**HONORS LINEAR ALGEBRA, FALL 2021**[1]

MATH-UA 148 at CAS, MATH-UY 3054 at Tandon
Lectures: Mon/Wed 9:30–10:45, WWH 201
Recitations: Fri 2:00–3:15, WWH 109

Instructor: Professor Robert Kohn
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In-person office hour: Monday after class (10:45-11:45) in our classroom or WWH 502
Zoom office hour: Thursday 3-4pm

Teaching Assistant: Paco Rilloraza, fpr2017@nyu.edu
Office hour: Tuesday 5:30-6:30pm in WWH 605.

**Please note**: This honors version of Linear Algebra is intended for well-prepared students who have already developed some mathematical maturity. Its scope includes the usual Linear Algebra (MATH-UA 140) syllabus, *however this class is different* from MATH-UA 140: we will move faster and go deeper; we will place more emphasis on proofs; and we will use a different textbook.

**Are you ready for this class**? The official prerequisites are listed at https://math.nyu.edu/dynamic/courses/undergrad/math-ua-148/. However students who meet these prerequisites may still find this class too difficult, since it assumes more mathematical maturity than is typically developed in a high school AP Calculus class or a single semester of college-level calculus. Potential sources of the desired level of mathematical maturity include summer or after-school enrichment programs, or additional exposure to college-level mathematics. Prior exposure to linear algebra is *not* expected, though students with such exposure will naturally find it helpful.

Mathematical maturity is difficult to define, and difficult to assess. But students who are ready for this class would, for example, have experience with proofs by mathematical induction; with arguments involving summation and rearrangement; and with turning word problems into math problems. Thus if you're ready for this class you should find the following questions straightforward:

(1) Use induction to show that $\sum_{j=1}^{n} j^3 = \frac{n^2(n+1)^2}{4}$ for any integer $n \geq 1$.

(2) Show that for any real numbers $a_1, \ldots, a_n$ (not necessarily positive), $\sum_{i,j=1}^{n} a_i a_j \geq 0$.

(3) Recall that an "opening-upward parabola centered at 0" has the equation $y = ax^2 + b$. Show that the points $(1, 1)$ and $(2, 2)$ lie on a unique opening-upward parabola centered at 0, and find the equation of that parabola.

**Textbook**: Our primary text will be *Introduction to Linear Algebra*, Fifth Edition (2016), by Gilbert Strang. The publisher is Wellesley-Cambridge Press. The NYU bookstore is presently renting it for $50.70 and selling it for $78 new. (The book's website http://math.mit.edu/~gs/linearalgebra/ has links for other ways to order it, including some that work outside the US, but I doubt that any of them will be significantly less expensive.) *Be sure you get the correct book*,

[1]This version differs from v2 only by including information on office hours. Version 2 differed from the Aug 11 preliminary syllabus mainly by having new or revised sections on *Course requirements*, *What to expect of the lectures and recitations*, *What else is there besides Lectures and Recitations?*, and *What if you can't come to class?*, and by including a tentative semester plan (including the dates of our exams).

not the text for MATH-UA 140 (which is a different book by the same author – its title is Linear Algebra for Everyone).

Strang's book is great. It is rich with links to applications; its explanations are clear; and it has lots of problems – with solutions available on the book's website! But it isn't perfect. The greatest deficiency, in my view, is that Strang postpones discussing linear transformations and change-of-basis until very late in the book. For this topic, and for an alternative viewpoint on other topics as well, we'll draw from another fine book: *Linear Algebra Done Wrong* by Sergei Treil. It is available as a free pdf download (subject to the Creative Commons licence) from the author's website (The most recent version there is dated Jan 11, 2021; we'll use that.)

**What we'll cover**: Briefly, we'll do (most of) the material in Chapters 1-8 of Strang's book, though linear transformations (Chapter 8) will come after Orthogonality (Chapter 4). This means our main segments will be

- Solving linear equations
- Vector spaces and subspaces
- Orthogonality
- Linear transformations and change of basis
- Determinants
- Eigenvalues and eigenvectors
- Singular value decomposition

Various applications will be taken up when we discuss the tools they use. (Strang embeds many applications in Chapters 1-8, but we'll also do a few from Chapter 10.) For more detail, see the tentative semester plan at the end of this syllabus.

