Chirality in Modelling of Nodal Cilia Movement

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Introduction

Purpose
Modelling and analyzing the expression of *chirality* in the whirling movement of *nodal cilia*

Definitions
*Nodal cilia:* finger like structures in embryos
*Chirality:* handedness

Significance
Breaking of left/right symmetry in embryos

Structure of Non-Nodal Cilia

Cilia Structure: “9+2” [2]

Model: Single Cilium Unit

Central pair of microtubules = Central vector
9 peripheral doublet microtubules
Dynein motor protein links = Wires with tension

Modelling Assumptions
Inextensible microtubules
Central vector tip centroid

Geometric Constraint
Upper polygon moves only by translation

\[
\frac{dX}{dt} = \frac{1}{\zeta} \left( A(t) - \frac{1}{H^2} (A(t)X(t) + B(t)Y(t))X(t) \right)
\]
\[
\frac{dY}{dt} = \frac{1}{\zeta} \left( B(t) - \frac{1}{H^2} (A(t)X(t) + B(t)Y(t))Y(t) \right)
\]
\[
\frac{dT_j}{dt} = \beta \left( KL_j^2(t) - T_j(t) \right)
\]

**Model**

**Notation**

\(T_j(t)\): Tensions in wire \(j\)

\(X(t), Y(t)\): Central vector coordinates

\(H\): Length of central vector

\(L_j(X(t), Y(t))\): Length of wire \(j\)

\(n\): Number of wires

\(R\): Radius of circle

\(\zeta, \beta, K\): constants
Simulation of Movement

Whirling of nodal cilia visualization [3]  
Our simulation of cilia movement

Unstable Cycles

Case A: goes to steady state solution or vertical position

Case B: goes to maximum amplitude oscillations
Bistability

Single Cilium X,Y (Runge-Kutta)

Single Cilium X,Y (Runge-Kutta)
**Subcritical Hopf Bifurcation**

**Bi-stability**
Parameter interval in which stable max-amplitude cycle coexists with stable steady state

**Notation**
\[ \gamma = \frac{2rC}{H^2 + C^2} \]
\[ r = \text{radius of cycle} \]

\[ \frac{K}{\zeta} \frac{2nC^2}{\beta \sqrt{H^2 + C^2}} \]
Chirality Not Expressed in Movement

Simulation 1
- $X(0) = X^0$
- $Y(0) = Y^0$
- $T_j(0) = T_j^0$

Simulation 2
- $X(0) = -X^0$
- $Y(0) = Y^0$
- $T_j(0) = T_{-j-1}^0$
Future Work

1. Stabilizing Limit Cycles

\[ G = (0,0,g); \quad g: \text{non-zero constant} \]

Find non-maximum amplitude limit cycles by applying constant force to central vector in to vertical position

2. Expressing Chirality

\[ \phi: \text{twist angle} \]

- Nodal cilia structure “9+0”
- Reduce geometric constraint:
  - Twisting upper polygon
  - Resistance to twisting in wires
QUESTIONS?

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