Chirality in Modelling of Nodal Cilia Movement

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Introduction

Purpose

Modelling and analyzing the expression of *chirality* in the whirling movement of *nodal cilia*

Definitions

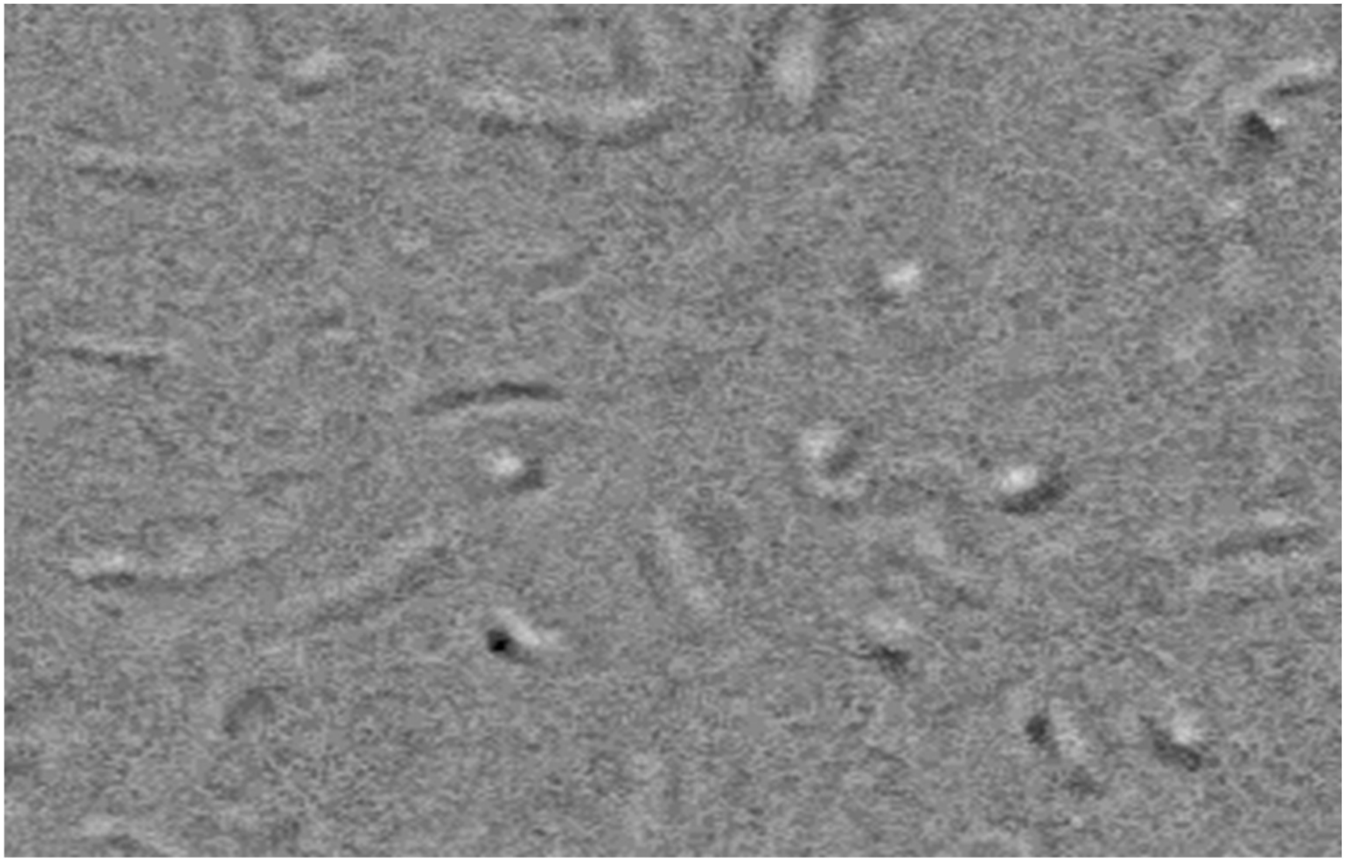
Nodal cilia: finger like structures in embryos

Chirality: handedness

Significance

Breaking of left/right symmetry in embryos

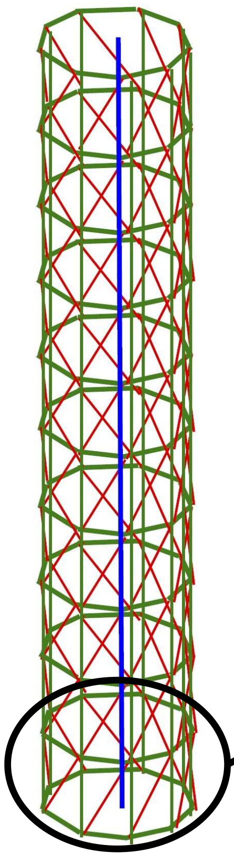
Whirling of nodal cilia [1]