**Course requirements**: There will be weekly homework assignments, two midterm exams, and a final exam. They will be weighted equally (1/4 HW, 1/4 each midterm, 1/4 final exam). In calculating your HW score for the semester, one week's HW (the lowest) will be excluded, and the other weeks will be weighted equally.

Homework assignments will usually be distributed sometime Wednesday. You'll upload your solutions using Gradescope. Each HW will have a due date and time (usually Tuesday midnight the next week). Late homework will be accepted with a 20% penalty until a late-submission deadline (usually Friday noon). Not every problem will be graded, however you won't be told which problems will be graded. HW solution sheets will be distributed a few days after the late-submission deadline.

Our midterm exams will be *Tuesday 10/12* (which is a class day, since NYU classes meet that day on a Monday schedule) and *Monday 11/22*. According to the NYU Registrar's website, our Final Exam will be *Monday Dec 20, 10-11:50am*.

Makeup exams will be given only for appropriate reasons. If the reason for requesting a makeup is known in advance (for example a religious holiday or an athletic competition), permission to take a makeup must be requested *before* the exam date. Requests based on personal convenience (e.g. flights home are cheaper before the final) will *not* be granted.

**What to expect of the lectures and recitations:** The lectures are not a substitute for reading the textbook. Rather, the main goal of each lecture is to help you see the overall shape of that day's topic, providing intuition and emphasizing the most important points. To get the most out of the lectures, you are strongly encouraged to read ahead, so you'll already have some familiarity with that day's topic. After a topic has been discussed in lecture, you'll want to read about it again (probably more than once, maybe in Treil's book as well as Strang's) to gain full mastery.

One learns new ideas by using them. Therefore the recitations will mainly be problem-oriented: Paco will challenge you with problems or questions related to recent material. You'll have time to do them, then the answers will be discussed. The recitations are also an opportunity for you to ask questions, of course.

Let me repeat: *one learns new ideas by using them.* A particular strength of our textbook is that it has *many* problems, with solutions available at the book's website. Use those problems to reinforce your understanding. For maximum benefit, *don't look at a problem's solution until after you have done your best to solve it.* Examples in the book can also be treated as problems, by closing the book and trying to reproduce its discussion on your own.

**What else is there, besides Lectures and Recitations?** Briefly, the answer is: a weekly recap, and office hours.

Each Wednesday evening a 10-15 minute *weekly recap* zoom recording will be posted. It will briefly indicate what was covered in that week's lectures and provide guidance about what to read to stay up to date.

Students who missed the week's lectures are especially encouraged to join Professor Kohn's zoom office hour on Thursday. However, all students are welcome at any office hour, and *you don't need to have a specific question to join either an in-person or zoom office hour.*

Another resource: each lecture will be recorded using the ceiling-mounted webcam at the back of the room; you'll have access to the recording through the class website. While watching a recording is not like being in class, it is better than nothing. And if you want to review a particular segment of the lecture, the recording makes it easy to do that.

**What if you can't come to class?** As NYU resumes in-person classes this Fall, we understand that students may sometimes need to stay home. (In particular, *please don't come to class if you have covid symptoms!*) The additional resources just discussed – the weekly recap, Professor Kohn's zoom office hour, and the recording of lectures – should help you stay up to date. Repeating for emphasis: *students who missed one or both of the week's lectures are especially encouraged to join Professor Kohn's zoom office hour* on Thursday.

**Computing**: Programs such as Matlab and Mathematica are useful tools for solving linear algebra problems. (Even Excel can do basic linear algebra.) You'll be introduced to Matlab a few weeks into the semester. (All NYU students have access to this software through the NYU site license.) We'll basically use it as a calculator that can do linear algebra problems. It can do much more – Matlab is a programming language – but more sophisticated uses of Matlab lie beyond the scope of this class. When a HW problem calls for by-hand calculation, you are still welcome to check your work using a program.

