

# Algorithmic Approach to Taxable Investing

**Betterment**

# Betterment



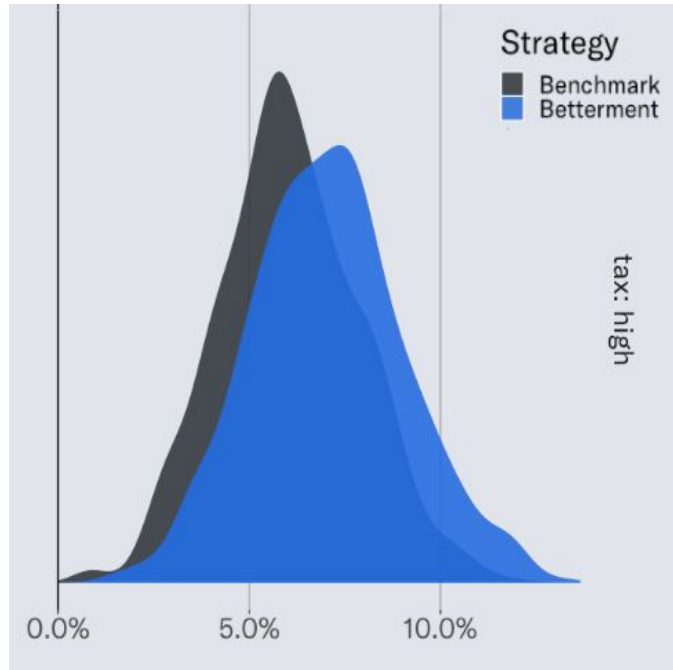
## Automated, online financial advising platform

- Goal-based advice and investment management
- Higher returns at a lower cost by using technology to automate optimal investing practices

# Robo-advisor

“provides financial advice or investment management online with moderate to minimal human intervention. They provide digital financial advice based on mathematical rules or algorithms, executed by software.”

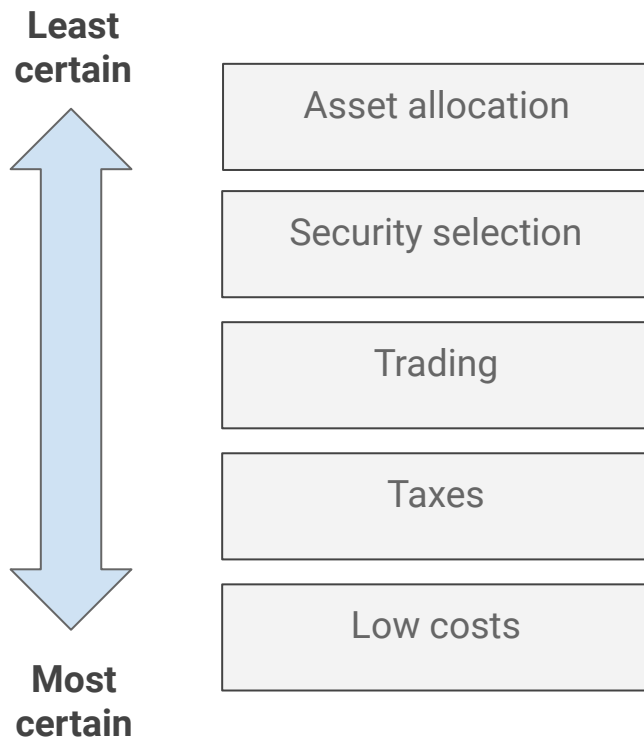
# Better after-tax, after-fee investing outcomes



- Low costs
- Diversified global portfolio
- Automated portfolio management
- Tax optimization
- Good investing behavior

Annual IRR over 30 year investment horizon vs  
US-only stock/bond portfolio

# Determinism in investment management



# Taxes are a huge consideration for retail investors

- Taxes can be a 0.5% - 2.0% annual drag on returns depending on portfolio turnover and tax bracket
- Presents additional opportunities to generate outperformance for investors
- Easier than “beating the market”

