



Free motion of a rotating cylinder under gravity in a
viscous incompressible fluid

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Motivation

- No spinning: Fall straightly to the ground
- Backspin: Gliding for a long distance

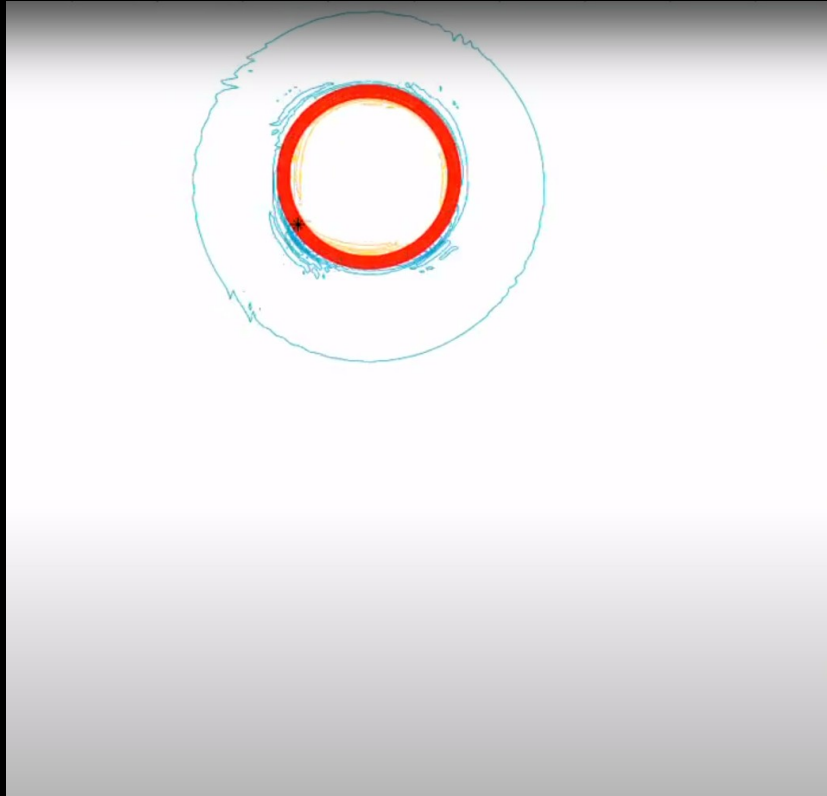


Credit: Veritasium

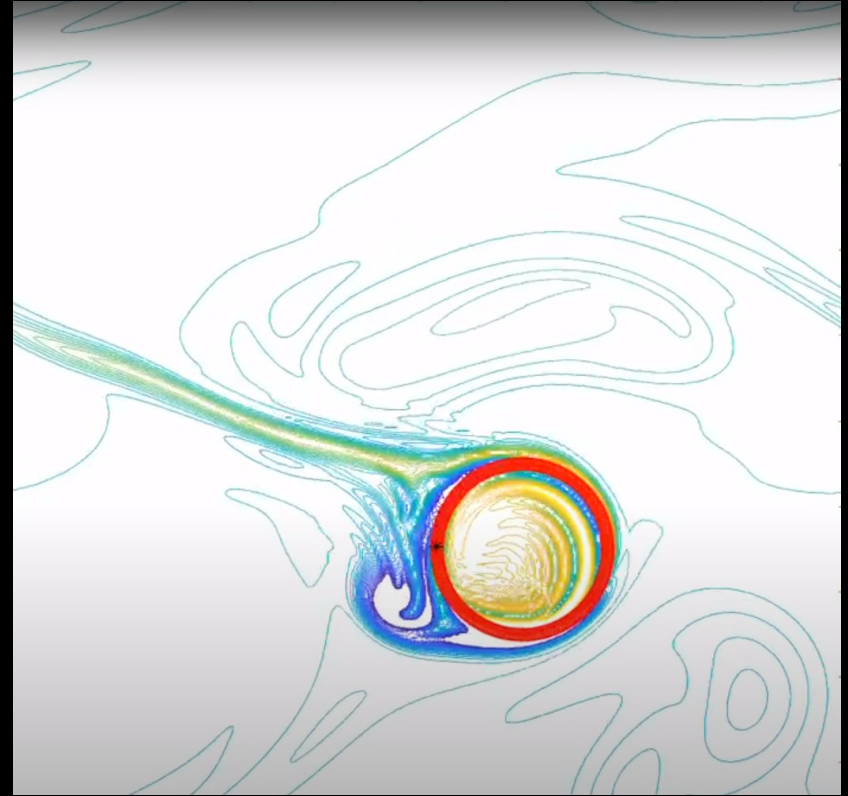
Can we simulate a simplified basketball?

