simple case in which only one of the agents, agent 1, increases his μ_1 and thus his π_1. When only agent one increased its μ_1 and thus π_1, he would not be able to transact at current price if all the other agents are content with their portfolios. Therefore, agent 1 should keep bidding higher prices to buy stocks until he achieve his new proportion π_1 of his wealth in stocks. When agent 1 pushes the price up from S to $S+dS$, wealth of agent k, W_k, will increase by $dW_k = N_k dS$. As π_k has not changed for $k \neq 1$, the change in the optimal stock value for agent k would be $dX_k = \pi_k N_k dS$. If agent k buys dN_k number of stocks to rebalance to the new optimal allocation, the value of his stock holdings would change from $N_k S$ to $(N_k + dN_k)(S + dS)$. Therefore, the following equation holds for each agent other than agent 1:

$$dX_k = \pi_k dW_k = \pi_k N_k dS = (N_k + dN_k)(S + dS) - N_k S = N_k dS + S dN_k$$

Rearranging the equation above, the number of stocks that agent k should buy is:

$$dN_k = -\frac{1-\pi_k}{S} N_k dS$$

This suggests that in response to agent 1's optimism, all the other agent will sell stocks (assuming that $\pi_k \leq 1$)to maintain their Merton proportion of their wealth in stocks.
By separating the variables, I get:

$$\frac{dN_k}{N_k} = -(1-\pi_k)\frac{dS}{S}$$

By solving this equation and plugging in the initial condition, I derive the number of stocks that agent k will hold in equilibrium:

$$N_k(t) = N_k(0)\left(\frac{S(0)}{S(t)}\right)^{1-\pi_k(t)} \quad \forall k \neq 1$$

As total number of stocks remains invariant, the number of stocks owned by the agent 1 who changed his views can be expressed as:

$$N_1(t) = N - \sum_{k=2}^{n} N_k(t) = N - \sum_{k=2}^{n} N_k(0)\left(\frac{S(0)}{S(t)}\right)^{1-\pi_k(t)}$$

From the identities and the stock holdings derived above, I now derive the new equilibrium price given that only agent 1 revised its μ_1 and thus π_1.

At the new equilibrium, both identities 1 and 2 should hold:

$$\sum_{k=1}^{n} W_k(t) = \sum_{k=1}^{n} (B_k(t) + N_k(t)S(t)) = \sum_{k=1}^{n} \frac{1}{\pi_k(t)} N_k(t)S(t)$$

As total cash in the economy is invariant,

$$\sum_{k=1}^{n} (B_k(t) + N_k(t)S(t)) = \sum_{k=1}^{n} B_k(0) + \sum_{k=1}^{n} N_k(t)S(t)) = \sum_{k=1}^{n} \frac{1}{\pi_k(t)} N_k(t)S(t)$$

As total number of stocks is invariant,

$$\sum_{k=1}^{n} B_k(0) + NS(t) = \sum_{k=1}^{n} \frac{1}{\pi_k} N_k(t)S(t)$$

Now, I separate agent 1 from others:

$$\sum_{k=1}^{n} B_k(0) + NS(t) = \frac{1}{\pi_1(t)} N_1(t)S(t) + \sum_{k=2}^{n} \frac{1}{\pi_k(t)} N_k(t)S(t)$$

Plugging in the stock holdings of agents in new equilibrium,

$$\sum_{k=1}^{n} B_k(0) + NS(t) = \frac{1}{\pi_1(t)} \left(N - \sum_{k=2}^{n} N_k(0) \left(\frac{S(0)}{S(t)} \right)^{1-\pi_k(t)} \right) S(t) + \sum_{k=2}^{n} \frac{1}{\pi_k(t)} N_k(0) \left(\frac{S(0)}{S(t)} \right)^{1-\pi_k(t)} S(t)$$

By dividing up the sum, I get:

$$\sum_{k=1}^{n} B_k(0) + NS(t) = \frac{1}{\pi_1(t)} NS(t) - \sum_{k=2}^{n} \frac{1}{\pi_1(t)} N_k(0) \left(\frac{S(0)}{S(t)} \right)^{1-\pi_k(t)} S(t) + \sum_{k=2}^{n} \frac{1}{\pi_k(t)} N_k(0) \left(\frac{S(0)}{S(t)} \right)^{1-\pi_k(t)} S(t)$$

By rearranging the terms, I get the following equation for S(t):

$$\left(\frac{1}{\pi_1(t)} - 1 \right) NS(t) + \sum_{k=2}^{n} \left(\frac{1}{\pi_k(t)} - \frac{1}{\pi_1(t)} \right) N_k(0)S(0)^{1-\pi_k(t)} S(t)^{\pi_k(t)} - \sum_{k=1}^{n} B_k(0) = 0$$

I confirm that the equation above is consistent with the simulation.