# Tax minimization strategies

- Tax loss harvesting
- Asset location
- Lot sorting

# Tax loss harvesting

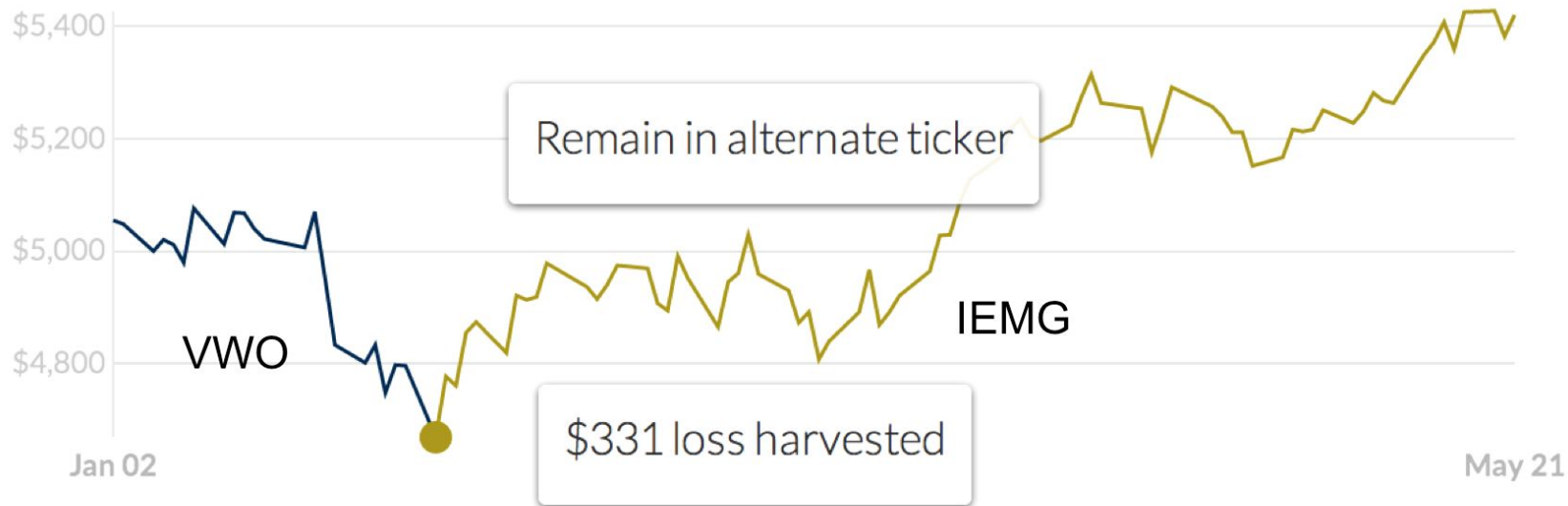
- Sell security at a loss to realize capital loss for tax purposes
- Buy correlated security to keep market exposure





# Tax Loss Harvesting: An example

*Emerging Markets, 1/2/2014 – 5/21/2014*



# How losses can be used

- Defer tax on capital gains
- Shift capital gains from short-term to long-term
  - Short-term losses against short-term gains
  - “Net the nets” -- excess long-term losses against short-term gains
- Reduce taxable income up to \$3,000
- Carry excess losses forward into subsequent tax years
- Permanent tax avoidance in certain situations -- charitable donation, bequest

# Setting up the problem

Maximize losses harvested within a tax year while maintaining target allocation

- Look for harvesting opportunities daily
- Determine when to harvest vs wash sale lockout period
- Avoid wash sales

# Wash sales are a big constraint

The **wash sales rule** disallows a loss from selling a security if a “substantially identical” security is purchased 30 days after or before the sale.

“Substantially identical” not explicitly defined by the IRS.

