
Credit: Met Office

Moist Rayleigh Benard Convection—

Investigation of Different Convective Regimes

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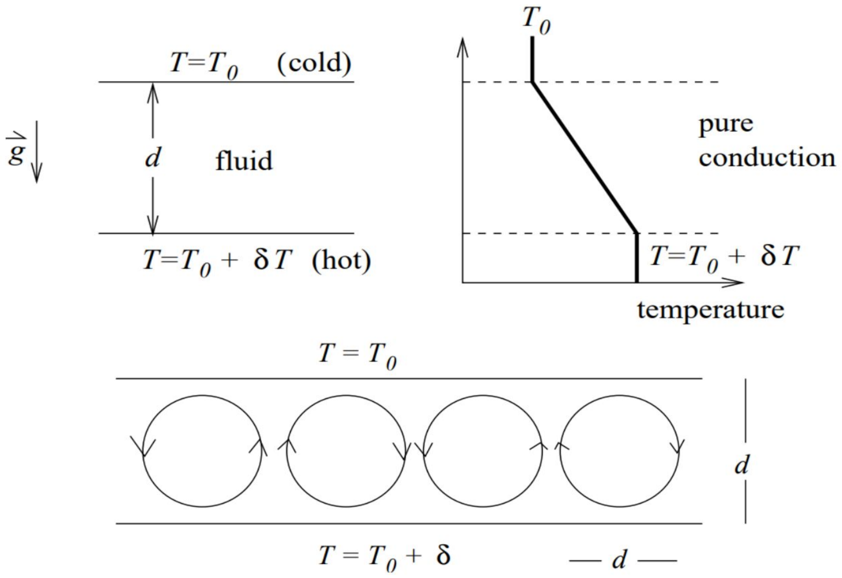
NYU

PART 01

Rayleigh Benard Convection (RBC)

Problem Setting:



D Rothman, 2006

Governing Equations:

$$\frac{D\vec{u}}{Dt} = B\hat{k} - \nabla P + \nu\nabla^2\vec{u}$$

$$\nabla \cdot \vec{u} = 0$$

$$\frac{DB}{Dt} = \kappa\nabla^2 B$$

Disadvantage: only for dry air; no energy variation due to phase change of water.

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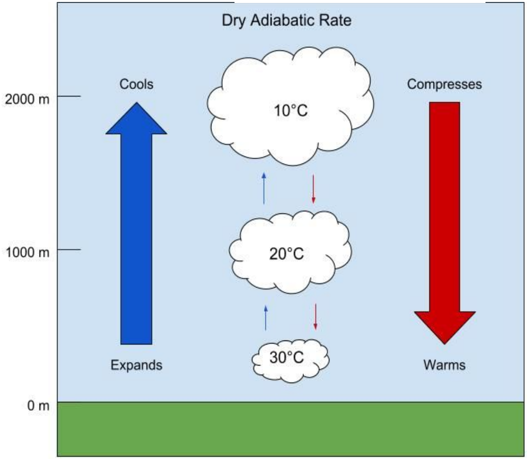
RBC to MRBC

- Two new variables D & M: f(qT, S)

$$D = B_{S,u}(S - S_{ref}) + B_{q_T,u}(q_T - q_{T,ref})$$
$$M = B_{S,s}(S - S_{ref}) + B_{q_T,s}(q_T - q_{T,ref}).$$

D: “Dry Buoyancy,” M: “Moist Buoyancy”

