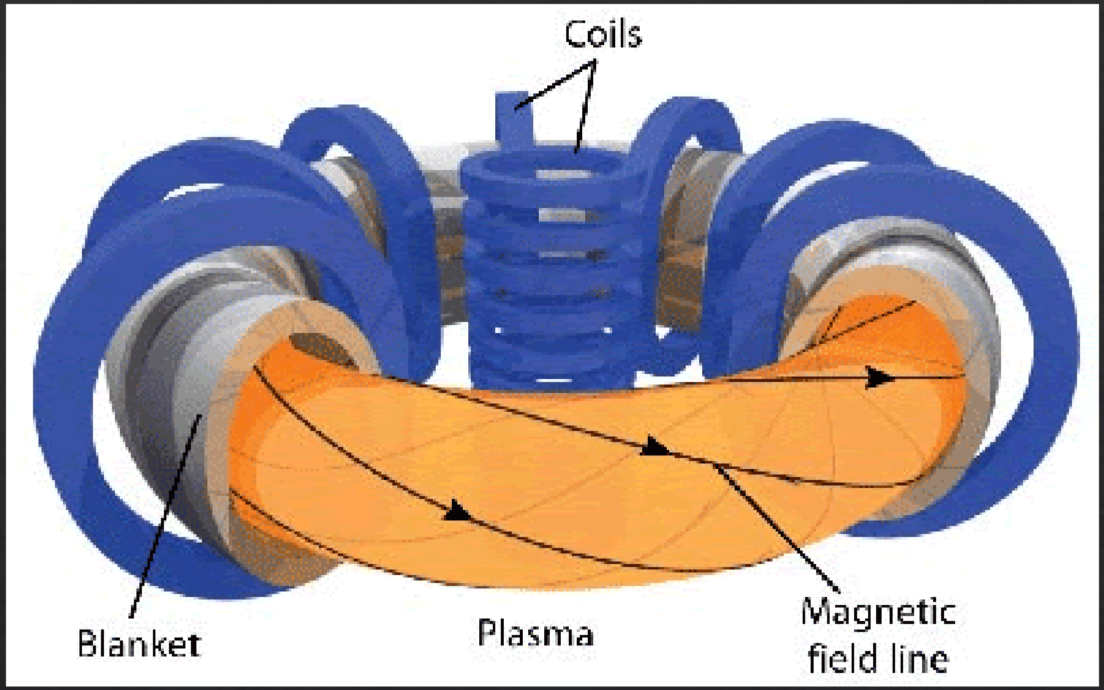

Simulating Poloidal Impurity Density Variation in the Tokamak Edge

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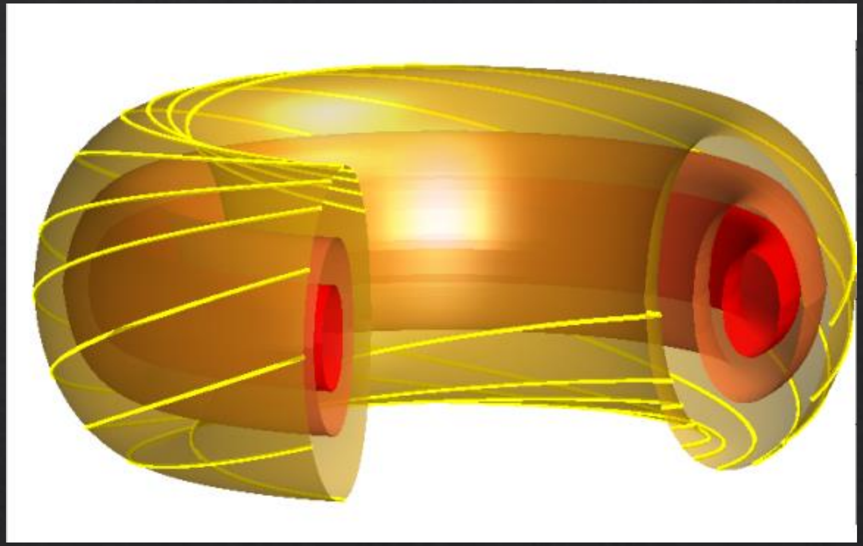
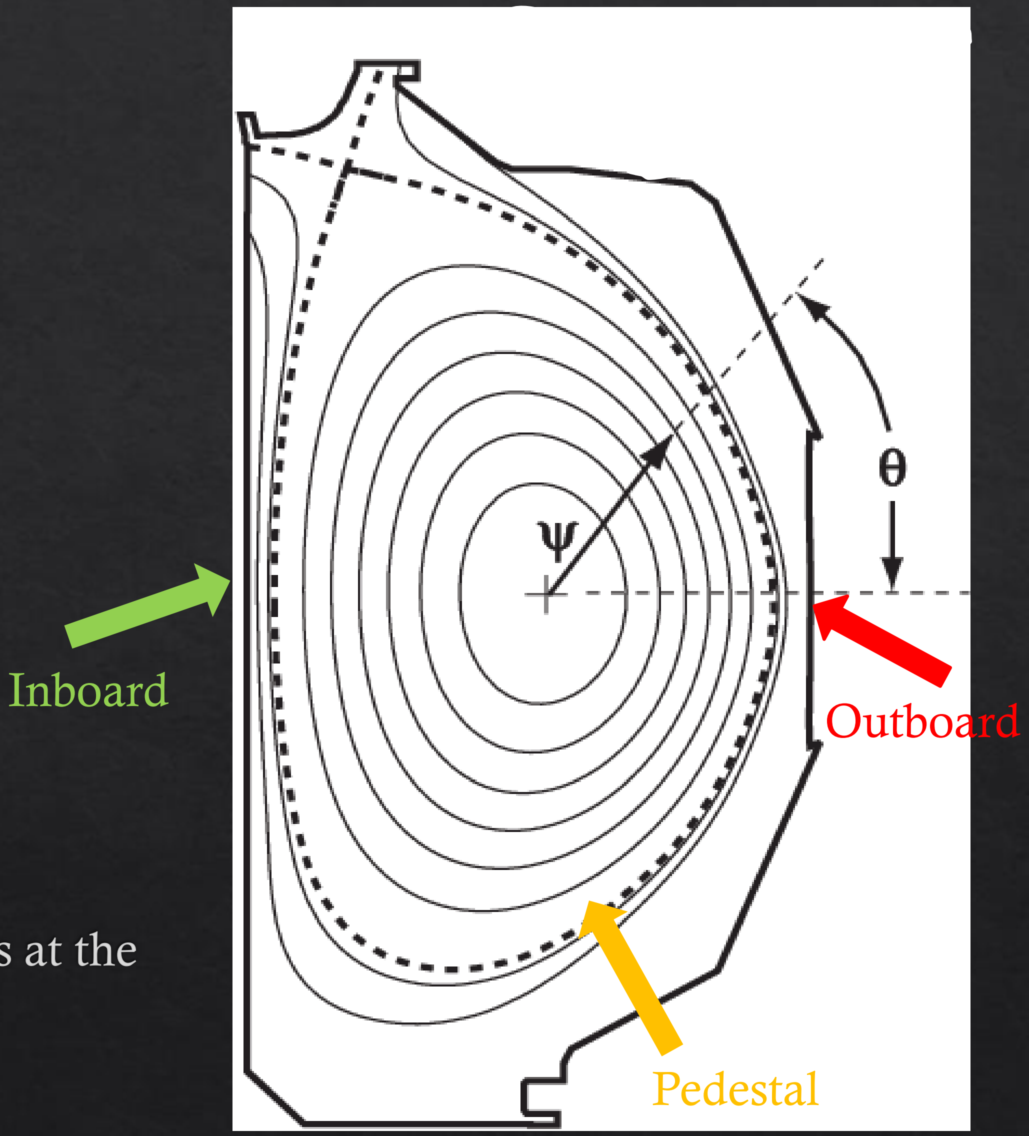
Motivation

- Why fusion?
  - The Sun