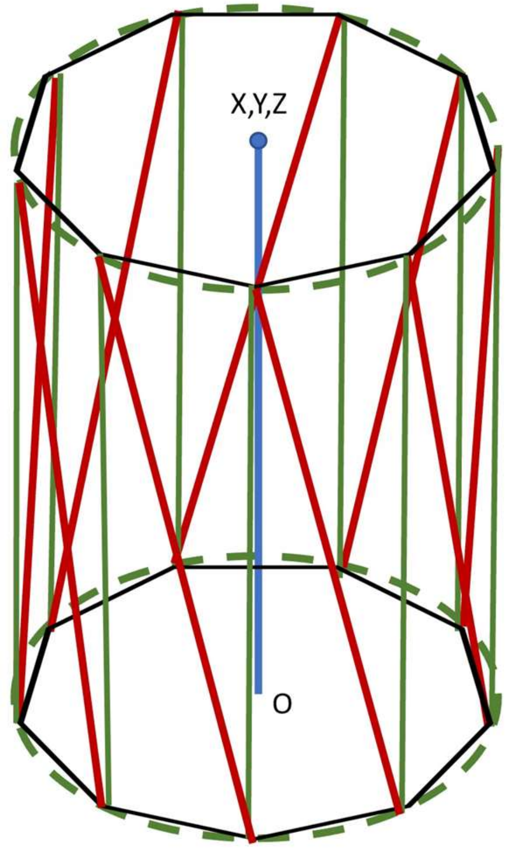
[1] Nonaka S, Yoshiba S, Watanabe D, Ikeuchi S, Goto T, Marshall WF & Hamada H. (2005). De novo formation of left-right asymmetry by posterior tilt of nodal cilia. *PLoS Biol.* , *3*, e268, https://embryology.med.unsw.edu.au/embryology/index.php/Nodal_Cilia_Movie.

- Central pair of microtubules = Central vector
- 9 peripheral doublet microtubules
- Dynein motor protein links = Wires with tension

Structure of Non-Nodal Cilia



Cilia Structure: “9+2” [2]

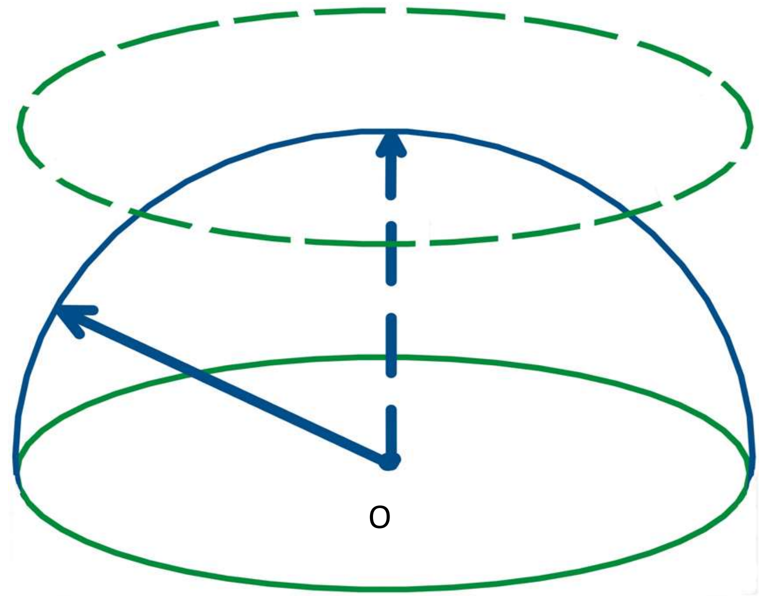


Model:Single Cilium Unit

Modelling Assumptions

Inextensible microtubules

Central vector tip centroid



Geometric Constraint

Upper polygon moves only by translation

[2] Han and Peskin. Spontaneous Oscillation and Fluid-Structure Interaction of Cilia, *PNAS*, 2018.

Model

$$\frac{dX}{dt} = \frac{1}{\zeta}\left(A(t) - \frac{1}{H^2}\big(A(t)X(t) + B(t)Y(t)\big)X(t)\right)$$

$$\frac{dY}{dt} = \frac{1}{\zeta}\left(B(t) - \frac{1}{H^2}\big(A(t)X(t) + B(t)Y(t)\big)Y(t)\right)$$

$$\frac{dT_j}{dt} = \beta\left(KL_j^2(t) - T_j(t)\right)$$

$$A(t) = \sum_{j=0}^{n-1}\left\{\frac{T_j(t)}{L_j(t)} R\left(cos\left(\frac{2\pi}{n}(j+1)\right) - cos\left(\frac{2\pi}{n}j\right)\right)\right\}$$

$$B(t) = \sum_{j=0}^{n-1}\left\{\frac{T_j(t)}{L_j(t)} R\left(sin\left(\frac{2\pi}{n}(j+1)\right) - sin\left(\frac{2\pi}{n}j\right)\right)\right\}$$

