Applied Stochastic Analysis 2022 (MATH-GA.2704-001): Course Information

Wedensdays, 1:25pm-3:15pm, WWH 1302
Instructor: Miranda Holmes-Cerfon (she/her), holmes@cims.nyu.edu
Office Hours: Monday 2:30-3:30pm, Tues 3-4pm, WWH 1111

Prerequisites
Basic Probability (or equivalent masters-level probability course), and good upper level undergraduate or beginning graduate knowledge of linear algebra, ODEs, PDEs, and analysis.

Description
This course will introduce the major topics in stochastic analysis from an applied mathematics perspective. Topics to be covered include Markov chains, Gaussian processes, stochastic differential equations, numerical algorithms for solving SDEs and simulating stochastic processes, forward and backward Kolmogorov equations. It will pay particular attention to the connection between stochastic processes and PDEs, as well as to physical principles and applications. The class will attempt to strike a balance between rigour and heuristic arguments: it will assume that students have seen some analysis before, but most results will be derived without using measure theory. The target audience is PhD students in applied mathematics, who need to become familiar with the tools or use them in their research.

The course will be divided roughly into two parts: the first part will focus on stochastic processes, and the second part will focus on stochastic differential equations and their associated PDEs.

Homework will be a critical part of the course. Lectures will mostly be theory, and examples or extensions will be assigned as homework problems. You must do these if you want to learn something from the course. Homework will require some computing, in a high-level language such as Matlab, Python, or Julia. Students without programming experience will have to put in extra effort in the first few weeks.

Website and communication
All lecture notes, announcements, and other class information will be posted to the **Brightspace** webpage. Homework will be posted and collected on **Gradescope**, which you can access from Brightspace. Lectures will be recorded using Panopto and recordings will be available after class on Brightspace.

Homework
There will be weekly homework assignments due **Wednesdays at 12pm.** Late homework will be accepted for up to 7 days, and will be penalized 5% per day. The lowest-scoring homework will be dropped. You should hand in the homework as a pdf on Gradescope, and include any codes in your pdf. Make to to label your pages with the correct question numbers.

Homework will be a mixture of theoretical questions and computational questions. You do not need to have experience coding beforehand, but you must be willing to learn basic programming during the class. A high-level software package that lets you easily plot things, such as Python, Julia, or Matlab, is recommended.

You are *encouraged to work with others*, on the homework problems and to study. However, you ***must write up your own solutions***. Solutions that are identical, or nearly so, will be considered as plagiarism and will be treated accordingly. The best way to ensure this doesn't happen is, (once you have discussed the problems with others), to find a place on your own to sit and write your solutions, away from the input of others.

Grading

Grading will be based on two exams and weekly homework assignments. The grading scheme will be:

Midterm (March 23)	25%
Final Exam (May 11)	40%
Homework (weekly)	35%

References

There are three textbooks that are not required, but that are highly recommended:

- G. A. Pavliotis. *Stochastic Processes and Applications*.
- G. Grimmett and D. Stirzaker, *Probability and Random Processes*.
- C. Gardiner, *Stochastic Methods: A Handbook for the Natural and Social Sciences* (Springer) (won't be assigned direct readings, but this is a generally extremely useful book to have on the shelf.)

You can download the first book (and all other Springer books) for free from the Springer website if you are a member of NYU, and/or have it printed in softcover for 25$. It will also be available at the bookstore.

Other good references include:

- L. Koralov and Y. G. Sinai, *Theory of Probability and Random Processes* (Springer) (Part I; more rigorous/theoretical construction of stochastic processes)
- B. Oksendal, *Stochastic Differential Equations* (Springer) (Part II; excellent introduction to stochastic calculus)
- I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus* (Springer) (Parts I and II; more rigorous/theoretical construction of Brownian motions and its various properties.)
- R. Durrett, *Essentials of Stochastic Processes* (Springer) (Part I; highly accessible)
- R. Durrett, *Stochastic Calculus: A Practical Introduction* (Part II; more theoretical construction

than we will see, but useful and accessible text)

- P. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations* (Springer)

These will be on reserve at the library. Many of them you can download for free from Springer.

Relationship to other courses

Some students wonder what is the difference between this course, and related ones like Stochastic Calculus or the Limit Theorems series.

Regarding Stochastic Calculus: while there is a lot of overlap in the topics, this course is PhD level course and hence it moves faster and covers more material (such as Markov chains, the relationship to PDEs, and numerical algorithms), and demands more independent study. Stochastic calculus is a master's level course, and puts more emphasis on the properties of Brownian motion, matingales, and stochasic calculus. In addition, this course focuses on applications from physics and chemistry; for example it emphasizes the concept of detailed balance in various settings. Stochastic calculus, on the other hand, is aimed in large part at finance students, and hence draws many applications from mathematical finance, a topic we probably won't mention at all.

Regarding the Limit Theorems series: while the names of the topics covered may have some overlap, the approach to these topics is generally quite different. Limit Theorems I & II develop the theory behind these topics rigorously; while this course assumes the mathematical objects we work with exist (Brownian motion, etc), and that they have sufficiently nice properties that we can use them the way we wish, and then goes on to use them to solve more concrete problems. Many students take both this course, and the Limit Theorems series, and learn sufficiently distinct things from each of them; no one who has taken both courses has complained so far about there being too much overlap.

Outline

This is a rough outline of the topics we will cover in the course. It is subject to change.

Week	Day	Topics
1	Jan 26	Introduction / Review / Markov Chain introduction
2	Feb 2	Markov chains I (discrete-time Markov chains)
3	Feb 9	Markov chains II (detailed balance, Markov Chain Monte Carlo)
4	Feb 16	Markov chains III (continuous-time Markov chains)
5	Feb 23	Stochastic processes I (finite-dimensional distributions, Gaussian processes)
6	March 2	Stochastic processes II (stationary processes, spectral theory)
7	March 9	Stochastic processes III (Brownian motion)
-	March 16	*no class – spring break*
8	March 23	*Midterm Exam*
9	March 30	Stochastic Integration
10	April 6	Stochastic Differential Equations
11	April 13	Numerically solving SDEs
12	April 20	Forward and backward equations for SDEs
13	April 27	Detailed balance and eigenfunction methods
14	May 4	Some applications of the backward equations
Exam	May 11	*Final Exam*