  - Clean Energy
- How do we do it?
  - High-z Materials
  - Magnetic Fields



- What's the Problem?
  - High-z walls → Impurities
  - Steep impurity density gradients → Poor understanding of physics at the edge



ψ: Radius Minor

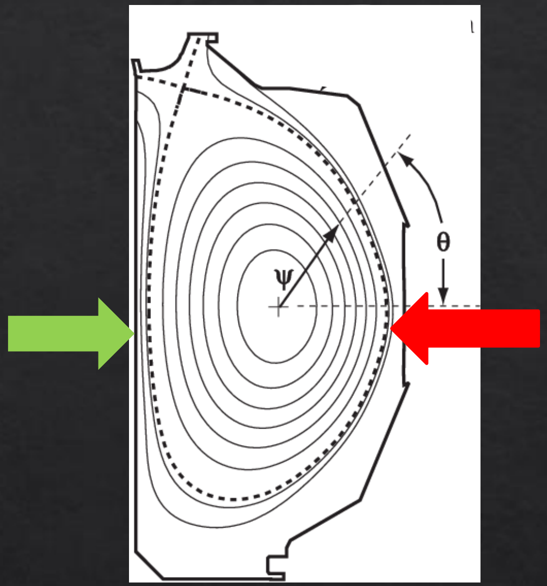
What's New?