Notation

$\boldsymbol{T_j(t)}$: Tensions in wire j

$\boldsymbol{X(t)}$, $\boldsymbol{Y(t)}$: Central vector coordinates

$\boldsymbol{H}$: Length of central vector

$\boldsymbol{L_j(X(t), Y(t))}$: Length of wire j

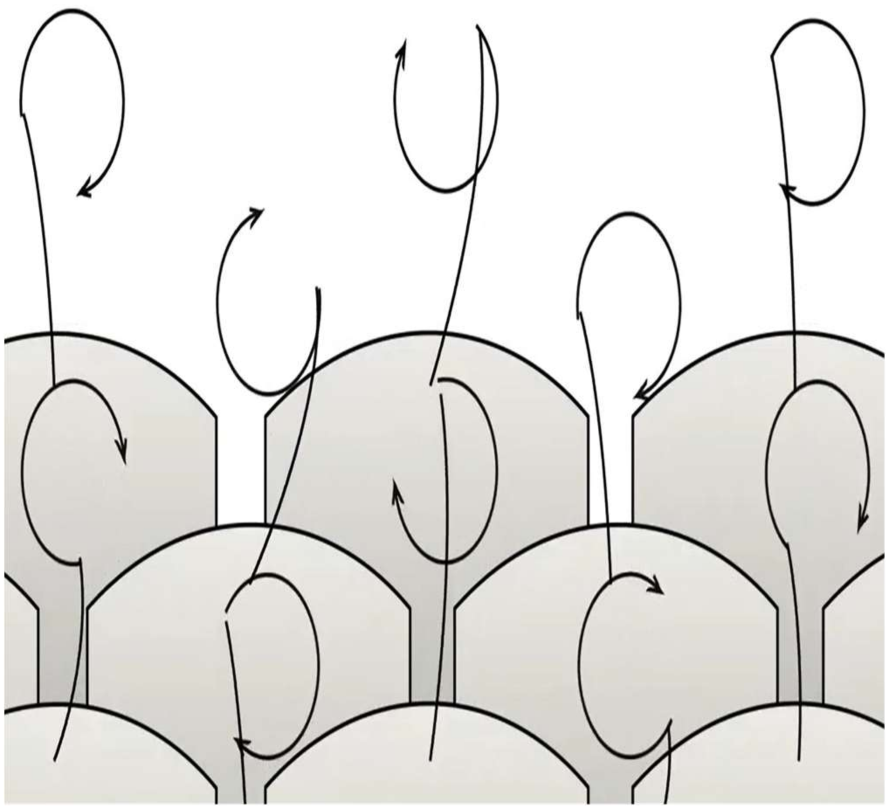
$\boldsymbol{n}$: Number of wires

$\boldsymbol{R}$: Radius of circle

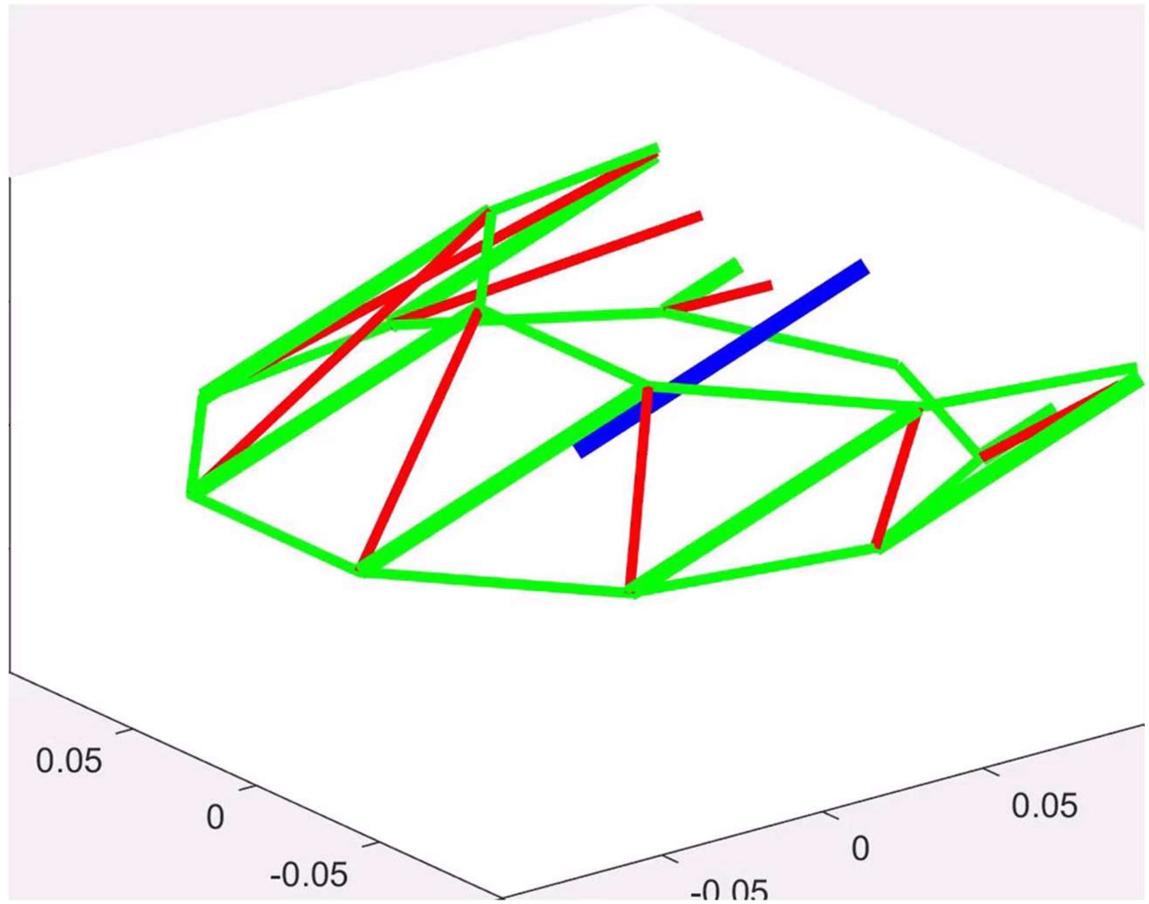
$\boldsymbol{\zeta, \beta, K}$: constants

Simulation of Movement

Whirling of nodal cilia visualization [3]