2.2 Model Extension 1: Changing Beliefs

Now, consider a more realistic case in which agents adjust their beliefs based on the arrival of news information. I assume that news information arrives through Poisson process and the time between each of them is exponentially distributed. Each agent's reaction to news information can be decomposed into two parts: the common response η_t and agent-specific response $\xi_{k,t}$.

$$\mu_k(t + \Delta t) - \mu_k(t) = \eta_t + \xi_{k,t}$$

The common response can be considered as the original news information shared by all agents, and is denoted by η_t. I assume that η_t is normally distributed with mean μ_η and variance σ_η^2. In this setting, μ_η would be the average level of response of agents to the news information and σ_η^2 would be the variability among the news.

$$\eta_t \sim N(\mu_\eta, \sigma_\eta^2)$$

Then, for a given original new information, each agent reacts differently as would actual investors in the market. I assume that this idiosyncratic response, denoted as $\xi_{k,t}$, is distributed normally with mean μ_ξ and variance σ_ξ^2. As I want this term to capture only the effect specific to a certain agent, I have the mean μ_ξ equal to 0 and set σ_ξ^2 as the parameter that determines the level of heterogeneity among agents' reactions to news information.

$$\xi_{k,t} \sim N(0, \sigma_\xi^2)$$

3 Simulation Results

I simulated the extended model with news information. Starting from an identical distribution of wealth, stock holdings, and beliefs for all agents, I had the news information arrive in exponentially distributed intervals.

