New York University
Courant Institute of Mathematical Sciences
Mathematics Department
Summer Undergraduate Research Experience (SURE) Program

Summer 2025 Projects

1. **Ben Carpenter**, *Algebraic K-Theory and Its Number-Theoretic Applications*
2. **Jialin Chen**, *Develop a Stable Machine Learning Emulator for Chaotic Dynamical Systems in Climate Modeling*
3. **Hanjun Deng**, *The Logarithmic Minkowski Problem for Pentagons*
4. **Krisha Garg**, *Efficient Prediction of Rare Events with Temporal Difference Learning in the Online Setting*
5. **Edoardo Gargiulo**, *Radial vector orders and order-preserving Lipschitz extensions of maps between Hilbert spaces*
6. **Daniel Kenigsberg and Avi Ray**, *Markov Chain Monte Carlo on SO(n) via Representation Theory*
7. **Yizheng Li**, *Exploring Stochastic Lifting with physical constraints*
8. **Siyong Liu**, *Exploring Physics-Informed Neural Networks*
9. **Sixiong Shao**, *Open-Form Stochastic Differential Equations via Numerical PDE Transformations and Symbolic Learning*
10. **Armin Ulrich**, *PDE‑Constrained Coupling of Micromagnetics and Magnetostatic Optimization for Permanent‑Magnet Stellarators*
11. **David Veerman**, *Experiments, modelling and simulations of droplet dynamics on fibers*
12. **Zichu Wang**, *Optimal transport with Density-dependent Lagrangian*
13. **Fangzheng Yu**, *Noise in Equations: An exploration to BM, SDEs, and SPDEs*
14. **Thomas Zhang**, *Fractal Geometry*
15. **Sifan Sylvia Zhang**, *Application of optimal transport to biomedical data*

SURE Participant	Project Title	Faculty Mentor
Ben Carpenter	*Algebraic K-Theory and Its Number-Theoretic Applications*	Sylvain Cappell

We present an overview of algebraic K-theory. First, we develop the notions of K0(R) and K1(R) and describe their connections to properties of matrices in GL(R). These allow us to describe two fundamental theorems of algebraic number theory: the finiteness of the class group of a ring of integers, as well as the Dirichlet unit theorem. Next, we examine K2(R) in two different ways: one relating to matrices in GL(R) (similar to the approaches for K0(R) and K1(R)), and another, more abstract perspective informed by category theory. We work with these to obtain several results relating to number fields, including a K-theoretic proof of the law of quadratic reciprocity.

Jialin Chen	*Develop a Stable Machine Learning Emulator for Chaotic Dynamical Systems in Climate Modeling*	Sara Shamekh

Machine–learning (ML) emulators are increasingly used to approximate chaotic dynamics, yet good one–step accuracy does not guarantee dynamical faithfulness over long rollouts. Using the one-tier Lorenz–96 (L96) system as truth, we study when emulators reproduce hallmark behaviors of chaos—short-time amplification of perturbations, saturation of spread, and their dependence on forcing F. We ask: (i) do emulators exhibit the same error-growth behavior as the numerical solver, and how do onset, rates, and saturation compare? (ii) does adding memory via an LSTM lookback q help near-term prediction without making the model overly history-dependent? (iii) how do past context and explicit knowledge of F affect generalization and divergence? (iv) how do target choices (X vs. ∆X) and numerical–ML hybridization (AB3 vs. direct substitution) shape outcomes? and (v) what happens as F varies in time—can surrogates track a drifting attractor, and how should that be measured?

Hanjun Deng	*The Logarithmic Minkowski Problem for Pentagons*	Gaoyang Zhang

We investigate a constrained version of the logarithmic Minkowski problem in the planar, discrete setting: Let $\{(v_i, a_i)\}_{i=1}^{5}$ be prescribed unit vectors $v_i \in S^1$ ordered counter-clockwise and positive numbers $a_i > 0$. In addition, let q, r be two points in $\mathbb{R}^2$ and assume $v_2 \times v_5 < 0$. The problem is to determine whether there exists a convex pentagon $P \subset \mathbb{R}^2$ containing the origin in its interior such that

$$V(P, \cdot) = \sum_{i=1}^{5} a_i \, \delta_{v_i},$$

and such that q and r are vertices of P corresponding respectively to the normal pairs (v_2, v_3) and (v_4, v_5).