- A hollow cylinder slightly heavier than water (~3%), in a periodic domain
- Fix rotation rate but leave the center of mass free
- We found:
 - * Frequency(f) is **high, steady** horizontal motion
 - * Frequency is **low, unsteady** gliding motion
 - * The transition is sharp, there exists critical f
- Key result: quantitative relation between U and f during **steady state**

Animations



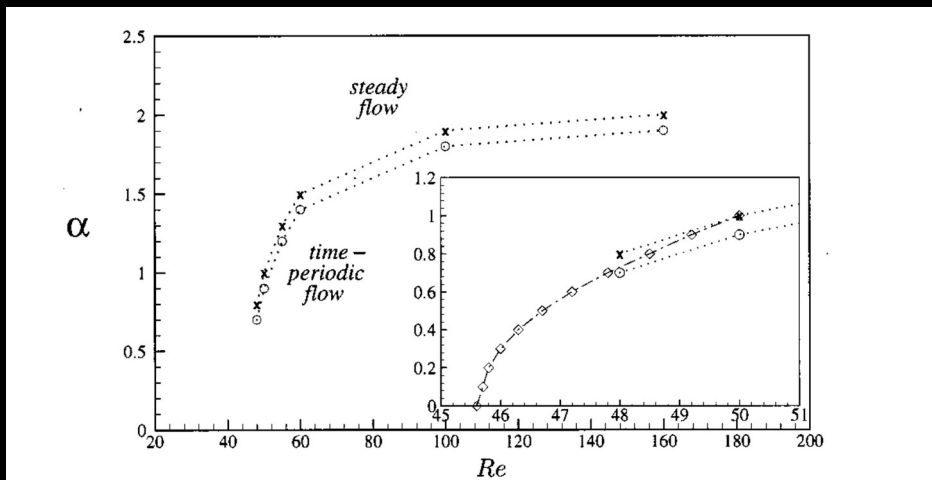
Steady, 5.5 rotations/second



Unsteady, 2 rotations/second

Literatures

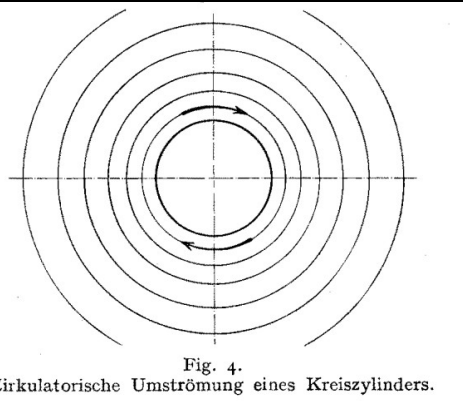
- Flow past a rotating cylinder which was hold in place (experiments/simulations)
- Novelties:
 - * Effect of gravity
 - * Degree of freedom, cylinder itself finds its U
- Re - α phase diagram. Line shows separation between steady and unsteady state.



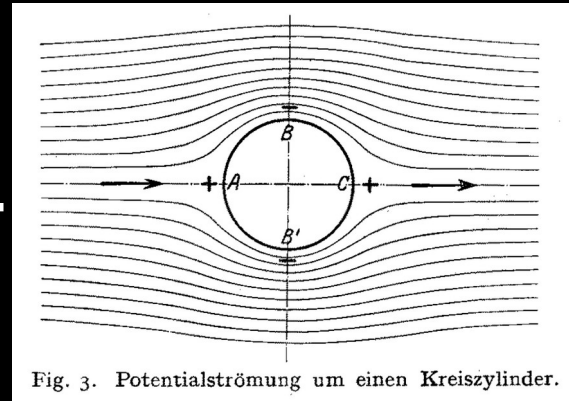
Kang et al. 1999

A model

- Analyze: superposition of two steady flows in an infinite domain
(Ask me if you want more details)
- **Good** at predicting $U(f)$ when f large enough
- **Not good** at predicting the critical transition frequency