Saturation Condition: $M > D - N_s^2 z$

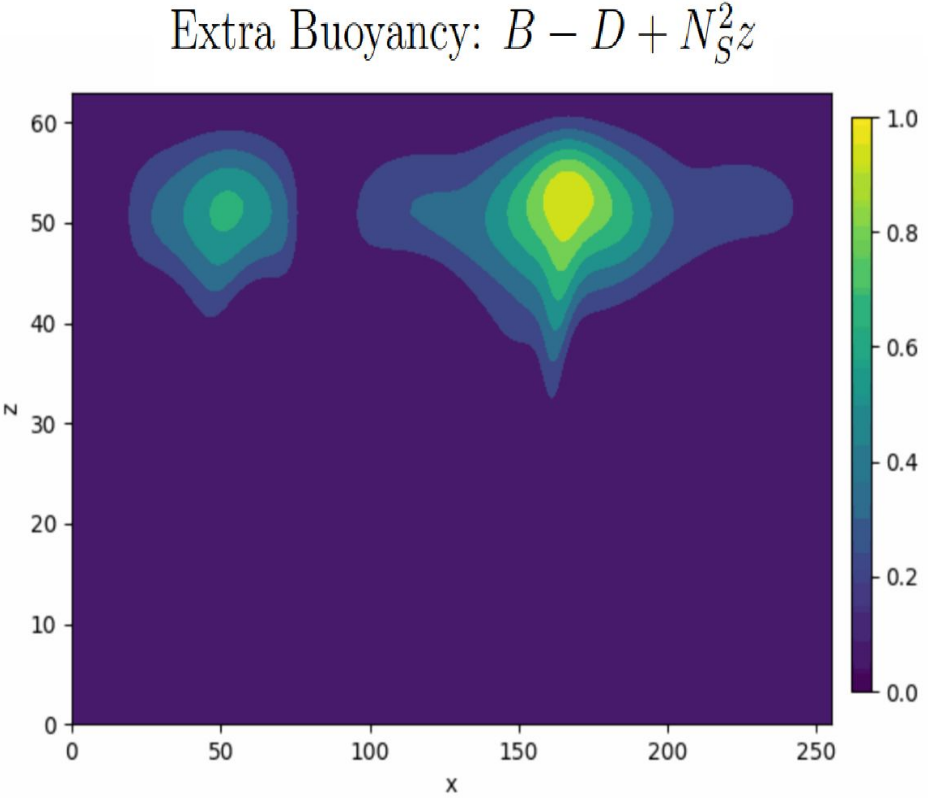


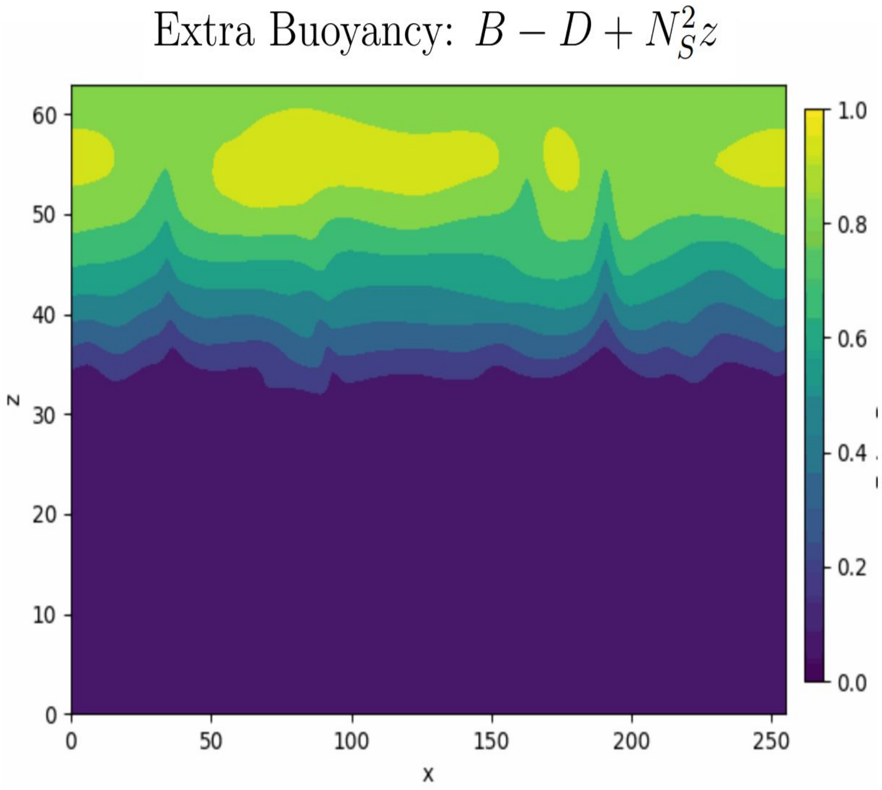
Credit: Britt Seifert

$$\frac{d\vec{u}}{dt} = B\hat{k} - \nabla P + \nu \nabla^2 \vec{u}$$
$$\nabla \cdot \vec{u} = 0$$
$$\frac{dD}{dt} = \kappa \nabla^2 D$$
$$\frac{dM}{dt} = \kappa \nabla^2 M$$
$$B(M, D, z) = \max(M, D - N_s^2 z))$$

d/dt : material derivative
Ns : Brunt-Väisälä frequency

Different Environmental Profiles



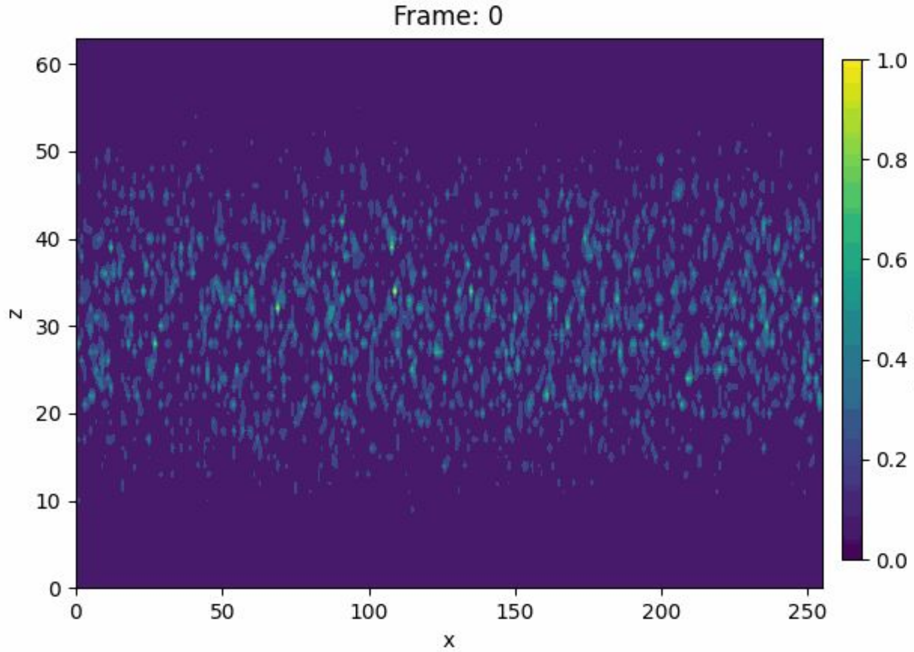


Clouds Visualization

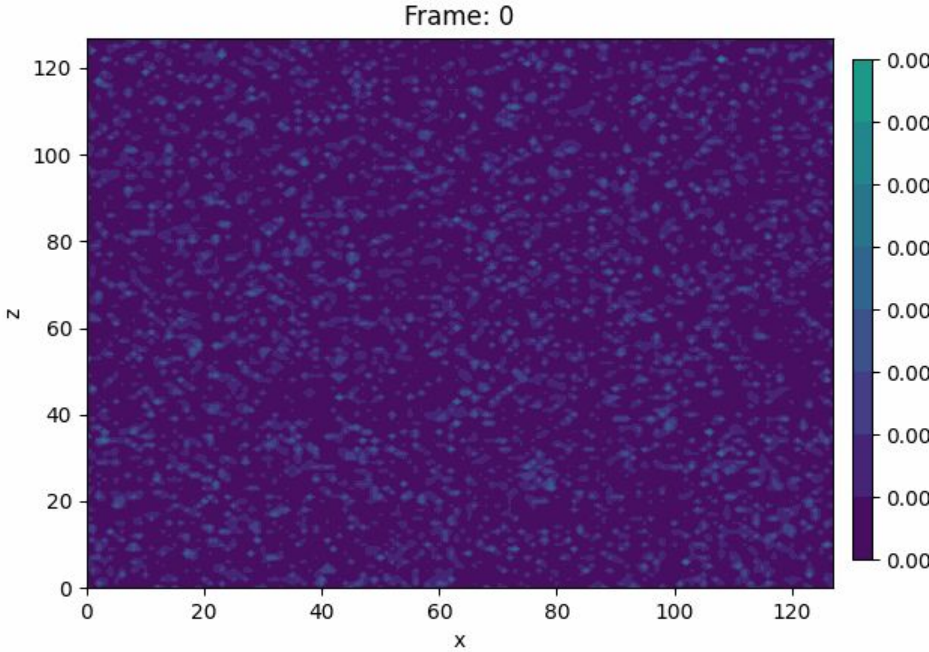
$$\text{Extra Buoyancy: } B - D + N_S^2 z$$

