

HONORS LINEAR ALGEBRA, FALL 2019
MATH-UA 148 at CAS, MATH-UY 3054 at Tandon
Mon/Wed 8:55–10:45, WWH 512

Professor Robert Kohn
kohn@cims.nyu.edu, 8-3217, WWH 502
Fall 2019 office hrs: Mon 2-3, Wed 5-6
Updated 9/4/2019: changed the Wed office hour

Please note: This honors version of Linear Algebra is intended for well-prepared students who have already developed some mathematical maturity. Its scope includes the usual Linear Algebra (MATH-UA 140) syllabus, and it uses the same text as MATH-UA 140, *however this class is different* from MATH-UA 140: we will move faster and go deeper; also, we will place more emphasis on proofs and on computing.

Prerequisites: The official prerequisite is a grade of A- or better in MATH-UA 121 (Calculus I) or MATH-UA 211 (Math for Economics I), or the equivalent. Students may meet these prerequisites by sufficiently strong performance in an Advanced Placement exam, an A level exam, or an International Baccalaureat exam; for information about the the adequacy of such credentials, contact the Mathematics Department (djaggi@cims.nyu.edu).

Students with the the official prerequisites may still find this class too difficult, since it assumes more mathematical maturity than is typically developed in just a year of AP calculus or a single semester of college-level calculus. Potential sources of the desired level of mathematical maturity include summer or after-school enrichment programs, or additional exposure to college-level mathematics. Prior exposure to linear algebra is *not* expected, though students with such exposure will naturally find it helpful.

Are you ready for this class? Mathematical maturity is difficult to define, and difficult to assess. But students who are ready for this class would, for example, have experience with proofs by mathematical induction; with arguments involving summation and rearrangement; and with turning word problems into math problems. Thus if you're ready for this class you should find the following questions straightforward:

- (1) Use induction to show that $\sum_{j=1}^n j^3 = \frac{n^2(n+1)^2}{4}$ for any integer $n \geq 1$.
- (2) Show that for any real numbers a_1, \dots, a_n (not necessarily positive), $\sum_{i,j=1}^n a_i a_j \geq 0$.
- (3) Recall that an “opening-upward parabola centered at 0” has the equation $y = ax^2 + b$. Show that the points $(1, 1)$ and $(2, 2)$ lie on a unique opening-upward parabola centered at 0, and find the equation of that parabola.

Textbook: Our text is *Introduction to Linear Algebra*, Fifth Edition (2016), by Gilbert Strang. The publisher is Wellesley-Cambridge Press. Ordering directly from the publisher (www.wellesleycambridge.com) appears to be cheaper than from Amazon (at least for new copies). The NYU bookstore will have copies at the beginning of the Fall semester. Please get the 5th edition (not a prior edition).

Course requirements: There will be weekly homework assignments, two midterm exams, and a final exam. They will be weighted equally (1/4 HW, 1/4 each midterm, 1/4 final exam). In calculating your HW score for the semester, one week's HW (the lowest) will be excluded, and the other weeks will be weighted equally.

Exam dates: There will be two *midterm exams*, on Mon Oct 7 and Mon Nov 18. You will have the entire class period (8:55-10:45) for each midterm. The scheduling of our final exam is up to the NYU Registrar.

Some Policies

Preparing for class: You will be asked to read a section of the book in advance of each class. Our class time is for discussing ideas, doing examples, etc. It is intended to *supplement* the book, not to replace it. Ideas and examples are important; thus, to do well in this course you are strongly advised to *come to every class*.

Collaboration on homework: Collaboration is encouraged. However each student must write up his/her own solutions (this is an important part of the learning process). If you work closely with someone else, get help from a book, take a solution from the web, etc, please *identify your collaborators and/or sources*. Direct copying of another student's solutions is *not* permitted – both because it amounts to cheating, and because it defeats the entire purpose of the homework (which is to gain practice and familiarity with new concepts and techniques).

Late HW: Each HW assignment will have a due date (usually by the beginning of a particular class). Late HW will not be accepted. Please turn in hard copy if possible; if this isn't possible, a single PDF sent by email is OK. (Separate jpg's of each page are *not* acceptable.)