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$$Lift = 2\rho U \pi \omega R^2$$

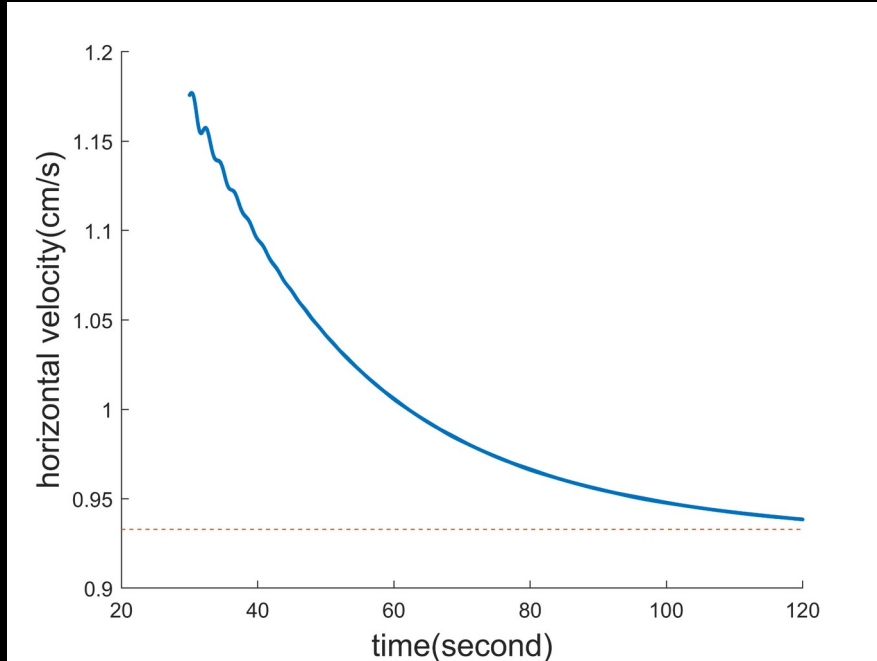
$$2\rho U \pi \omega R^2 = mg$$

$$U(f) = \frac{mg}{4\pi^2 \rho R^2 f}$$

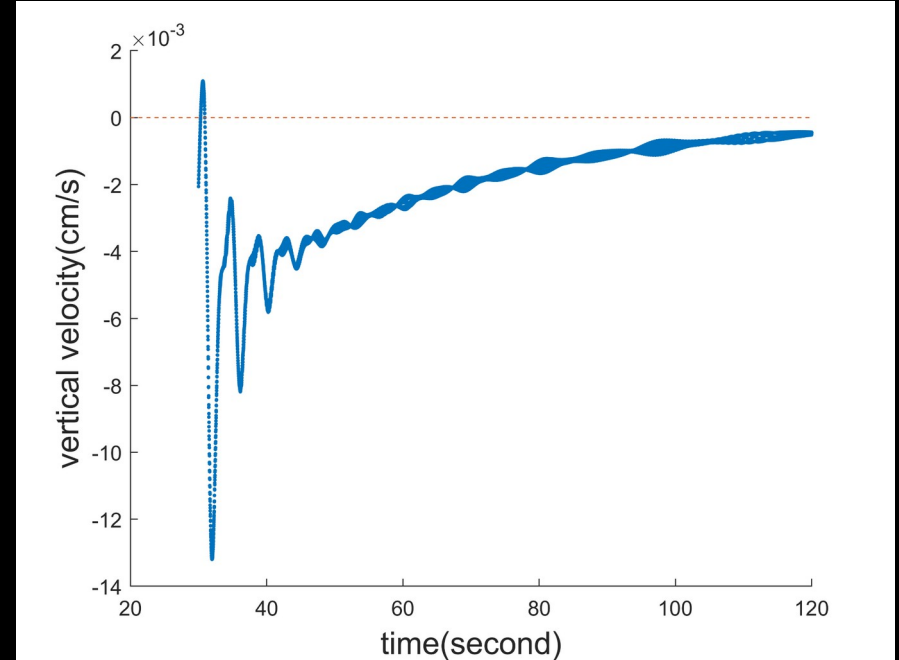
Von L. Prandtl, 1925