Corresponds to the condensed water (clouds); visualize this quantity to see cloud evolution.

2D XZ View, A=4, RA=1e6

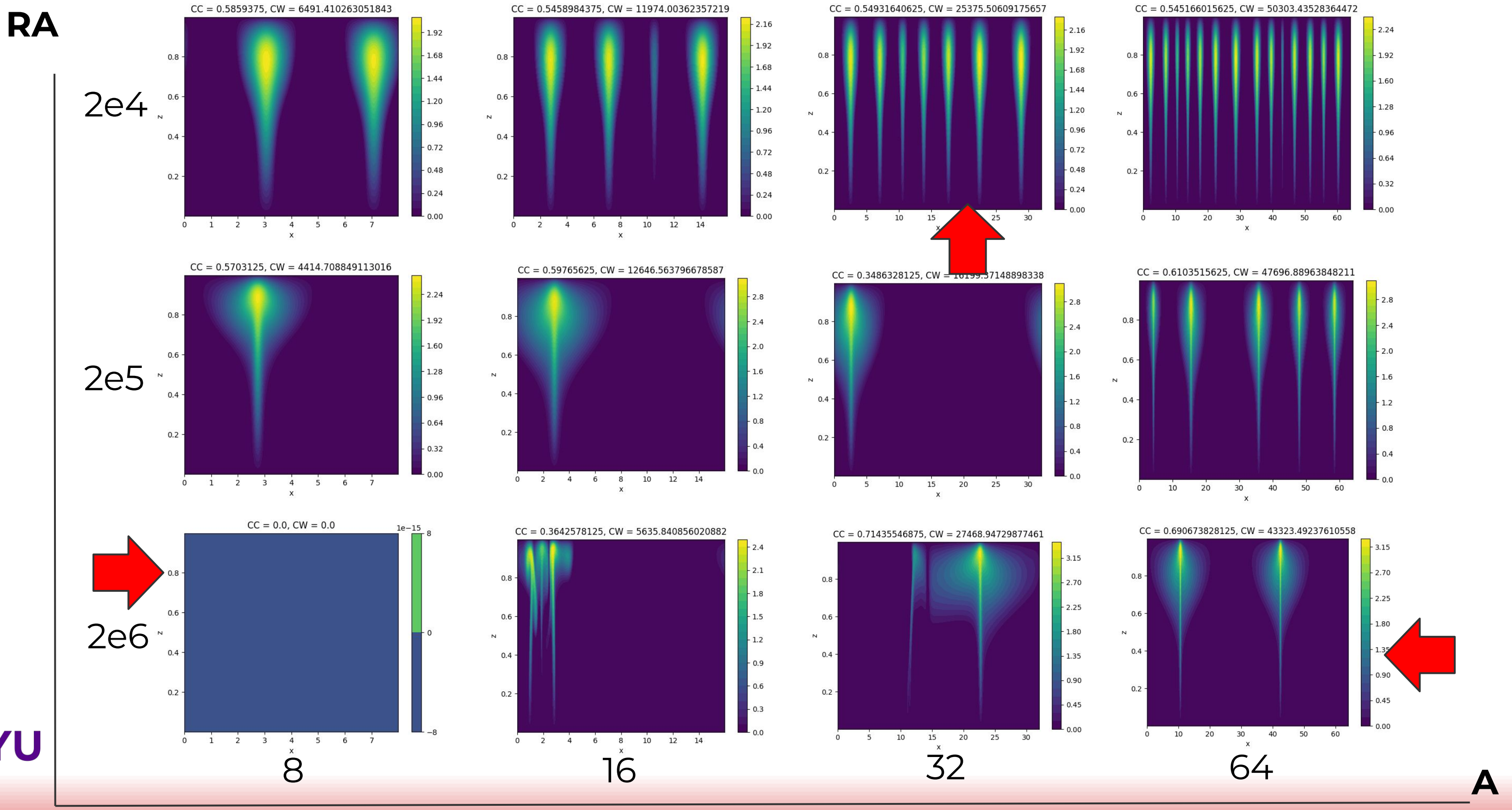


3D XY View, A=4, RA=1e6



PART 02

2D MRBC Parameter Space (A, RA)



RA: Rayleigh Number

A: Aspect Ratio

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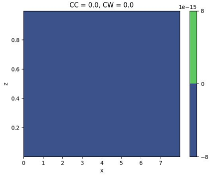
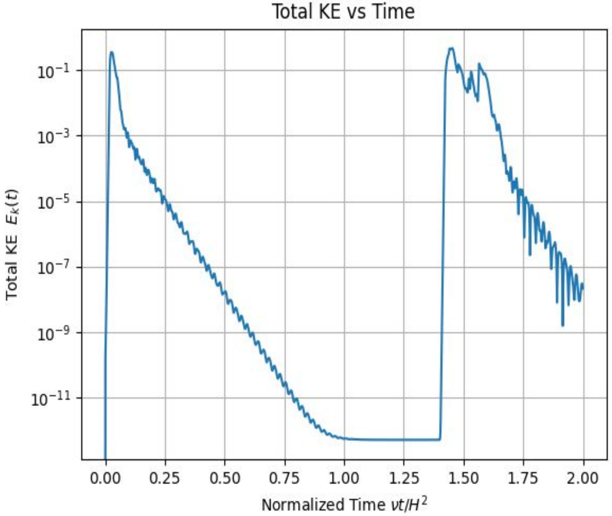
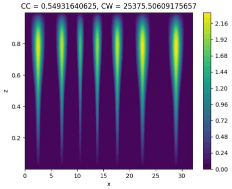
Patterns of Total KE vs Time