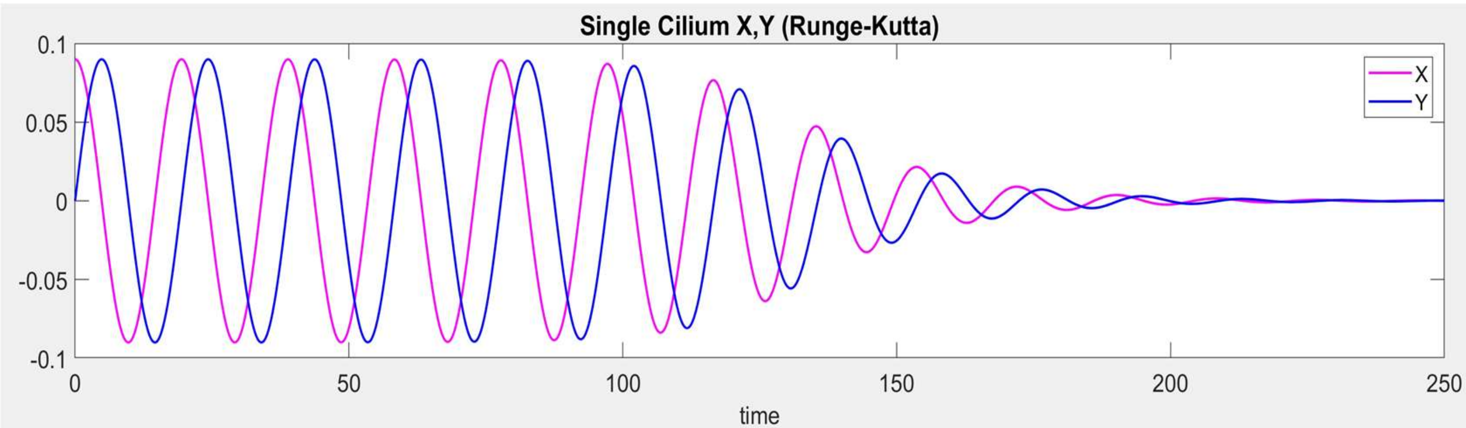


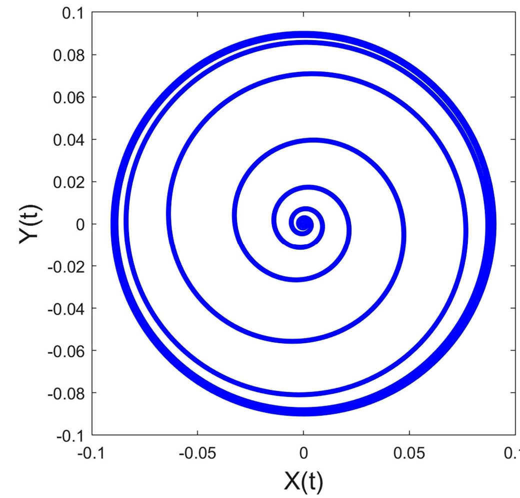
Our simulation of cilia movement



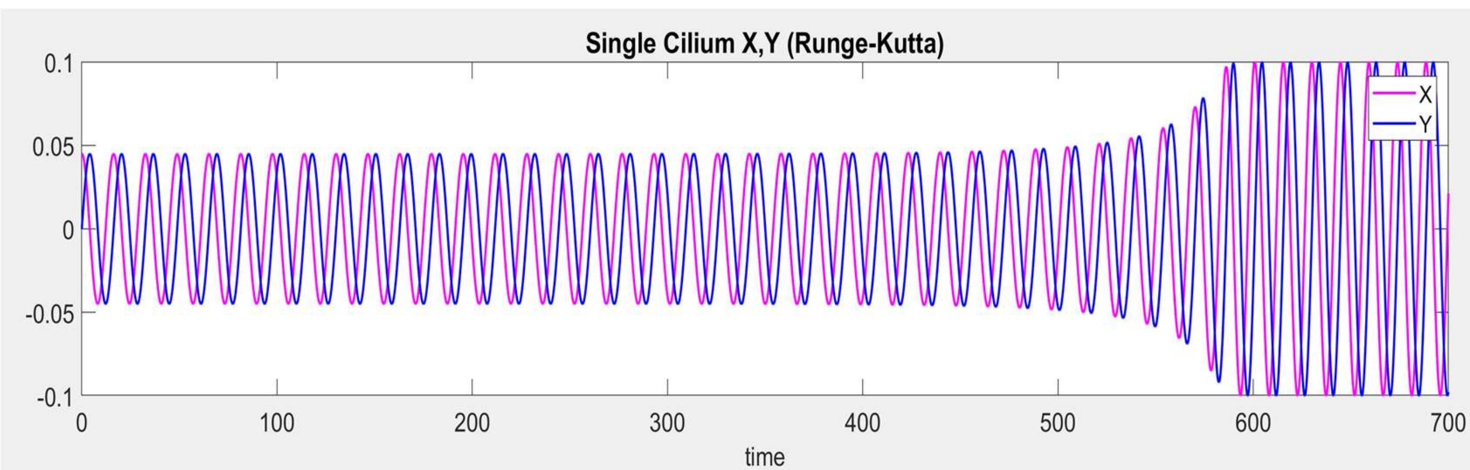
[3] "How Your Body Knows Left From Right." *YouTube*, uploaded by It's Okay To Be Smart, 19 July 2014, https://www.youtube.com/watch?v=oGahCXE18Ko&t=216s.