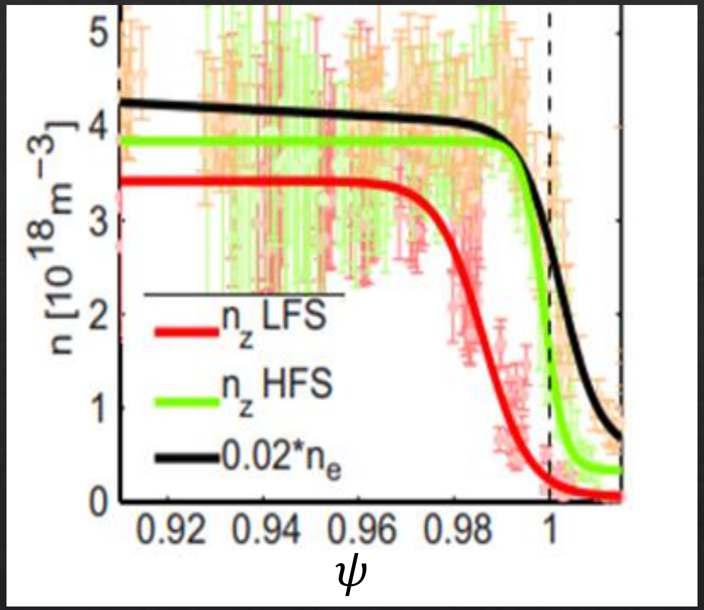
Previous state-of-the-art models

- Ignore radial and diamagnetic flow effects
- Underpredicts the inboard impurity accumulation.

This model

- Includes the radial and diamagnetic flow effects.
- Correctly predicts the magnitude of the inboard impurity accumulation





n_z: Dimensionless Impurity Density
ρ: Minor Radius

Model and Notation

$$\alpha n \frac{dn}{d\theta} + k + (b^2 - 1)m = \left[-\frac{d}{d\theta} + k \right] n$$

Experimentally Derived Terms(Constants)

D(ψ) : Impurity Diamagnetic Friction

k and m depend on D

Functions of Theta

$n(\psi, \theta)$: Impurity Density (Unknown)