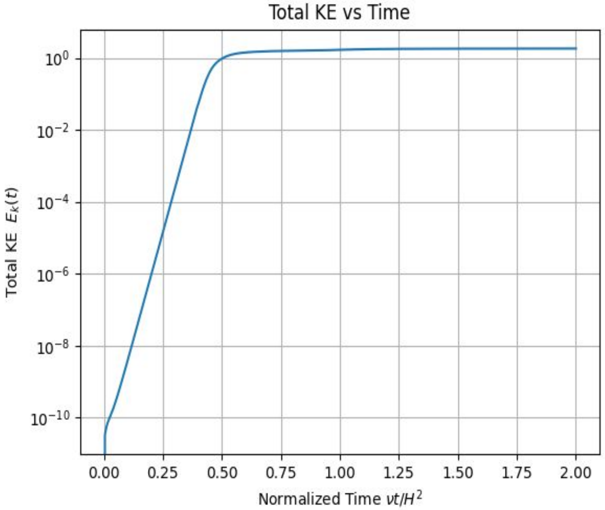
A=8, RA=2e6



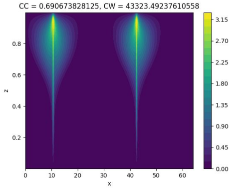


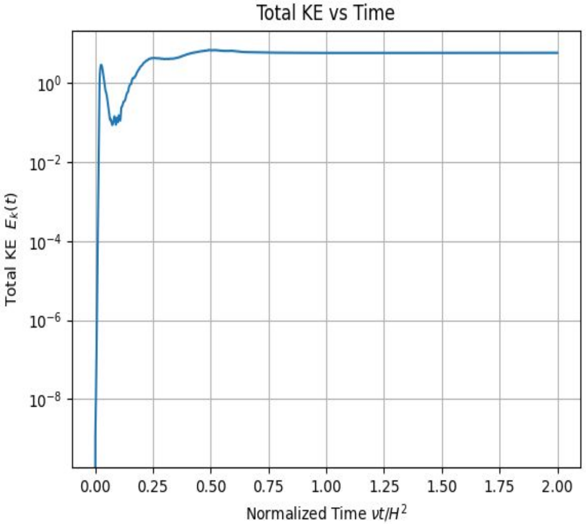
A=32, RA=2e4





A=64, RA=2e6





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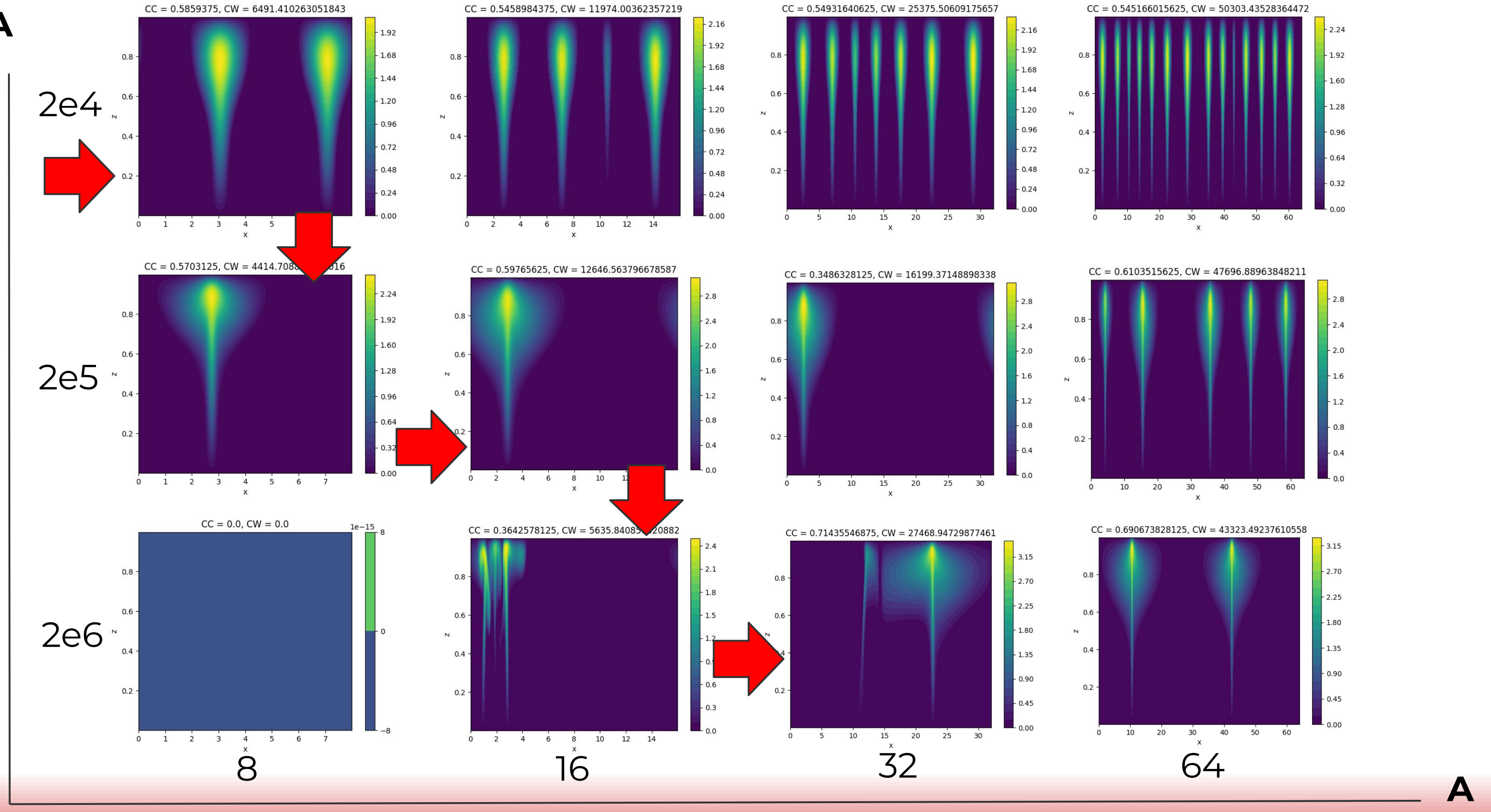
PART 02