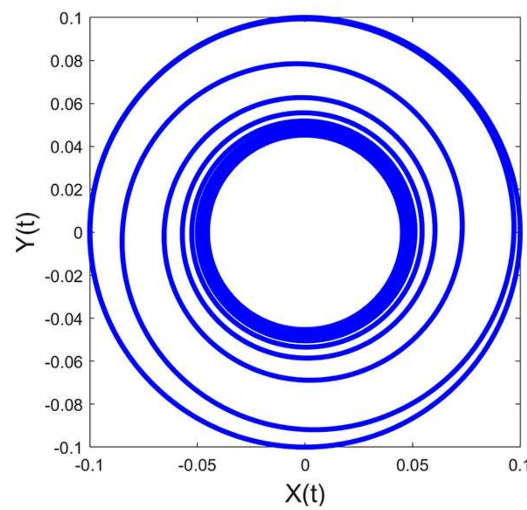
Unstable Cycles





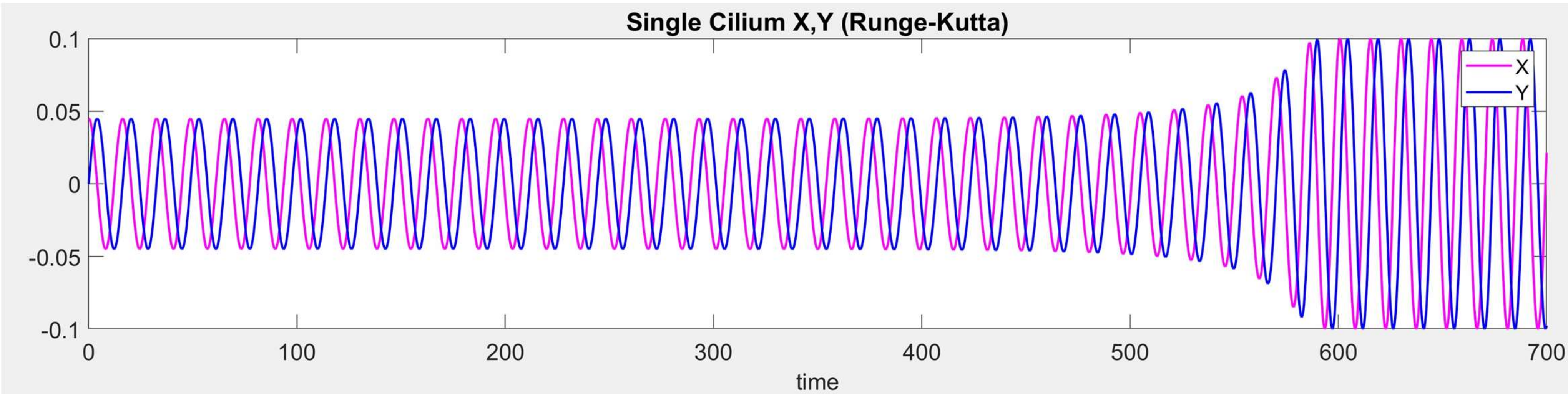
Case A: goes to steady state solution or vertical position