Krisha Garg	*Efficient Prediction of Rare Events with Temporal Difference Learning in the Online Setting*	Jonathan Weare

Temporal difference (TD) learning has been proven to be powerful for rare event prediction. Existing analysis, however, only quantifies the statistical efficiency of TD in the offline setting—where all data is processed simultaneously. In this work, we investigate the online setting—where data arrive sequentially during training by Richardson iteration—and examine whether the TD estimator remains efficient in this setting. In the rare-event setting, the iteration matrix has a small spectral gap induced by long escape times, which severely slows down the convergence of Richardson iteration. To overcome this, we integrate subspace iteration techniques into Richardson through a Galerkin correction. Specifically, we use simultaneous iteration to build a lower-dimensional basis approximating the dominant eigenvectors of the TD matrix. The Galerkin step then helps correct the Richardson iterates along these dominant directions, which are most responsible for slow convergence. We further stabilize the iteration by averaging along steps before applying the correction. Preliminary numerical experiments demonstrate that this subspace-accelerated scheme significantly improves convergence speed. These findings suggest that incorporating subspace acceleration into LSTD offers a practical pathway toward real-time rare event prediction in reinforcement learning applications.

Edoardo Gargiulo	*Radial vector orders and order-preserving Lipschitz extensions of maps between Hilbert spaces*	Efe A. Ok

We investigate the problem of extending monotone Lipschitz maps between a subset of a partially ordered Hilbert spaces and a second partially ordered Hilbert space, without increasing the Lipschitz constant and preserving monotonicity. In other words, we ask whether a monotonic analogue of Kirszbraun's extension theorem can hold. We show that, under a reasonable compatibility between the order and the inner product in the codomain, a condition known as radiality—introduced by Ok—is necessary on the domain order. Furthermore, we prove that, in normed spaces of dimension two or higher, the only closed radial vector order is equality. This establishes that, under natural assumptions, Kirszbraun's theorem does not admit a monotonic generalization.

Daniel Kenigsberg and Avi Ray	*Markov Chain Monte Carlo on SO(n) via Representation Theory*	Jonathan Goodman

A large problem with MCMC is determining how quickly your Markov Chain can converge to its stationary distribution. This is our motivation behind studying the spectral gap, as it determines the rate of convergence and mixing times of our samplers. This paper aims to establish spectral gap bounds on Markov Chain Monte Carlo samplers of $SO(n)$.

We utilize the MCMC algorithm as outlined in Professor Goodman's MCMC on Manifolds. To build intuition, we first study random walks on finite groups, such as D_n, where the role of representation theory is especially transparent. We then consider the case of sampling on the compact Lie group $SO(2)$. Extending these methods to the general case of $SO(n)$ is the focus of ongoing work.

Yizheng Li	*Exploring Stochastic Lifting with physical constraints*	Benjamin Peherstrofer

We explore a novel generative modeling scheme, Stochastic Lifting, on chaotic/turbulent systems and methods to control quantities of interest(QoI) that contain physical knowledge. Stochastic Lifting pairs each data point with a randomly drawn label $\xi \sim N(0, Id)$, which enables the training of a single smooth map that predicts $\hat{x}_{t+1} = F_\theta(x_t, \xi_t)$ and rolls out autoregressively. This approach adds variation through high-dimensional labels. To respect underlying physics, we apply distributional and per-frame constraints on physical quantities of interest. First, we look at a scalar that depends on the entire trajectory, e.g., the "first time the random wave hits the boundary." In training we build the histogram of these first-hit times in data and nudge our model so its rollout histogram matches (via a Wasserstein/CDF loss) the population QoI distribution. Second, at each time t, we compare simple batch statistics such as integrated mass over the frame—between our predictions and the training set. We add a distributional loss on these scalars, alongside a mean-squared error loss, to align the per-time distributions while keeping frames accurate. Third, to preserve conserved quantities such as mass and energy over time predictions, we enforce a hard constraint via a nonlinear projection step.