Velocity components of the cylinder

- Mass/length = 0.5 gram/cm, 10 rotations/second, radius = 0.5 cm



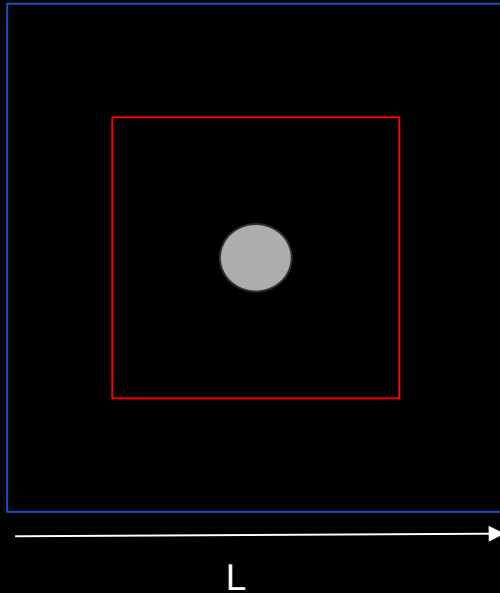
Horizontal velocity(cm/second) vs. time(second)



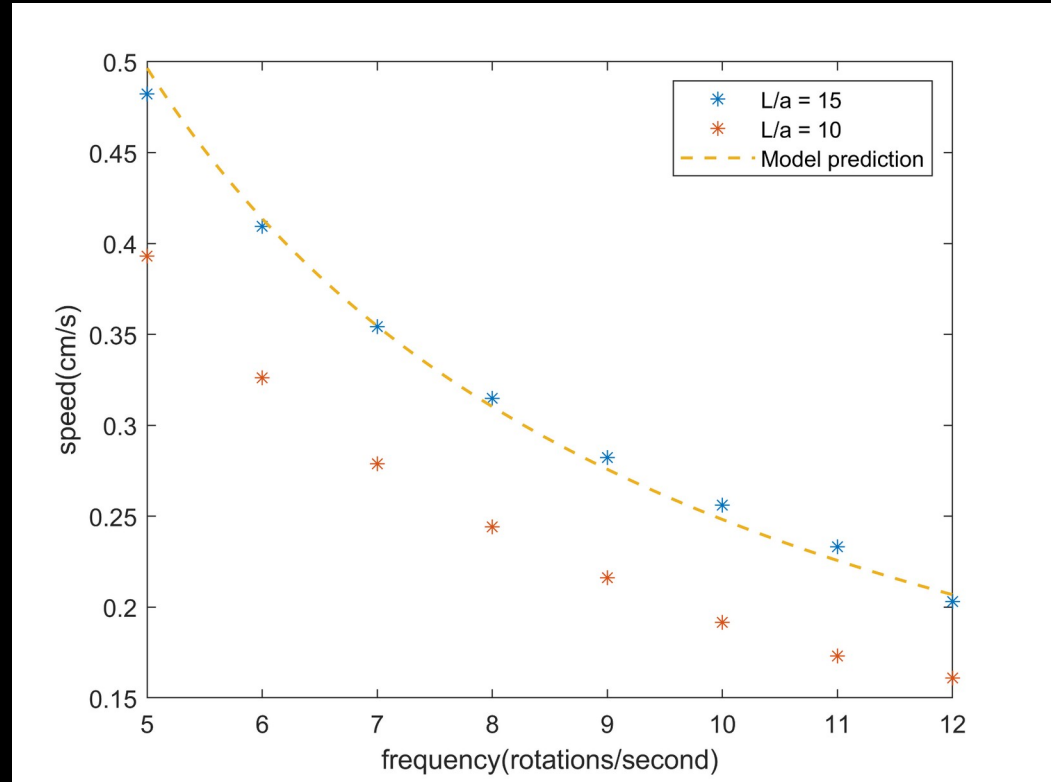
Vertical velocity(cm/second) vs. time(second)

Key result !