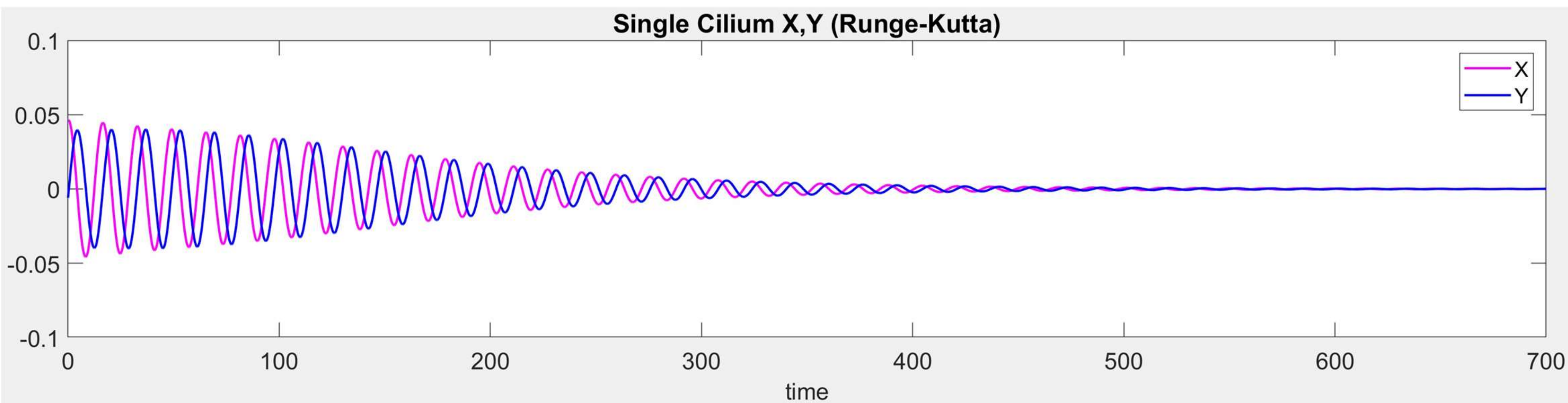




Case B: goes to maximum amplitude oscillations

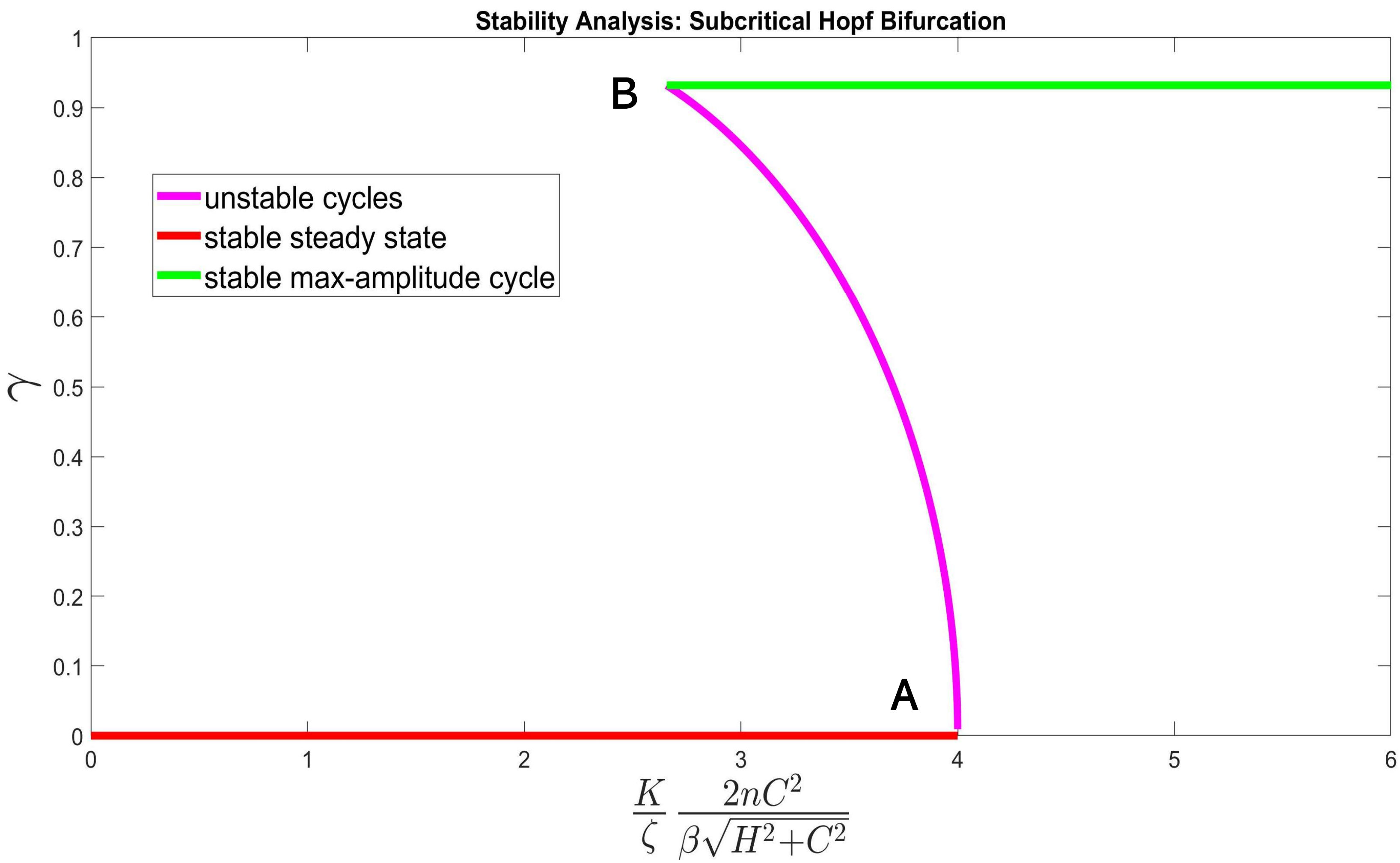
Bistability





Subcritical Hopf Bifurcation

Bi-stability
Parameter interval in which stable max-amplitude cycle coexists with stable steady state