Siyong Liu	*Exploring Physics-Informed Neural Networks*	Tristan Buckmaster

PINNs are a relatively new approach for finding approximate solutions to PDEs. In contrast to traditional solvers, they are able to work with incomplete boundary data and fit unknown parameters in the PDE. This makes PINNs useful for discovering self-similar solution profiles to certain PDEs. We examine some factors that affect their performance. We explore some methods to prove that exact solutions exist in a neighbourhood of an approximate solution. We review some recent advances in PINNs and their applications.

Sixiong Shao	*Open-Form Stochastic Differential Equations via Numerical PDE Transformations and Symbolic Learning*	Wenzheng Shi

Discovering stochastic differential equations (SDEs) directly from data is difficult because both drift and diffusion must be inferred from noisy trajectories. We develop a symbolic genetic–algorithm (SGA) pipeline for open–form SDE discovery and compare three routes: (i) a direct extension of SGA–PDE with an explicit Gaussian term; (ii) symbolic learning of the Fokker–Planck equation (FPE) followed by back–out of drift/diffusion; and (iii) a Kramers–Moyal (KM) route that estimates the first two conditional moments from increments and then fits symbolic forms for drift and diffusion separately. We also introduce a multi–pool evolutionary scheme for SGA that prevents single–term collapse and stabilizes the symbolic search.

Empirically and analytically, route (i) identifies drift only in vanishing–noise regimes and fails to recover diffusion; route (ii) succeeds when diffusion terms are simple, but degrades for state–dependent diffusion due to higher–order operator couplings, ill–conditioning, and sensitivity to density–smoothing; route (iii) is robust and accurate across diverse SDEs because it decouples the estimation problems, avoids confounding.

Finally, we outline an extension to stochastic PDEs (SPDEs): after spatial discretization, KM ideas remain applicable but require principled treatment of conditional expectations in a finite, local feature space; alternatively, one can exploit the reliable learning of (constant) diffusion to denoise the dynamics and recover the drift via our optimized SGA–PDE. Together, these contributions provide a practical and statistically grounded framework for data–driven discovery of stochastic dynamics.

Armin Ulrich	*PDE-Constrained Coupling of Micromagnetics and Magnetostatic Optimization for Permanent-Magnet Stellarators*	Alan Kaptanoglu

Permanent magnets provide an attractive path for shaping stellarator magnetic fields. Previous work has shown that greedy permanent magnet optimization (GPMO) can produce sparse, grid aligned arrays that match target surfaces with high accuracy. Here we extend this approach by introducing a macromagnetic model that accounts for finite permeability and demagnetizing interactions at the block level. We integrate this model into the greedy loop through a winner only refinement step that updates the active set of magnets using a linear system solve. We then apply the method to the published MUSE magnet grid. The results show that finite permeability effects produce small tilts and magnitude changes in individual blocks, yet the overall normal field error and plasma facing metrics remain nearly unchanged. In direct comparisons, macromagnetic GPMO and classical GPMO generate almost identical $B \cdot n$ distributions even though the underlying dipole patterns differ. These findings confirm that macromagnetic corrections are small but quantifiable and that they can be absorbed into the synthesis step without loss of computational efficiency. Our formulation provides a fast and practical tool for designing permanent magnet stellarators that are robust to finite permeability effects.

David Veerman	*Experiments, modelling and simulations of droplet dynamics on fibers*	Leif Ristroph

Many emerging applications in chemical and biological testing require fluidic systems capable of transporting and manipulating small, discrete volumes of liquid. One promising approach is to use droplets sliding along fibers, where droplets act as discrete packets or digital currents that can be driven by gravity through networks of fibers. However, the fundamental dynamics of such systems remain poorly understood. In this project, we study the seemingly simple case of a train of droplets deposited at a controlled frequency along an inclined fiber and allowed to propagate under gravity. We developed a mathematical model to describe the motion and interactions of the droplets and implemented a droplet-tracking program to extract quantitative trajectories from experimental data. Preliminary tests have established the feasibility of the setup and provided parameter calibration for both the model and the tracking system. While full experimental datasets are still being collected, this work establishes a framework for systematic comparison between theory and experiment. Ultimately, our study lays the groundwork for predictive models of droplet transport along fibers, with potential relevance to droplet-based mixing, delivery, and assay technologies.