**Collaboration on homework**: Collaboration is encouraged, however *each student must write up their own solutions* (this is an important part of the learning process). If you work closely

with someone else, get help from a book, take a solution from the web, etc, please *identify your collaborators and/or sources*. Direct copying of another student's solutions is *not* permitted – both because it amounts to cheating, and because it is defeats the entire purpose of the homework (which is to gain practice and familiarity with new concepts and techniques).

**Academic integrity**: Plagiarism and cheating will not be tolerated. NYU's College of Arts and Sciences has policies in this area, and they will be followed; see
http://cas.nyu.edu/academic-integrity.html.

**Tentative Fall 2021 Semester Plan**

**9/8** Getting started: vectors and linear combinations, lengths and dot products, matrices, systems of linear equations (Sections 1.1-1.3 and 2.1). HW1 distributed 9/8, due midnight 9/14.

**9/13, 9/15** Solving linear systems by Gaussian elimination (Sections 2.2-2.3); rules for matrix operations (Section 2.4); introduction to Matlab. HW2 distributed 9/15, due midnight 9/21.

**9/20, 9/22** Inverse matrices (Section 2.5); a matrix perspective on Gaussian elimination via $LU$ factorization (Section 2.6); transposes and permutations (Section 2.7). HW3 distributed 9/22, due midnight 9/28.

**9/27, 9/29** Vector spaces and subspaces (Section 3.1); the null space of a matrix, and the complete solution of $Ax = b$ (Sections 3.2-3.3). HW4 distributed 9/29, due midnight 10/5.

**10/4, 10/6** Independence, basis, and dimension (Section 3.4); dimensions of the four basic subspaces (Section 3.5); review. No HW assignment due to the upcoming midterm.

**10/12, 10/13** Note that this week there's no class on Monday 10/11; instead we meet Tuesday 10/12 (Legislative Day). *Our first midterm exam will be Tues 10/12,* covering material discussed through 9/29 and reinforced by HW1-HW4. On 10/13 we'll reinforce the material from 10/4-10/6; if time permits we'll also discuss graphs & networks (Section 10.1). HW5 distributed 10/13, due midnight 10/19.

**10/18, 10/20** Orthogonal complements and projections (Sections 4.1-4.2); introduction to Fourier series (Section 10.5). HW6 distributed 10/20, due midnight 10/26.

**10/25, 10/27** Least square approximation (Section 4.3); the Gram-Schmidt procedure for finding an orthogonal basis (Section 4.4). HW7 distributed 10/27, due midnight 11/2.

**11/1, 11/3** Matrix representations of linear transformations; change of basis (Treil's Section 2.8 and Strang's Chapter 8). HW8 distributed 11/3, due midnight 11/9.

**11/8, 11/10** Determinants (Sections 5.1-5.3). HW9 distributed 11/10, due midnight 11/16.

**11/15, 11/17** Eigenvalues and eigenvectors (Section 6.1); diagonalization, when it's possible (Section 6.2); the exponential of a matrix, and use of linear algebra to solve ODE's (Section 6.3); review. No HW assignment due to the upcoming midterm.

**11/22, 11/24** *Second midterm exam on Mon 11/22,* covering material discussed through 11/10 and reinforced by HW1-HW9. Our topic for 11/24 will be the Perron-Frobenius theorem with applications to Markov processes and economics (Section 10.3). HW10 distributed 11/24, due midnight 11/30.

**11/29, 12/1** Symmetric matrices, quadratic forms, and positive definiteness (Section 6.4-6.5). HW11 distributed 12/1, due midnight 12/7.

**12/6, 12/8** Singular value decomposition (Sections 7.1-7.2), and a bit about some data science applications (Section 7.3). HW12 distributed 12/8, due midnight 12/14.

**12/13** Wrap up SVD and review.

**12/20** According to the NYU Registrar's website, our final exam slot is Monday Dec 20, 10:00–11:50am.