Updraft and Stable Convection

RA: Rayleigh Number

A: Aspect Ratio



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Self Aggregation of Clouds

- Self-aggregation of clouds refers to the phenomenon where individual convective clouds or cloud clusters spontaneously organize and cluster together, forming larger-scale cloud structures.
- Self-aggregation would require a larger and larger aspect ratio as the diffusivity decreases/ RA increases.
- How to quantify Self-Aggregation?

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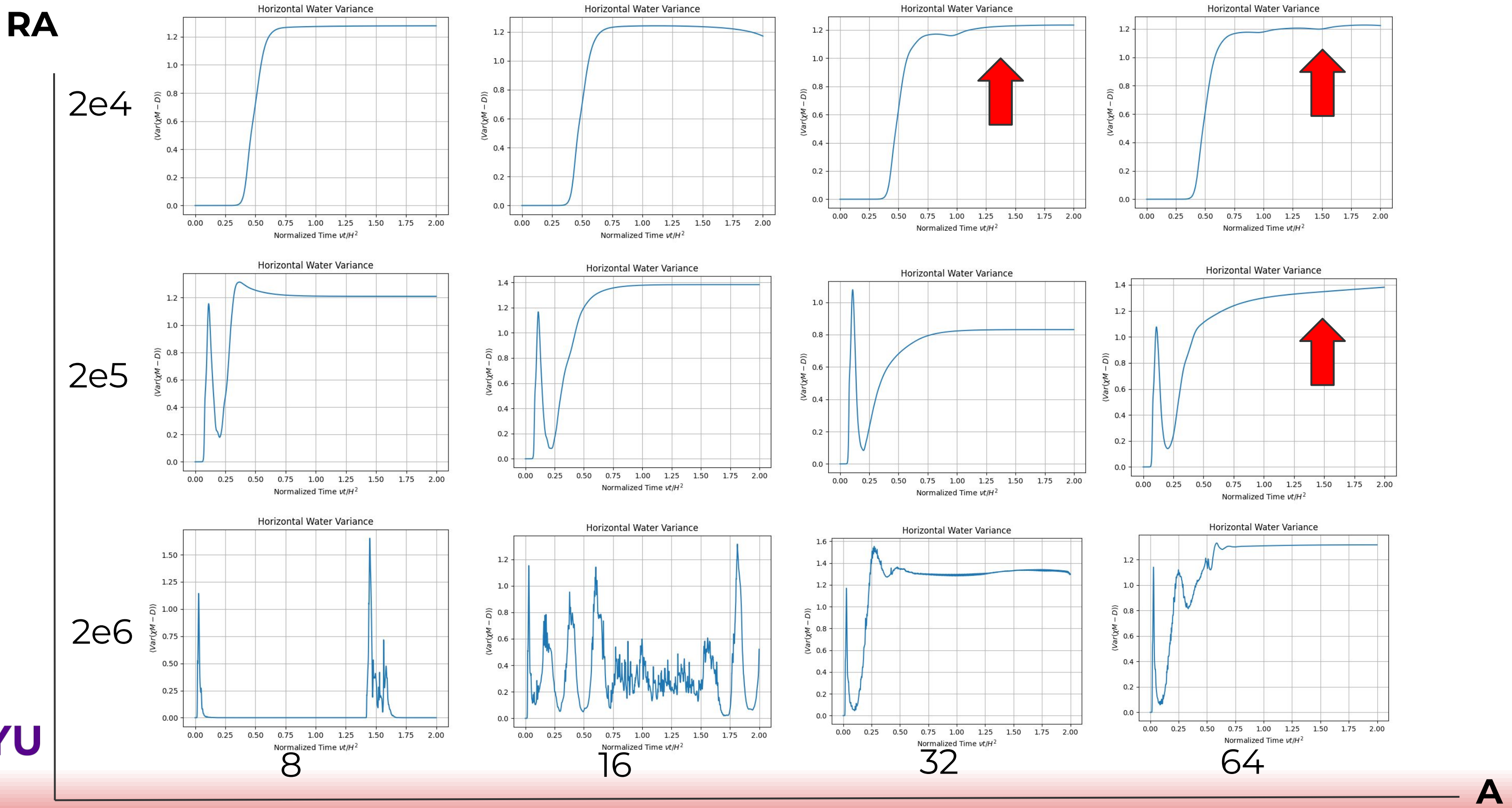
Answer: Horizontal Water Variance