Figure 2: Simulated Stock Price Path

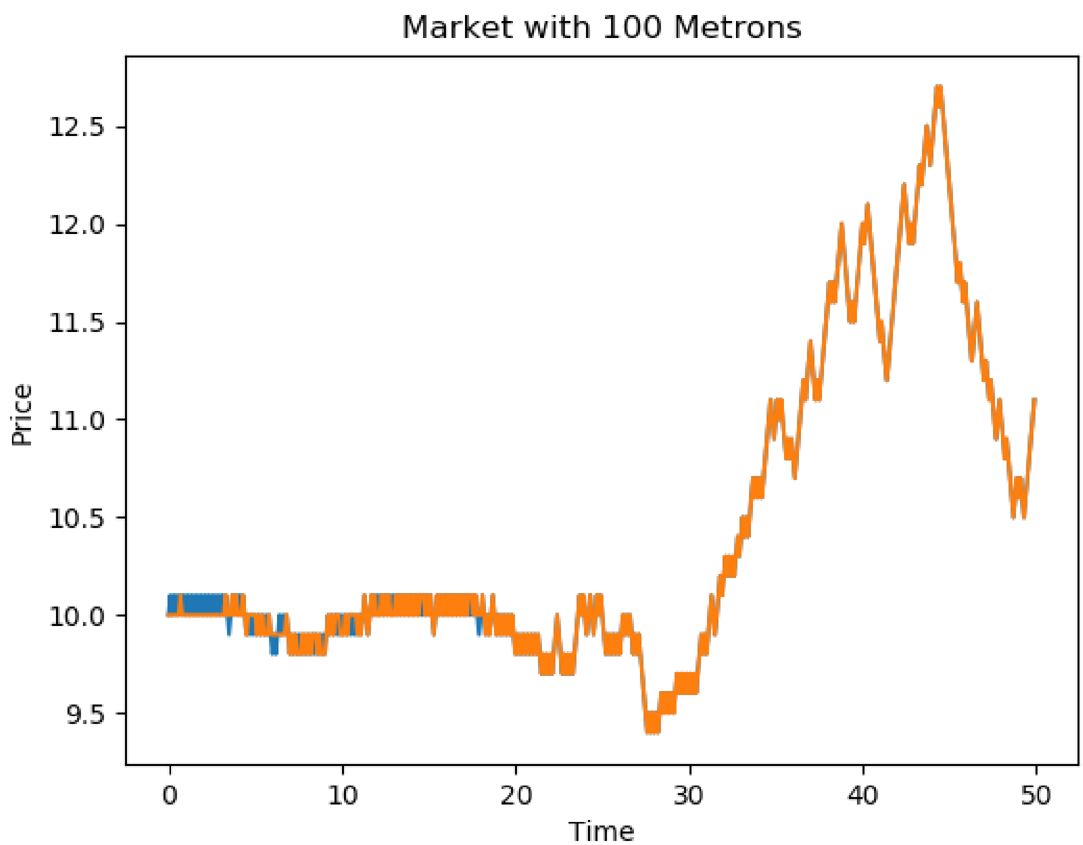


Figure 2 shows one of the stock price path generated by the simulation. The simulated stock price seems to become more volatile as heterogeneity across agent beliefs increases with the dispersion in reaction to news information accumulating over time.

Figure 3: Trade Volume and σ_ξ^2

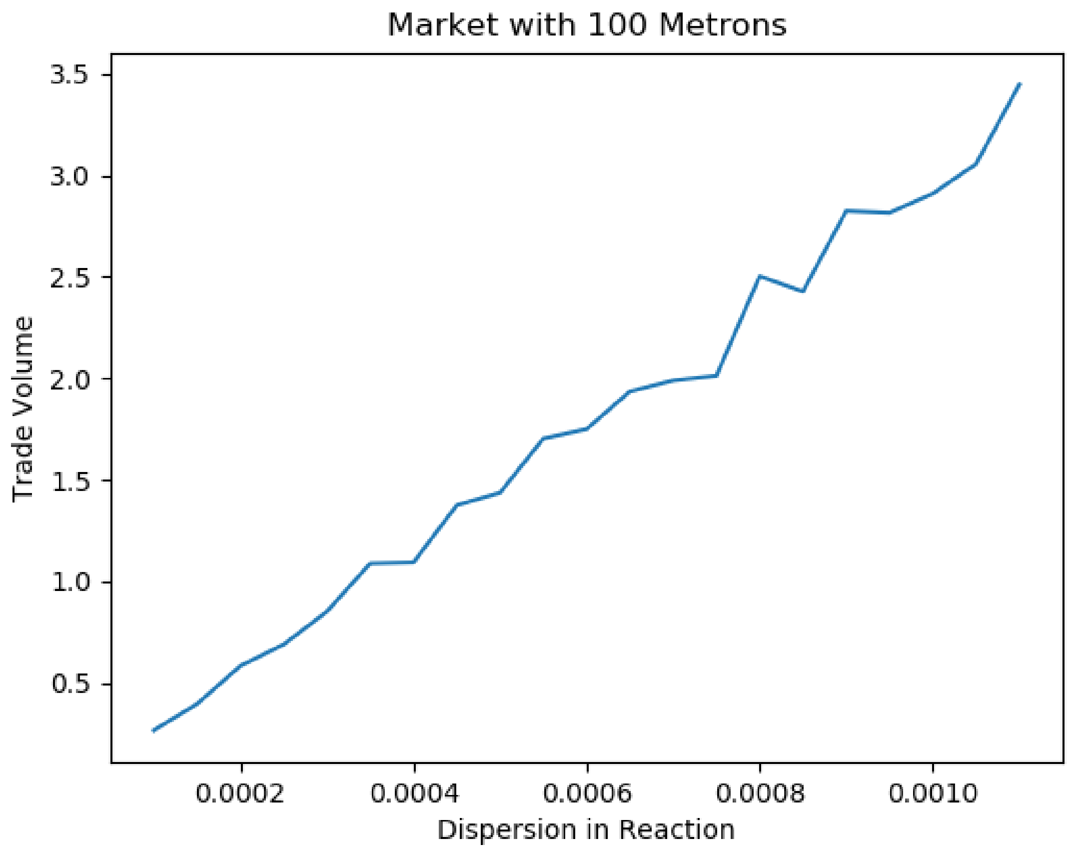


Figure 3 shows that the trade volume increases with dispersion in reaction to news information, which is the variance in agent-specific response (σ_ξ). This result is in line with my expectation, because this means that agents trade more when they respond to news information more differently. Thus in this model, heterogeneity in beliefs gives rise to trading in the market.