Notation

$$\gamma = \frac{2rC}{H^2 + C^2}$$

r = radius of cycle

Chirality Not Expressed in Movement

Simulation 1

$X(0) = X^0$

$Y(0) = Y^0$

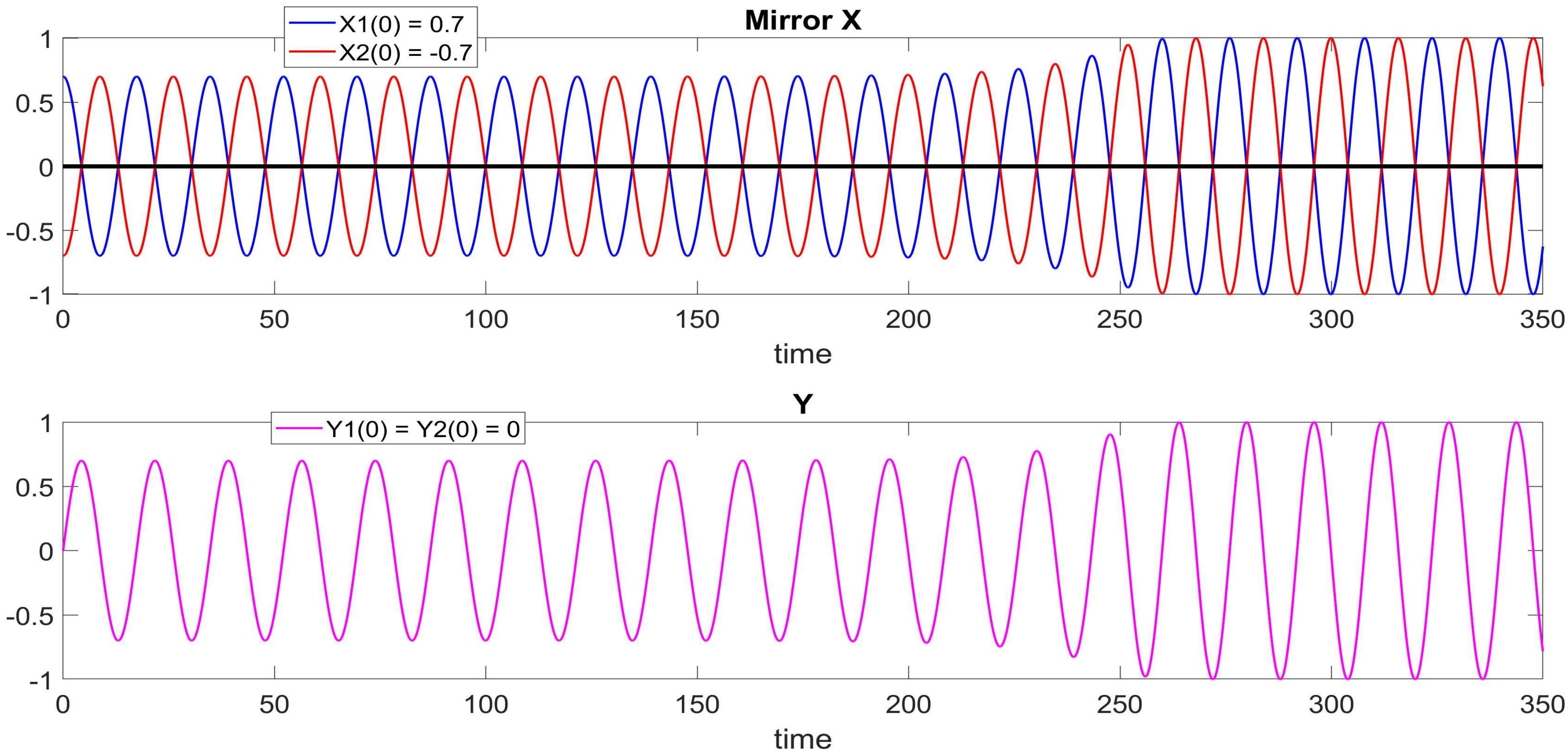
$T_j(0) = T_j^0$

Simulation 2

$X(0) = -X^0$

$Y(0) = Y^0$

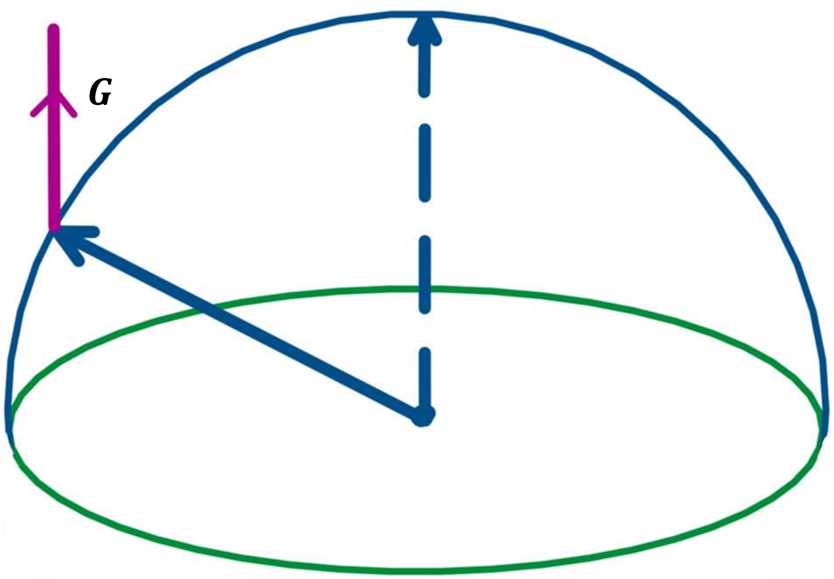
$T_j(0) = T^0_{-j-1}$



Future Work

1. Stabilizing Limit Cycles

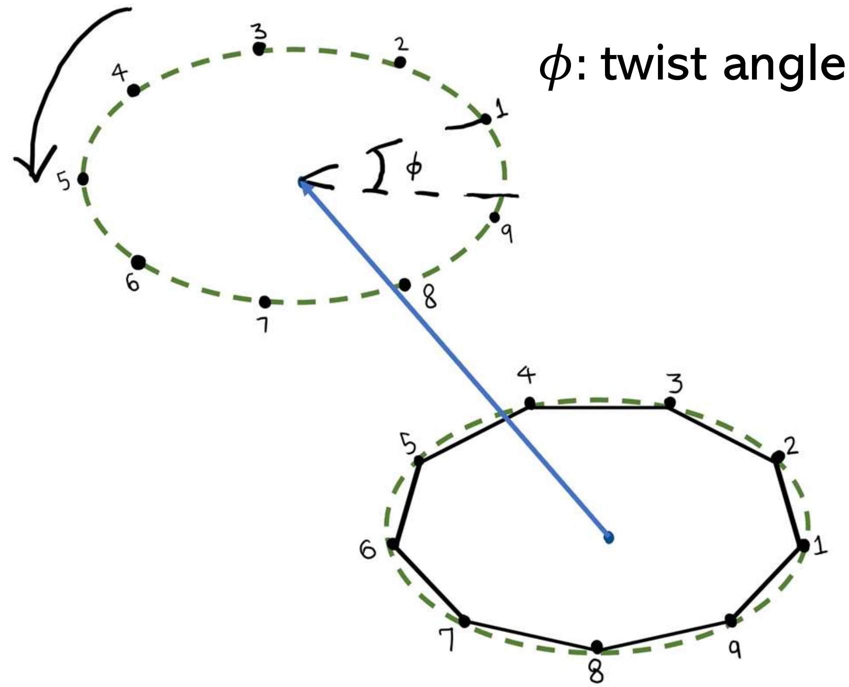
