Introduction to Mathematical Modeling

MATH-UA.0251 (CAS)
Monday & Wednesday 9:30-10:45am on Zoom (online-only)

Introduction

Plato's *Theory of Forms* holds that ideas and mathematical forms are the essential truth, while the things that inhabit our world (us, trees, clouds, …) are imperfect expressions of these truths. If there is a philosophy behind mathematical modeling, it is the basically the opposite of Plato's theory: we take data from the physical world[1] as the empirical truth, and seek to find ways to make sense of it. Inherent in this approach is the idea that

> "All models are wrong, but some are useful" [George Box]

A goal of the class is therefore to teach you to always be cautious and skeptical of model results. We will discuss the tradeoff between model complexity and model error.

The simplest form of modeling is pattern recognition: when a pattern repeats, we notice it, and naturally wonder *why does it reappear here and there?* We will cover examples of this through models that arise in a range of fields, including biology, epidemiology, economics, physics, and more. You will be introduced to the necessary mathematical tools along the way — the ordering of topics covered is somewhat determined by the need to systematically build this toolset.

The tools include linear algebra, probability & statistics, ordinary differential equations, partial differential equations, numerical analysis and stochastic processes. Some of you will have covered nearly all these subjects in other courses, others have seen very little of it. This presents a challenge in teaching it, but the former will certainly see many new applications, and the latter will find enough support for the new mathematics to make the course accessible.

Instruction team

Professor:	Shafer Smith (kss3@nyu.edu)
Shafer's office:	
Shafer's office hours:	Monday 12-1pm & Tuesday 5-6pm, or by appointment
Teaching Assistant:	Evan Toler (eht247@cims.nyu.edu)
TA's office hours/location:	[TBD]
Zoom monitor:	Siva Thambiran

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[1] By "physical" I'm including everything in our world, from sociology to atomic interactions.

Resources (* indicates text available online from NYU IP address)

- Gardiner (2004): Stochastic methods: a handbook for the natural and social sciences (Springer)
- Holmes (2009): Introduction to the Foundations of Applied Mathematics (Springer*)
- Maynard Smith (1968): Mathematical Ideas in Biology (Cambridge Core*)
- Murray (2002): Mathematical Biology I. An Introduction (Springer*)
- Riley, Hobson and Bence (2002): Mathematical Methods for Physics and Engineering (Cambridge*)
- Strogatz (1994): Nonlinear Dynamics and Chaos (Addison-Wesley)
- Tung (2007): Topics in Mathematical Modeling (Princeton*)
- Wilmott, Howison & Dewynne (1995): The Mathematics of Financial Derivatives (Cambridge University Press)

Python/JupyterHub: There will be a JupyterHub site set up for the class. You will have your own account, file space, and access to a server (no installation of Python on your computer required). We will teach you what's necessary along the way.

Assignments

Problem Sets & Computational Assignments: roughly one per week. Most of your time will be spent on these. These will be submitted electronically through NYU Classes. If you are not using a natively digital format to write your assignments out, you must use a scanning app (e.g. TurboScan) to scan your written assignment into a black & white PDF file before submitting. JPEGs and huge files will not be accepted.

Recitation participation and computational activities: Attendance is mandatory, and may involve activities/problems/quizzes for which you will be assessed.

Final project: You will choose from a set of possible modeling projects (or make your own, with approval from me), in which you will design a model for a real-world problem. The outcome of the project will be a short technical paper, describing the problem, rationalizing your choices, setting up the equations, explaining the nondimensional parameters that control it, describing any approximations, and if relevant, providing numerical calculations and figures. Due May 10.

Exams: As of now, I don't plan to give any exams for this class.

Schedule [this will likely change, it's ambitious]

Week	Topics	Reading
1	Scaling laws in nature: metabolism, power laws, least squares, fractal models	Maynard-Smith Ch. 1 Tung Ch. 2 Linear Algebra text
2	Dimensional analysis: projectile motion, drag, Buckingham Pi-theorem Discrete population models: difference equations, linearization, fixed points (FPs), connect to ODEs	Holmes Ch. 1 Maynard-Smith Ch. 1 Strogatz Ch 10
3	Population model with delay: second-order difference equations, Fibonacci series Discrete logistic model: iterated maps, cobwebbing, chaos	Maynard-Smith Tung Ch. 1 Chaos Ch. 1
4	First order autonomous ODEs: what they are, analytical solution, numerical solution, phase plane analysis, fixed points Part II: linear stability, uniqueness/existence, no oscillations on the line, nondimensionalization	Strogatz Ch. 1-2
5	Bifurcations: 1 and 2 parameters, catastrophes and critical values, fishery model, budworm outbreak, first glance at SIR model, snowball Earth	Strogatz Ch. 3
6	Flows on the circle: synchronization, fireflies, neurons	Strogatz Ch. 4
7	2D linear systems of ODEs: eigenvalue/eigenvector method, complex eigenvalues. Conservative systems, energy as constant of motion (pendulum). Beats and resonance.	Strogatz Ch. 5-6 + notes
8	2D nonlinear systems: linearization and phase plane analysis Competing species, predator-prey, thermohaline circulation, SIR model	Strogatz Ch. 5-6

9	Limit cycles, van der Pol oscillator, perturbation methods Deterministic nonperiodic flow: chaos, Lorenz model	Strogatz 7-9
10	Probability primer; Poisson processes and statistics Binomial distribution and random walks	
11	Mean field kinetic equations Diffusion equation/Heat equation: use and solution (intro to PDEs)	
12	Stochastic processes, stochastic differential equations	
13	Network theory: epidemiological models	
14	Parameter estimation/Gauss-Newton	