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PART 03

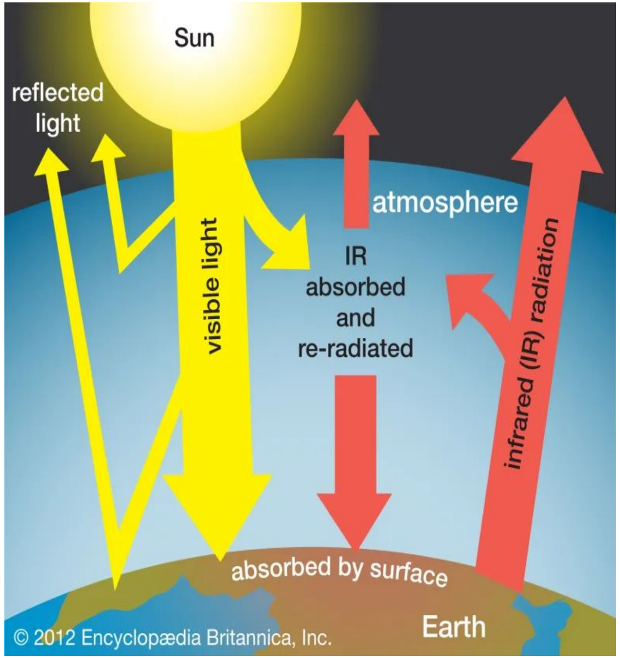
MRBC Augmentation–Radiative Cooling

Radiative cooling within the troposphere corresponds to a net loss of thermal energy without any change in the composition of moist air, which means the rate of change of entropy is solely determined by the cooling rate divided by temperature

$$\dot{S} = \frac{Q}{T}$$

, and from the previous definition of M and D, we can directly obtain the rate of change of M and D

$$\dot{D} = B_{S,u}(\frac{Q}{T}), \ \dot{M} = B_{S,s}(\frac{Q}{T})$$



Credit: Britannica

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MRBC with Radiative Cooling

Modified Equation:

$$\frac{d\vec{u}}{dt} = B\hat{k} - \nabla P + \nu\nabla^2\vec{u}$$

$$\nabla \cdot \vec{u} = 0$$

$$\frac{dD}{dt} = \kappa\nabla^2 D - Q_{rad} f(z)$$

$$\frac{dM}{dt} = \kappa\nabla^2 M - Q_{rad}^{M} f(z)$$

$$B(M, D, z) = \max(M, D - N_s^2 z)$$

f(z) is a function of altitude, serving as the radiation profile. For our idealized model, we can use f(z) = sin(πz/H)

Qrad(M) are equivalent as dry and moist lapse rate

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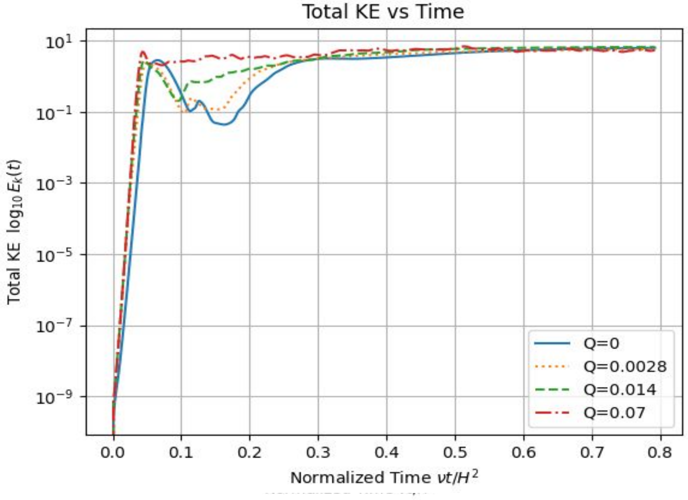
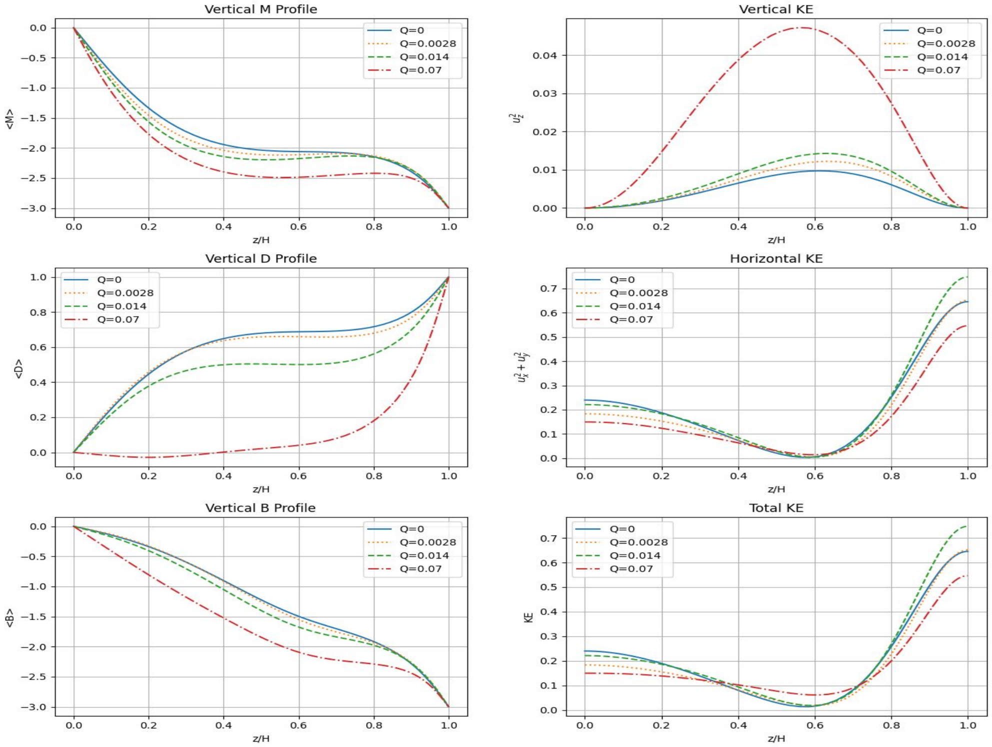
Boosting Recharge Process

- Different radiation strengths [0, 0.0028, 0.014, 0.07] in 2D MRBC



A=64, RA=4e5



Better mixing

More vertical convection

Impacts on Water Transport

- Characterize convective transport through a Nusselt number Nu, defined as the ratio of the total transport to the diffusive transport in the absence of fluid motion.
- Need to first extract water content from our model.

