MATH-UA 264.001 – CHAOS AND DYNAMICAL SYSTEMS
Spring 2021

Lectures: Monday, Wednesday, 11:00–12:15PM ET
Recitation: Friday, 12:30–1:45PM ET

Objectives

Dynamical systems theory is the branch of mathematics that studies the properties of iterated action of maps on spaces. It provides a mathematical framework to characterize a great variety of time-evolving phenomena in areas such as physics, ecology, and finance, among many other disciplines.

In this class we will study dynamical systems that evolve in discrete time, as well as continuous-time dynamical systems described by ordinary differential equations. Of particular focus will be to explore and understand the *qualitative* properties of dynamics, such as the existence of attractors, periodic orbits and chaos. We will do this by means of mathematical analysis, as well as simple numerical experiments.

List of topics

- One- and two-dimensional maps.
- Linearization, stable and unstable manifolds.
- Attractors, chaotic behavior of maps.
- Linear and nonlinear continuous-time systems.
- Limit sets, periodic orbits.
- Chaos in differential equations, Lyapunov exponents.
- Bifurcations.

Contact and office hours

- Instructor: Dimitris Giannakis, dimitris@cims.nyu.edu.
  Office hours: Monday 5:00-6:00PM and Wednesday 4:30–5:30PM ET.
- Recitation Leader: Wenjing Dong, wd583@nyu.edu.
  Office hours: Tuesday 4:00-5:00PM ET.

Textbooks

- Required:
  - Alligood, Sauer, Yorke, *Chaos: An Introduction to Dynamical Systems*, Springer.
    Available online at https://link.springer.com/book/10.1007%2Fb97589.
- Recommended:
  - Strogatz, *Nonlinear Dynamics and Chaos*.
  - Hirsch, Smale, Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*.
    Available online at https://www.sciencedirect.com/science/book/9780123820105.

Assessment

- Weekly homework sets (40%, lowest homework grade dropped).
- Midterm (30%).
- Final (30%).

Assignment submission

Assignments will be nominally issued on Wednesday of each week, and due the following week. We will use NYU Classes for homework submission and grading. Please submit assignments as a *single PDF*.

If you are unable to return your homework, or participate in the lectures, for reasons such as religious observance, medical issues, etc., I request that you notify me in advance or as soon as reasonably possible. Otherwise, assignments which are not turned in on time will receive no marks.

Online lectures and forum

- Online lectures will be delivered on Zoom. See NYU Classes for the Zoom link and registration. Video recordings will be available for asynchronous viewing.
- We will be using a Piazza site for discussion and Q&A. Please use the online forum rather than email for questions about the assignments and material covered in lectures.

Academic integrity

Students are expected to abide by the University Policy on Academic Integrity.

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WEEKLY SCHEDULE

Lecture	Date	Topic	Textbook section (Alligood, Sauer, Yorke)
Lecture 1	1-Feb	Introduction, one-dimensional maps	1.1, 1.2
Lecture 2	3-Feb	Fixed points, periodic orbits	1.3, 1.4
Lecture 3	8-Feb	Logistic maps	1.5, 1.6
Lecture 4	10-Feb	Sensitive dependence on initial conditions	1.7
President's day	15-Feb		
Lecture 5	17-Feb	Two-dimensional maps	2.1, 2.2
Lecture 6	18-Feb	Linear maps	2.3
Lecture 7	22-Feb	Linearization and Jacobians	2.4, 2.5
Lecture 8	24-Feb	Stable and unstable manifolds	2.2
Lecture 9	1-Mar	Chaos, Lyapunov exponents	3.1, 3.2
Lecture 10	3-Mar	Basins of attraction	3.5
Lecture 11	8-Mar	Fractals, fractal dimension	4.1, 4.5
Lecture 12	10-Mar	Chaos in two-dimensional maps	5.1, 5.3
Lecture 13	15-Mar	Midterm	
Lecture 14	17-Mar	Continuous-time systems, review of linear ODEs	7.1
Lecture 15	22-Mar	Examples of nonlinear ODEs	7.2
Lecture 16	24-Mar	Limit sets, periodic orbits	8.1, 8.2
Lecture 17	29-Mar	Poincare-Bendixson Theorem	8.3
Lecture 18	31-Mar	Chaos in differential equations, Lorenz attractor	9.1, 9.2
Lecture 19	5-Apr	Lyapunov exponents in flows	9.6
Lecture 20	7-Apr	Stable manifolds, homoclinic/heteroclinic points	10.1, 10.2
Lecture 21	12-Apr	Crises	10.3
Lecture 22	11-Apr	Bifurcations	11.1, 11.2
Spring break	19-Apr		
Lecture 23	21-Apr	Continuability, bifurcations in one-dimensional maps	11.3, 11.4
Lecture 24	26-Apr	Bifurcations in differential equations	11.7
Lecture 25	28-Apr	Cascades	12.1
Lecture 26	3-May	Review/catchup	
Lecture 27	5-May	Review/catchup	