Figure 4: Return Volatility and μ_η

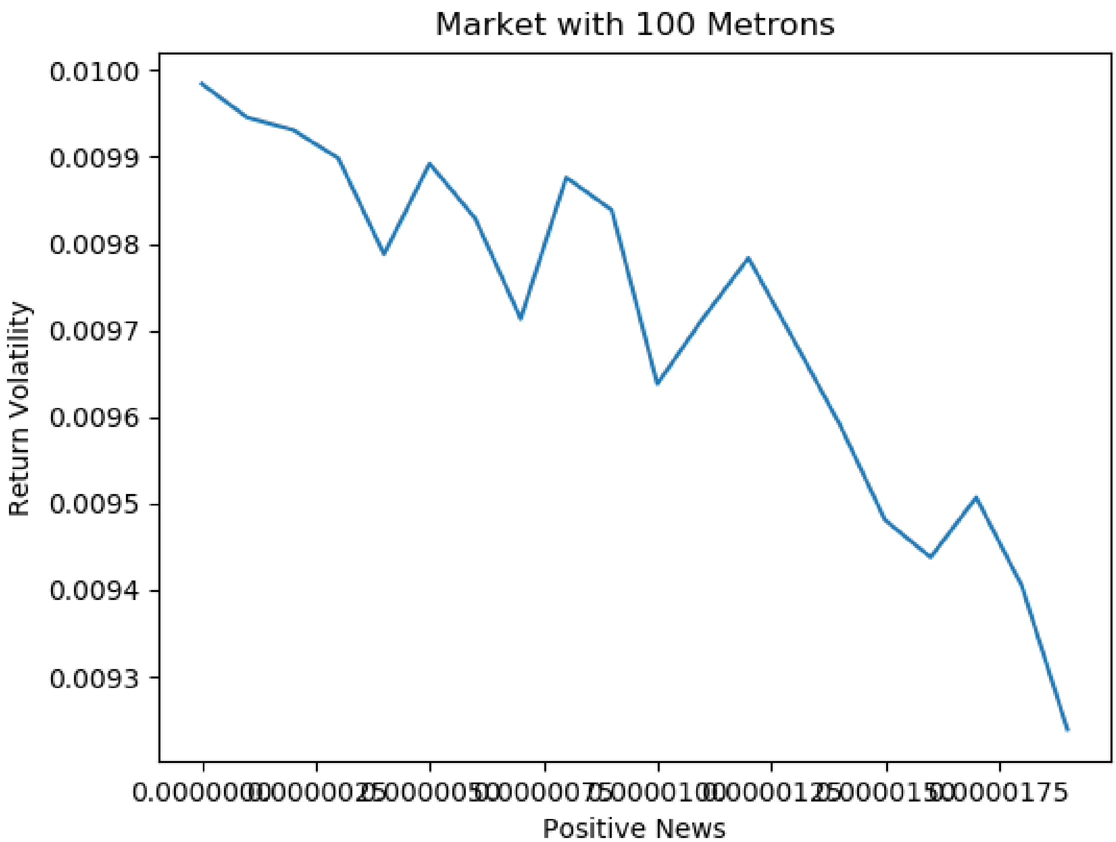


Figure 4 shows that return volatility decreases with more positive μ_η, which means that the average level of response by agents to news information is more positive. I also find that return volatility decreases with more negative μ_η, showing no asymmetry. This result is also within my expectation as my model implies that the stock price would be less volatile if all the investors are becoming more optimistic about the stock.