Variation of the total water content:

$$\frac{B_S^{(u)}}{B_S^{(s)}}M - D = \chi M - D = \left(\frac{B_S^{(u)} B_{q_T}^{(s)}}{B_S^{(s)}} - B_{q_T}^{(u)}\right)(q_T - q_{T0})$$

Water flux:

$$\langle u_z[\chi M(z) - D(z)]\rangle$$

Nu for water transport at the z=0:

$$\mathrm{Nu}_w = -\frac{H[\chi\partial_z\langle M(z=0)\rangle - \partial_z\langle D(z=0)\rangle]}{\chi(M_0 - M_H) - (D_0 - D_H)}$$

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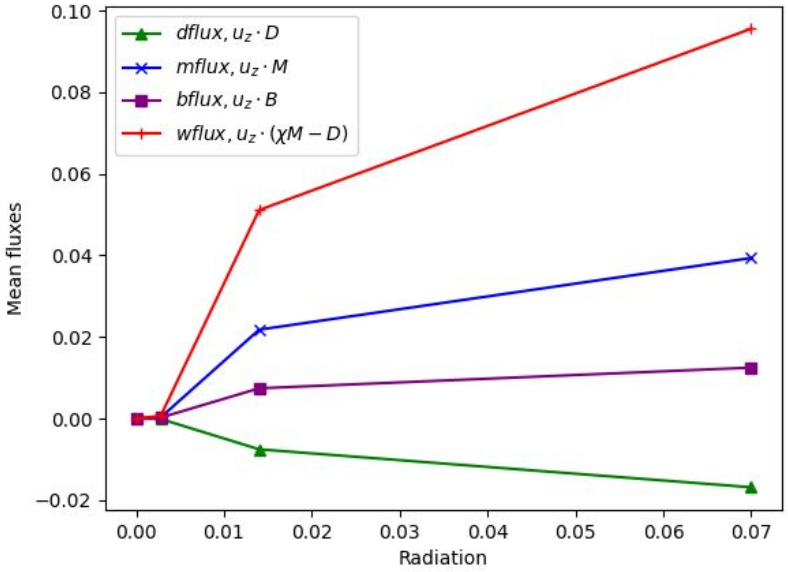
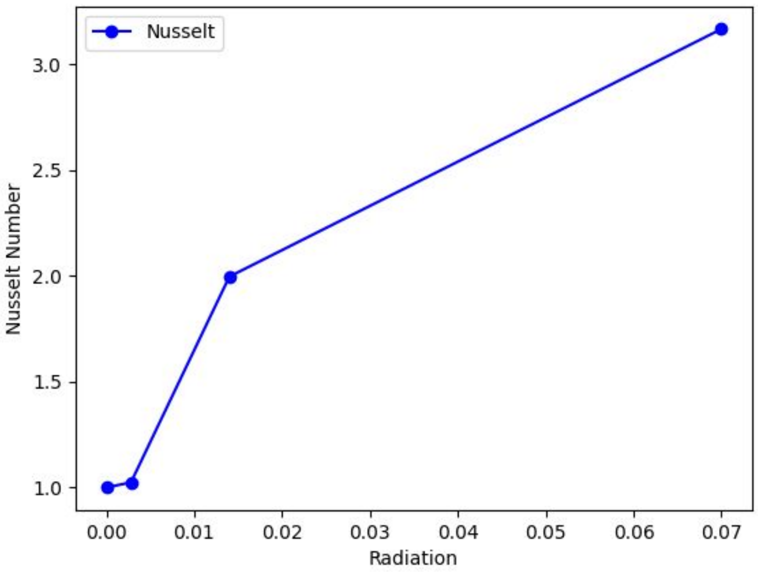
Impacts on Water Transport



Enhance upward water transportation



Increase rate of water entering domain

Extra modifications: Rotation

Modified Equation:

$$\frac{d\vec{u}}{dt} + f e_z \times u = B\hat{k} - \nabla P + \nu \nabla^2 \vec{u}$$