$b^2(\psi, \theta)$: Dimensionless Magnetic Field Squared,

$$b^2 = 1 - \frac{1}{3}\cos(\theta)$$

Problems

1. How to solve the non-linear?
   - Iterative Approach
2. How to represent differentiation?
   - Pseudospectral Method

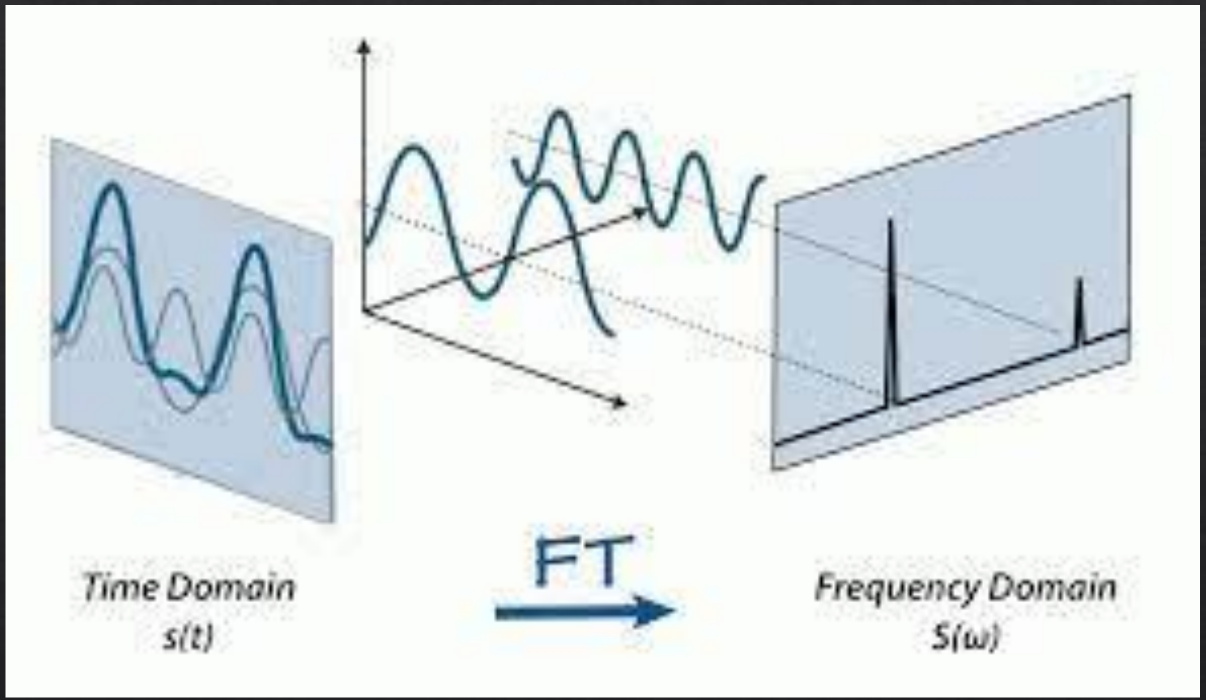
Discrete Fourier Transform

DFT: $\hat{n}_k = \Delta x \sum_{j=0}^{N-1} e^{-ikx_j} n_j, \quad k = -\frac{N}{2}+1, \ldots, \frac{N}{2}$

IDFT (Inverse DFT): $n_k = \frac{1}{2\pi} \sum_{k=-\frac{N}{2}}^{N/2} e^{ikx_j} \hat{n}_k, \quad j = 1, \ldots, N$

$$Example: DFT\left(\delta(k-j)\right) = \Delta x = x_j - x_{j-1}$$

$$IDFT(\Delta x) = S_N(x - x_k) = \frac{\sin(\frac{\pi x - x_k}{\Delta x})}{\left(\frac{2\,\pi}{\Delta x}\right) \tan(\frac{x - x_k}{2})}$$



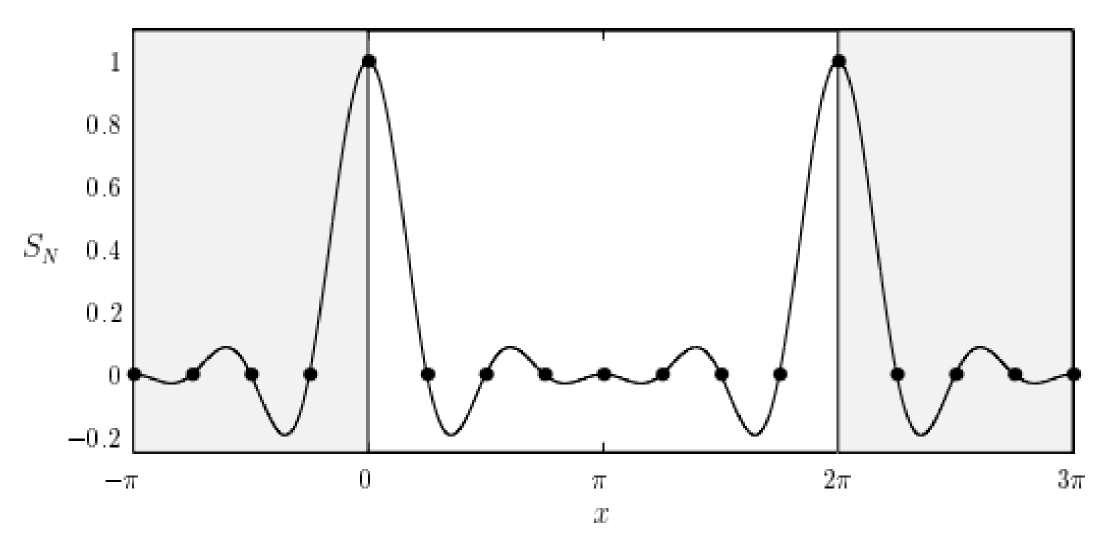
Discrete to Continuous and Differentiable

Interpolation Function

$$n_j = \sum_{k=0}^{N-1} n_k \delta(k - j) \rightarrow p(x) = \sum_{k=0}^{N-1} n_k S_N(x - x_k)$$

Matrix Vector Representation

$$\begin{pmatrix} S_N(x_0 - x_0) & \cdots & S_N(x_0 - x_{N-1}) \\ \vdots & \ddots & \vdots \\ S_N(x_{N-1} - x_0) & \cdots & S_N(x_{N-1} - x_{N-1}) \end{pmatrix} \begin{pmatrix} n_0 \\ \vdots \\ n_{N-1} \end{pmatrix} = \begin{pmatrix} p(x_0) \\ \vdots \\ p(x_{N-1}) \end{pmatrix}$$



Differentiation Matrix

$$\left(\boldsymbol{D}^{(\mathbf{1})}\right)_{ij} = \begin{cases} 0 & if\ i = j \\ \frac{1}{2}(-1)^{i+j} \cot\left(\frac{x_i - x_j}{2}\right) & if\ i \neq j \end{cases}$$