Figure 5: Trade Volume and News Arrival Frequency

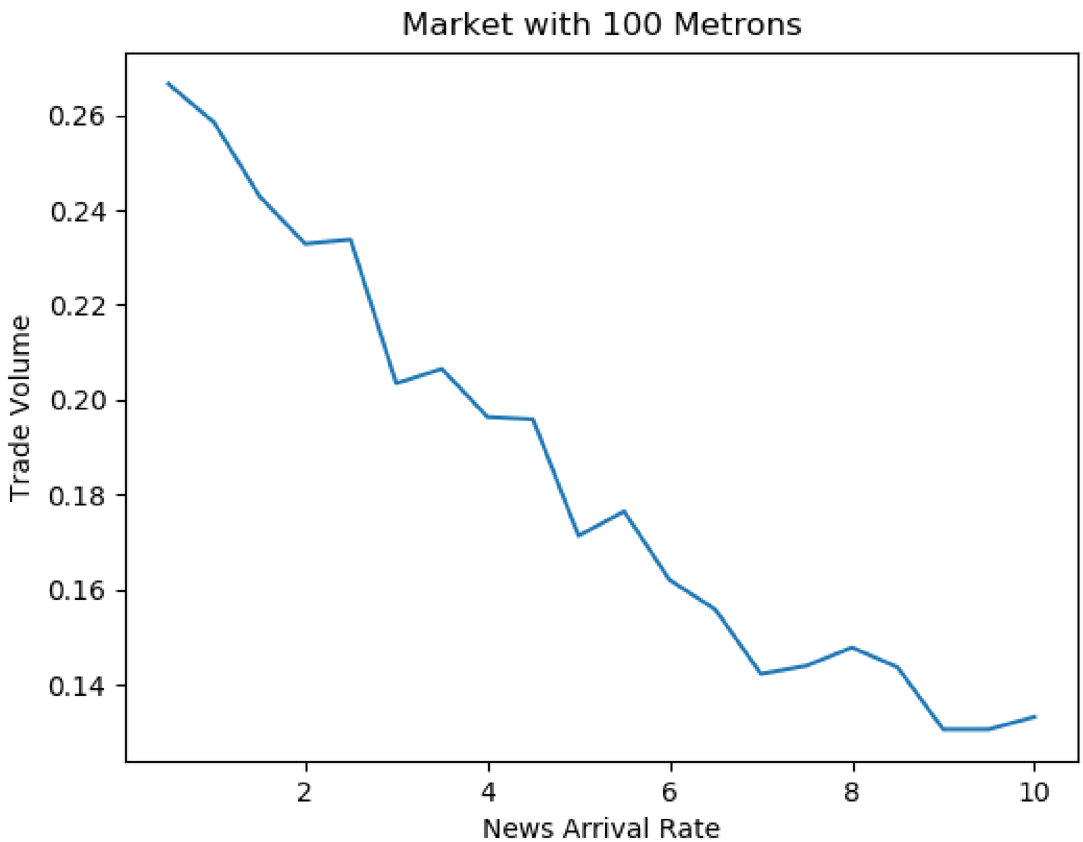


Figure 5 shows that trade volume decreases with more frequent news arrivals. This result is counterintuitive as I expect the heterogeneity in beliefs to accumulate faster with more frequent news arrivals. As of now, I do not have a clear explanation as to why agent trade less if news information arrives more frequently.

4 Future Research

As the second extension to the baseline model, I aim to add an extrapolative component to agent beliefs.

$$\mu_k(t + \Delta t) = (1 - \delta_k)(\mu_k(t) + \eta_t + \xi_{k,t}) + \delta_k \overline{\mu_t}$$

where δ_k denotes the degree to which belief of agent k reflects extrapolated market return $\overline{\mu_t}$ This would make the model more realistic, because it would be more reasonable that investors also adjust their beliefs based on the realized return, not just on news information. It would also make the belief dynamics more endogenous, since each agent's beliefs will be influenced by the realized return, which is determined by other agents' beliefs. The addition will make this ABM more valuable as the beliefs of agents will be affected by interaction (trading) with other agents with different beliefs, instead of relying solely on the arrival of news information, which is exogenous in this model.

5 Conclusion

In this paper, I construct a model in which the stock price is determined by investors who trade based on their heterogeneous and changing beliefs. By simulating this model, I find that trade volume is positively correlated with dispersion in investors' response to news information, suggesting that investors trade more when they have more different opinions. Moreover, I also find that the return volatility is negatively correlated with more positive (or more negative) average level of response by investors to news information, which means that the stock price is less volatile if all investors are becoming more optimistic (pessimistic). Finally, I find that trade volume is negatively correlated to news arrival frequency, contrary to my expectation that more frequent news arrival would accelerate the divergence in agent beliefs and thus increase trade volume.

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