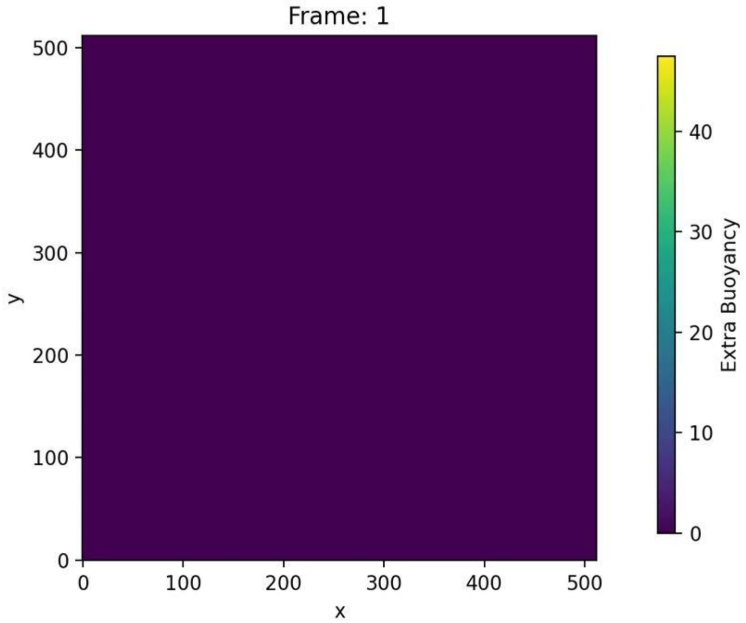
$$\nabla \cdot \vec{u} = 0$$

$$\frac{dD}{dt} = \kappa \nabla^2 D$$

$$\frac{dM}{dt} = \kappa \nabla^2 M$$

$$B(M, D, z) = \max(M, D - N_s^2 z)$$

3D XY view, A=16, RA=4e5, f=0.05



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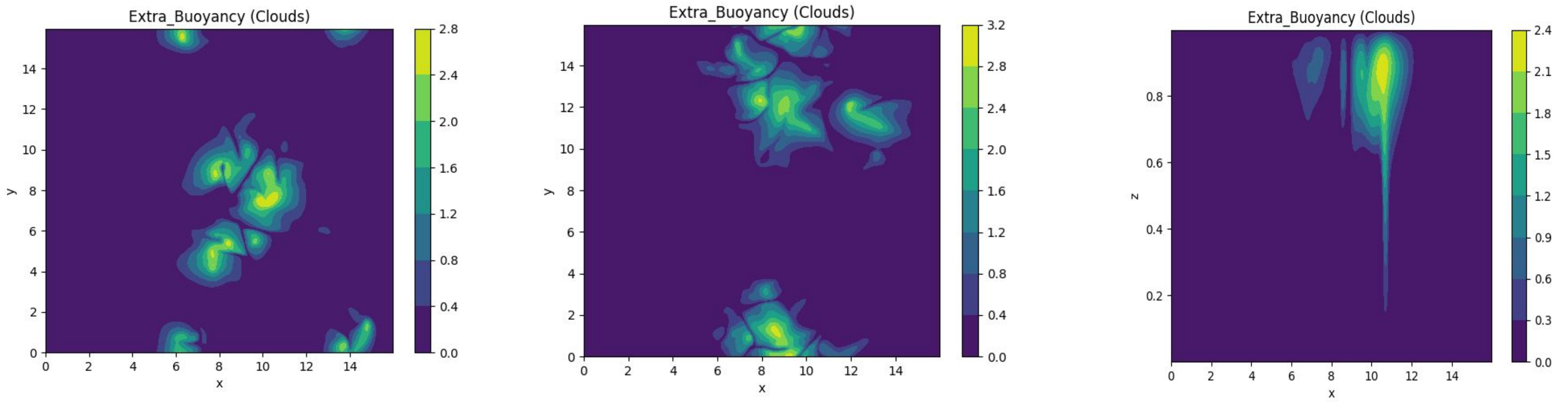
Future Work

- More post processing & analysis
- MRBC with Wind (vertical wind shear)
- 3D MRBC “Cloud Botany” (hypercube parameter space)



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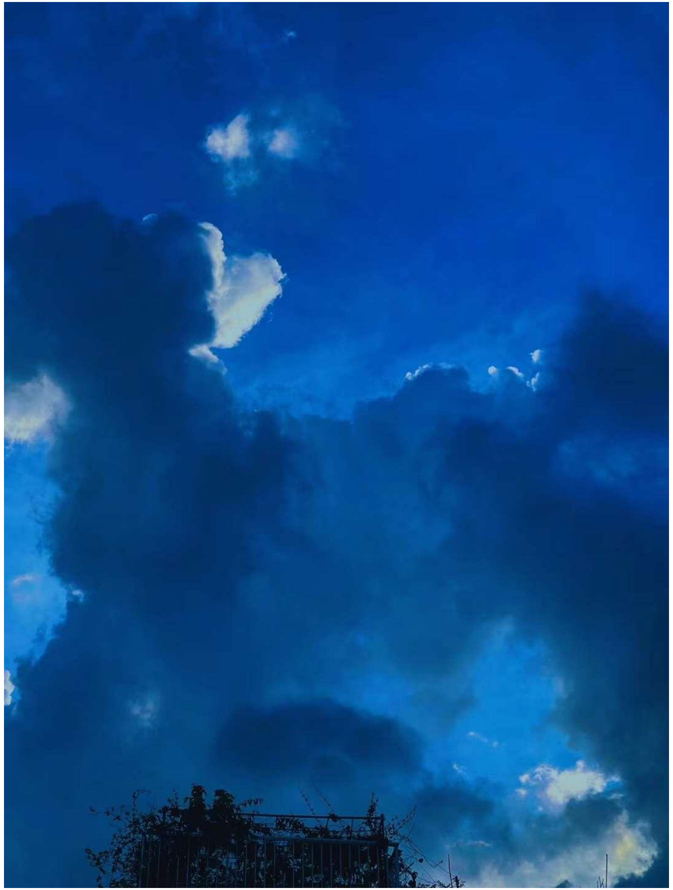

Any Questions?

References

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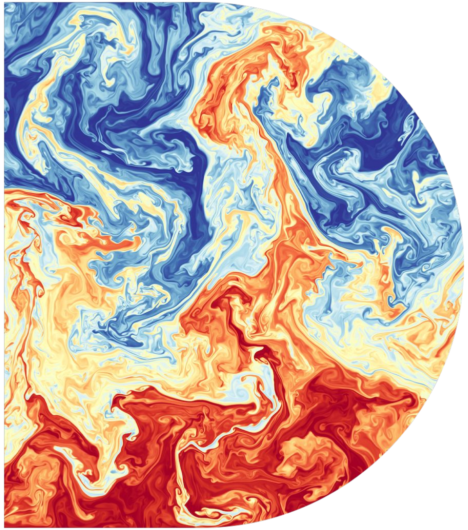
Pauluis, O., and Schumacher, J. Self-aggregation of clouds in conditionally unstable moist convection. Proc. Natl. Acad. Sci. USA 2011, 108, 12623–12628.

Muller, C. J., and Romps, D. M. (2018). Acceleration of tropical cyclogenesis by self-aggregation feedbacks. Proc. Natl. Acad. Sci. U. S. A. 115, 2930–2935. doi:10.1073/pnas.1719967115

Pauluis, O., and J. Schumacher, 2013: Radiation Impacts on Conditionally Unstable Moist Convection. J. Atmos. Sci., 70, 1187–1203, https://doi.org/10.1175/JAS-D-12-0127.1.

Chien, M.H., Pauluis, O., and Almgren, A.S. Hurricane-like Vortices in Conditionally Unstable Moist Convection. J. Adv. Model. Earth Syst. 2022, 14, e2021MS002846.

Additional Reference: Dedalus



- A flexible framework for numerical simulations with spectral methods (A general sparse tau method)
- Excellent for complex problems on simple domains
- Automatic MPI parallelization and efficient solutions

K. J. Burns, G. M. Vasil, J. S. Oishi, D. Lecoanet, and B. P. Brown,Dedalus: A flexible framework for numerical simulations with spectral methods, Phys. Rev. Res., 2 (2020),023068, https://doi.org/10.1103/PhysRevRes

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