Zichu Wang	*Optimal transport with Density-dependent Lagrangian*	Esteban Tabak

We develop a density–aware, action–based formulation of optimal transport that replaces the standard quadratic cost with the minimal action of a path under a Lagrangian whose speed penalty depends on an ambient probability density. This encodes the Fermat distance intuition: routes through likely regions are cheaper than routes through rare ones. We derive a practical solver by representing paths with endpoint–satisfying Chebyshev polynomials and evaluating the action with Gauss–Legendre quadrature. The construction yields simple, closed-form gradients: coefficient updates follow from the Euler–Lagrange structure and endpoint gradients are provided by the path momentum, enabling efficient optimization without shooting. To lift from pairs of points to full distributions, we parameterize families of paths in a reproducing kernel Hilbert space over anchor points and enforce alignment of learned endpoints with empirical source and target clouds using lightweight, feature-based penalties. We further generalize to a metric-weighted Lagrangian while preserving the same optimization pipeline. Experiments on Gaussian and ring-shaped backgrounds show that density-weighted geodesics bend along high-probability corridors and that the resulting transport correctly pairs and clusters data in settings where Euclidean Wasserstein couplings fail.

Fangzheng Yu	*Noise in Equations: An exploration to BM, SDEs, and SPDEs*	Jalal Shatah

This module develops a unified framework for equations driven by noise. It begins with Brownian motion—independent stationary increments, almost-sure nondifferentiability, quadratic variation [B]t = t, and Hölder regularity below 1/2—motivating Itô integration and Itô's formula with the drift correction from (dB) 2 = dt. Several useful facts are then presented: continuity criteria for random functions (e.g., Kolmogorov), the connection between Itô and Stratonovich integrals and the associated drift adjustment, and random change of variables for random fields via the Itô–Wentzell formula to account for cross-variation generated by the same noise. The discussion proceeds to stochastic differential equations (SDEs) dXt = b dt + σ dBt and basic numerics, and then to classifications of stochastic partial differential equations (SPDEs): viewing them both as stochastic equations and as PDEs with noise, together with the main notions of solutions (classical, weak/distributional, mild, and variational) and their relationships. For linear models, closed-form (kernel) representations are outlined for the heat, wave, and Poisson equations; a brief overview of nonlinear settings is also included. The result is a coherent pipeline

$$\text{BM} \rightarrow \text{Itô} \rightarrow \text{SDE} \rightarrow (\text{Itô–Stratonovich / Itô–Wentzell}) \rightarrow \text{SPDE}$$

with both linear and nonlinear entry points.

Thomas Zhang	*Fractal Geometry*	Lai-Sang Young

Fractal Geometry provides tools to analyze "non-smooth" or "irregular" sets, in contrast to the smooth and regular objects of classical calculus. We introduce two notions for defining fractal scaling, Minkowski and Hausdorff dimension. We extend these definitions with Minkowski content and Hausdorff measure, generalizations of Lebesgue measure which assign a "fractional measure" to sets of non-integral dimension. We discuss invariance of dimension under bi-Lipschitz maps and how they preserve geometric structure. Special attention will be paid to the key results of Marstrand, including almost-sure preservation of dimension under orthogonal projection and existence of densities for fractals. We will also cover some applications to dynamical systems, showing how fractals arise as invariant sets and attractors of iterated function systems.

Sifan Sylvia Zhang	*Application of optimal transport to biomedical data*	Esteban Tabak

Rare data is a critical challenge in data analysis problems, especially in biomedical settings. To address this problem, we apply the barycenter optimal transport problem (OTBP) framework to simulate additional data of the rare type. Our results demonstrate that applying OTBP methods on all data groups yields significantly more accurate simulation results than only using data from the rare class. These findings highlight the potential of OTBP in data simulation problems.

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