$\boldsymbol{G} = (0,0,g)$; g: non-zero constant



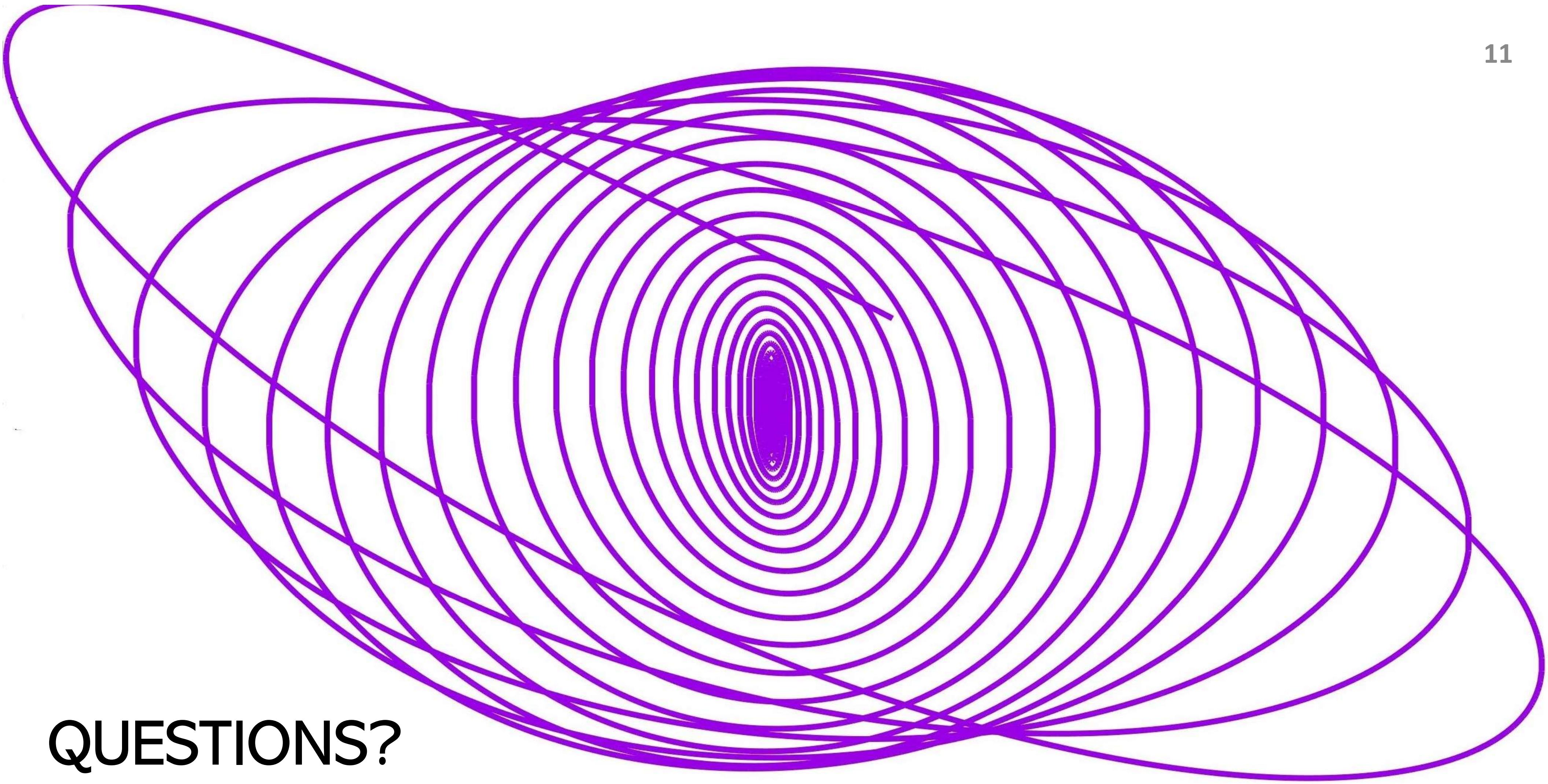
Find non-maximum amplitude limit cycles by applying constant force to central vector in to vertical position

2. Expressing Chirality

ϕ: twist angle



- Nodal cilia structure "9+0"
- Reduce geometric constraint:
  - Twisting upper polygon
  - Resistance to twisting in wires



QUESTIONS?

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