Non-Linear Solution, Algorithm

1) $\alpha n \frac{dn}{d\theta} + k + (b^2 - 1)m = \left[-\frac{d}{d\theta} + k \right] n$

$\mathrm{A} \coloneqq \left[-\frac{d}{d\theta} + k \right]$

$c \coloneqq k + (b^2 - 1)m$

2) $\mathrm{c} \coloneqq \alpha n_{old} \frac{dn_{old}}{d\theta} + k + (b^2 - 1)m$

Non-Linear Solution Algorithm

1) Compute Linear Solution

$$\boldsymbol{n} = \boldsymbol{A}^{-1}\boldsymbol{c}$$

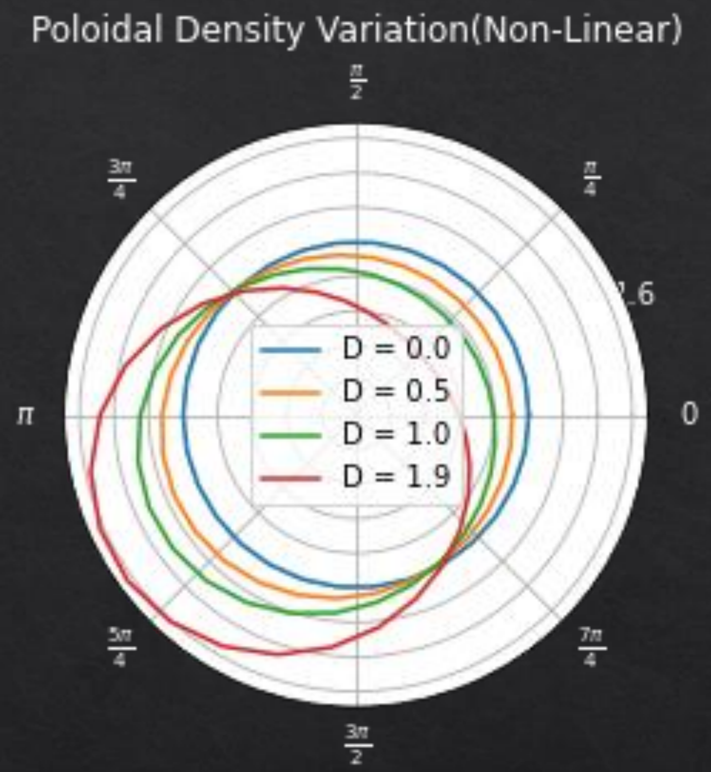
2) Set $\boldsymbol{n}_{old} \leftarrow \boldsymbol{n}$

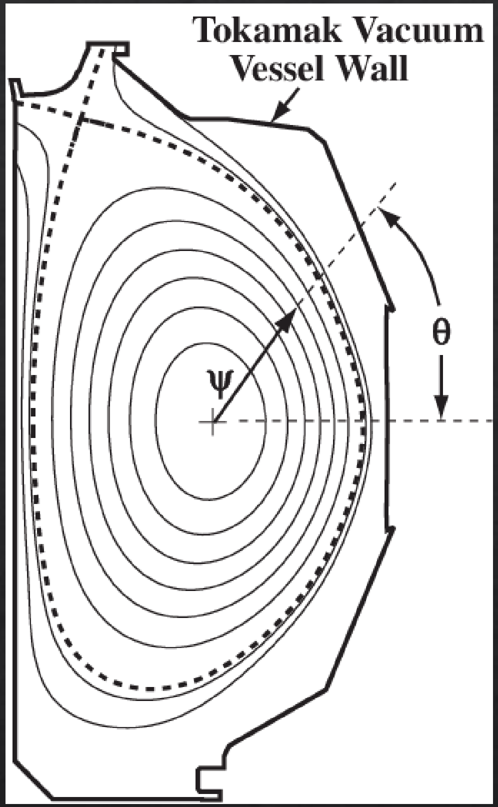
3) Compute new $\boldsymbol{n}$ using $\boldsymbol{n}_{old}$ for non-linear terms

4) Evaluate $||\boldsymbol{n} - \boldsymbol{n}_{old}||$

5) While above some error threshold,
   1) Set $\boldsymbol{n}_{old} \leftarrow \boldsymbol{n}$
   2) Compute new $\boldsymbol{n}$ using $\boldsymbol{n}_{old}$ for non-linear terms
   3) Evaluate $||\boldsymbol{n} - \boldsymbol{n}_{old}||$

Results





Model explains the experimentally found asymmetries

Future Work

- Change of Coordinates: Circle → D-Shape

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Thank You!!

Questions?