- Relationship between steady horizontal speed and rotational frequency

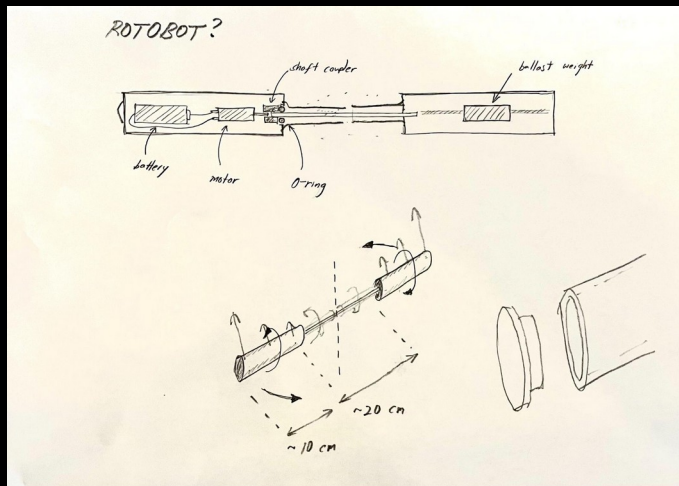


a : radius of the cylinder

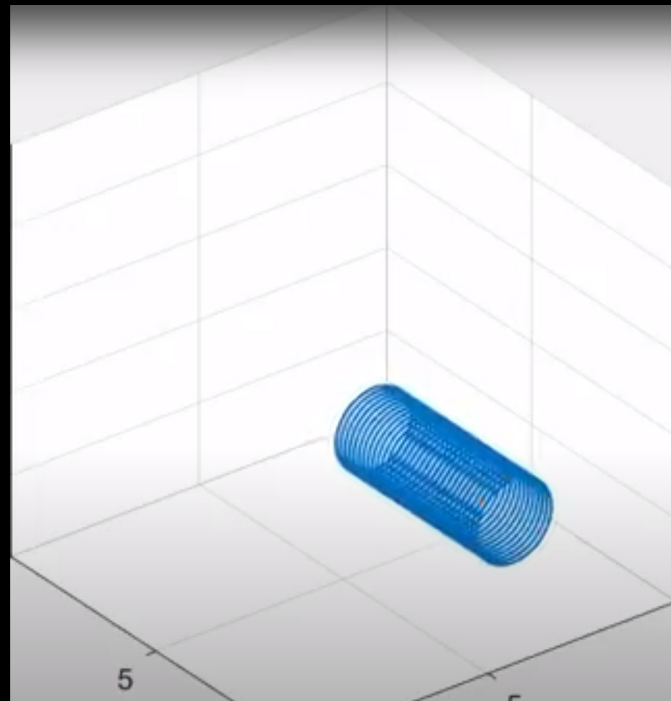


Current and future work

- Transition(boundary layer theory)
- Experiment
- 3D simulation(rigid body motion in 3d)

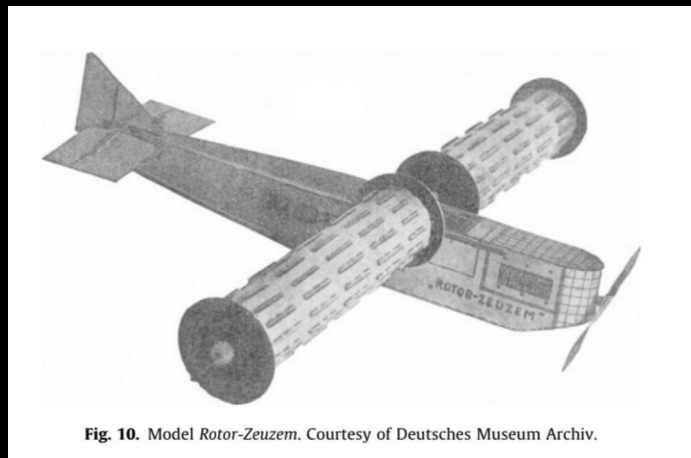


Credit: Leif Ristroph



A simple 3d simulation

Questions?



Jost Seifert, 2012

Numerical method

- A penalty immersed boundary method for a rigid body in fluid, Kim & Peskin 2016
- **Red dots** stick and move with **rigid body**
- **Green dots** move with the velocity of the fluid
- The center of rigid body moves according to

$$F = m a$$

where F is calculated from gravity and the
spring forces

