Honors Linear Algebra, Spring 2021

MATH-UA 148 (CAS) / MA-UY 3054 A (Tandon)

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Honors Linear Algebra is intended for well-prepared students with mathematical maturity. It is rigorous, i.e. proof-based: it is expected that the students enjoy reading/working out proofs and coming up with new ones!

Course topics:

- Vector spaces: Basic notions, linear combinations and independence, bases, linear transformations and their matrix representations, invertibility, range and kernel of transformations. Subspaces, isomorphisms.
- Systems of linear equations: Gauss-Jordan elimination, row and column operations, echolon forms, pivots, LU factorization, solution spaces, matrix inversion, rank-nullity.
- Determinants: Formal definition, basic properties.
- Eigenvalues and eigenvectors: Diagonalization, exponential of a matrix and other matrix functions, applications to differential equations.
- Inner product spaces: Orthogonality, orthogonal bases, orthogonal projections, Gram-Schmidt, least square and minimum norm solutions, isometries and unitary transformations
- Further spectral notions: Symmetric matrices and their diagonalization, singular value decomposition, application to principal component analysis.

Textbook: There are a lot of good textbooks on linear algebra and just as many ways to introduce the subject. Each route has its advantages, and sometimes, disadvantages, but at the end of the day, the material is the same. We will make use of two textbooks:

1. *Linear Algebra Done Right*, by Sheldon Axler (Third Edition), Springer, which is freely available to NYU users via Springer Link:
http://proxy.library.nyu.edu/login?url=http://link.springer.com/10.1007/978-3-319-11080-6

2. *Linear Algebra Done Wrong*, by Sergei Treil, made available freely by the author under the Creative Commons licence. Please see the author's homepage:
http://www.math.brown.edu/streil/index.html
(For definiteness, we will use the version from Jan 21, 2021.)

Homework, exams, grading and other course policies:

There will be weekly homework assignments which will be turned in electronically via NYU Classes. You will be asked to submit a single pdf file for each assignment. If you handwrite your solutions, the easiest way to produce these files is to use a scanning app on a mobile device. Please familiarize yourselves with a good app (such as Microsoft Office Lens or Adobe Scan). Your files should be reasonable in size (i.e. no more than a few MBs). You are also welcome to typeset your work if you enjoy doing so, but this is not expected at all -- you may find that it takes too much time anyway.

There will be two midterms and a final, as well as a number of quizzes placed suitably before the exams. All exams will be given in the form of assignments on NYU Classes, but they will be in real-time, video-proctored over Zoom, and recorded.

You are welcome to collaborate on homework assignments (and even encouraged), but you are expected to always write up your own individual solutions. You are also expected to acknowledge your collaborations as well as all other resources you have benefited from. Online sources of help (such as chegg.com) are discouraged, primarily for your own sake.

In contrast to homework assignments, all exams (including quizzes) will be closed-book, and required to be based solely on your own work. You can think of them as an online version of a typical in-class exam. Academic dishonesty (plagiarism and cheating) will not be tolerated. See, for example, the CAS policy on academic integrity:

https://cas.nyu.edu/academic-integrity.html

The course grade will be based on a weighted average of homework and exam scores, with the following tentative weights:

Homework: 15%
Quizzes: 15%
Midterm 1: 20%
Midterm 2: 20%
Final: 30%

Tentative course calendar:

(This list will be regularly updated on the NYU Classes course site.)

Week	Dates	Reading List
1	Feb 1, 3	Complex numbers, vector spaces and their properties, subspaces, direct sums. Axler: Ch. 1 Treil: Ch. 1 Sec. 1
2	Feb 8, 10	Linear combinations, span, spanning lists, linear independence, bases and dimension. Axler: Ch. 2 Treil: Ch. 1, Sec. 2
3	Feb 17, 18 (in place of Feb 15)	Linear transformations and their compositions, properties, examples. Axler: Ch. 2 (ctd), Ch. 3A Treil: Ch. 1, Sec. 3
4	Feb 22, 24	Null space and range of linear transformations, matrix representation and matrix algebra. Axler: Ch. 3B, 3C Treil: Ch.1, Sec. 4, 5.
5	Mar 1, 3 Quiz 1 (Mar 5)	Invertibility, isomorphism, dual space, adjoint of a transformation. Axler: Ch. 3D, 3E, 3F Treil: Ch. 1, Sec. 6.
6	Mar 8, 10	Solving systems of linear equations: row operations and Gauss elimination, echolon form, pivots, finding the inverse. Treil: Ch. 2, Sec. 1-4
7	Mar 15, 17 Midterm 1 (TBA)	Systems of linear equations continued, more on matrices, row/column rank, change of basis. Treil: Ch. 2, Sec 5-8
8	Mar 22, 24	Determinant, Trace, and other matrix characteristics Treil: Ch. 3 Axler: Ch. 10
9	Mar 29, 31	Eigenvalues and eigenvectors Axler: Ch. 5 Treil: Ch. 4
10	Apr 5, 7 Quiz 2 (Apr 9)	Eigenvalues and eigenvectors continued (eigenspaces and diagonalization, applications to differential equations, exponential of a matrix) Axler: Ch. 5 Treil: Ch. 4
11	Apr 12, 14	Inner product spaces, orthogonality, orthogonal bases, Gram-Schmidt orthogonalization Axler: Ch. 6 Treil: Ch. 5
12	Apr 21 Midterm 2 (TBA)	Least square and minimum norm solutions, applications Treil: Ch. 5
13	Apr 26, 28	Isometries and unitary transformations, self-adjoint transformations and the spectral theorem Axler: Ch. 7 Treil: Ch. 6
14	May 3, 5 Quiz 3 (May 7)	Singular value decomposition and its applications Axler: Ch. 7 Treil: Ch. 6
15	May 10	Closing discussions