Computing: Programs such as Matlab and Mathematica are useful tools for solving linear algebra problems. (Even Excel can do basic linear algebra.) You will be introduced to Matlab a few weeks into the semester. When a HW problem calls for by-hand calculation, you are still welcome to check your work using a program.

Makeup exams: Makeup exams will be given only for legitimate reasons such as religious holidays, conflicts with university-sponsored activities (eg athletics), or documented illness. If the reason for requesting a makeup is known in advance, permission to take a makeup *must* be requested *before* the exam date. Requests based on personal convenience (eg flights home are cheaper before the final) will *not* be granted.

Academic integrity: Plagiarism and cheating will not be tolerated. NYU's College of Arts and Sciences has policies in this area, and they will be followed. See <http://cas.nyu.edu/academic-integrity.html>

Tentative Semester Plan

- 9/4** Getting started: vectors and linear combinations, lengths and dot products, matrices, systems of linear equations (Sections 1.1-1.3 and 2.1). HW1 distributed 9/4, due 9/11.
- 9/9, 9/11** Solving linear systems by Gaussian elimination (Sections 2.2-2.3); rules for matrix operations (Section 2.4). HW2 distributed 9/11, due 9/18.
- 9/16, 9/18** Inverse matrices (Section 2.5); a matrix perspective on Gaussian elimination via LU factorization (Section 2.6); transposes and permutations (Section 2.7). HW3 distributed 9/18, due 9/25.
- 9/23, 9/25** Vector spaces and subspaces (Section 3.1); the null space of a matrix, and the complete solution of $Ax = b$ (Sections 3.2-3.3). HW4 distributed 9/25, due 10/2.
- 9/30, 10/2** Independence, basis, and dimension (Section 3.4); dimensions of the four basic subspaces (Section 3.5); review. HW5a distributed 10/2, due 10/16.
- 10/7, 10/9** *First midterm exam on Mon 10/7*, covering material discussed through 9/25 and reinforced by HW1-HW4. Our topic for 10/9 will be an introduction to linear programming (Section 10.4). HW5b distributed 10/9, due 10/16.
- 10/15, 10/16** *No class Mon 10/14 (Fall Break), but on Tues 10/15 classes meet on a Monday schedule (Legislative Day)*. Orthogonal complements and projections (Sections 4.1-4.2); introduction to Fourier series (Section 10.5). HW6 distributed 10/16, due 10/23.
- 10/21, 10/23** Least square approximation (Section 4.3); the Gram-Schmidt procedure for finding an orthogonal basis (Section 4.4). HW7 distributed 10/23, due 10/30.
- 10/28, 10/30** Determinants (Sections 5.1-5.3). HW8 distributed 10/29, due 11/6.
- 11/4, 11/6** Eigenvalues and eigenvectors (Section 6.1); diagonalization, when it's possible (Section 6.2); the exponential of a matrix, and use of linear algebra to solve ODE's (Section 6.3). HW9 distributed 11/6, due 11/13.
- 11/11, 11/13** Symmetric matrices and positive definiteness (Section 6.4-6.5); review. HW10 distributed 11/13, due Monday 11/25.
- 11/18, 11/20** *Second midterm exam on Mon 11/18*, covering material discussed through 11/6 and reinforced by HW1-HW9. Our topic for 11/20 will be the Perron-Frobenius theorem with applications to Markov processes and economics (Section 10.3). HW11a distributed 11/20, due 12/4
- 11/25** *No class Wed 11/27 due to Thanksgiving break*. Singular value decomposition (Sections 7.1-7.2). HW11b distributed 11/25, due 12/4.
- 12/2, 12/4** Application of SVD to principal component analysis (Section 7.3); linear transformations (Section 8.1-8.2). HW12 distributed 12/4, due 12/11.

12/9, 12/11 More on linear transformations (Section 8.3); review.

Final exam The date and time of our final exam will be fixed by the NYU Registrar, sometime during the week 12/16-20.