# Alt. tickers solve two wash sale challenges

## 1) Inflows

- Allows new inflows to be allocated while avoiding the buying back securities that were purchased 30 days before

## 2) Switch back to primary

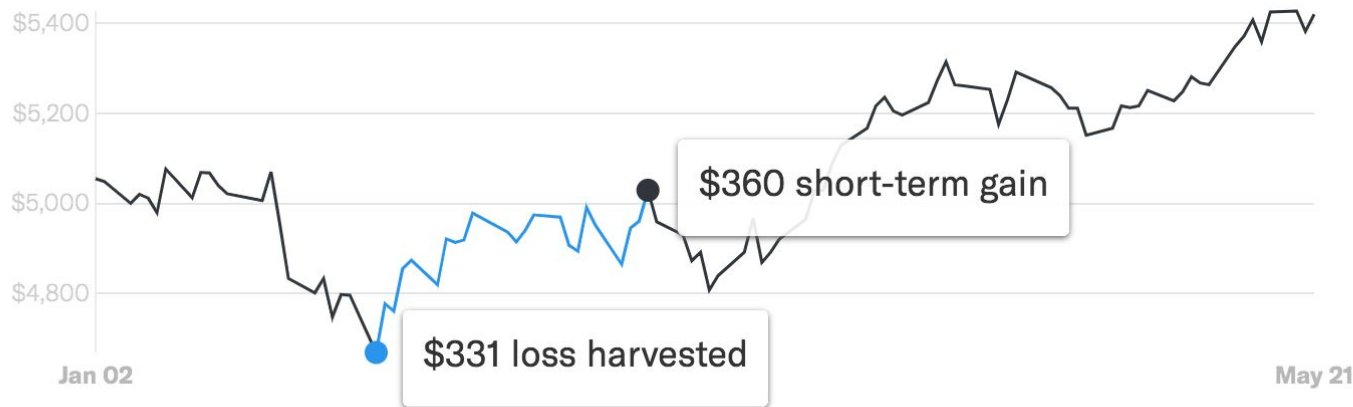
- Blindly switching back to primary security after 30 days can \*increase\* tax liability
- Standard tax loss harvesting implementation

# 30-day switchback can have neg. value

- If alternate ticker goes up, investor would realize STCG
- QDI only applies after 60 days

## TLH with 30-day Switchbacks

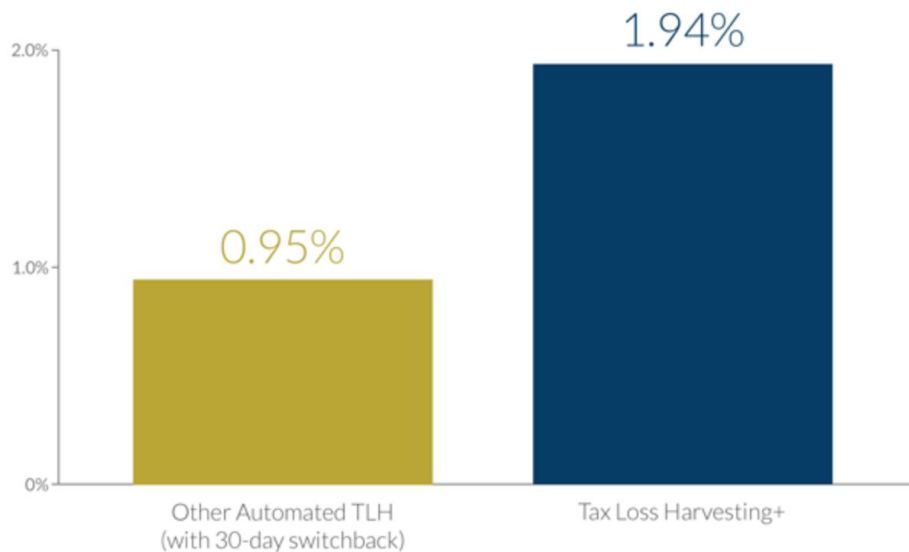
Emerging Markets, 1/2/2014 – 5/21/2014



# Avoiding negative tax alpha pays off

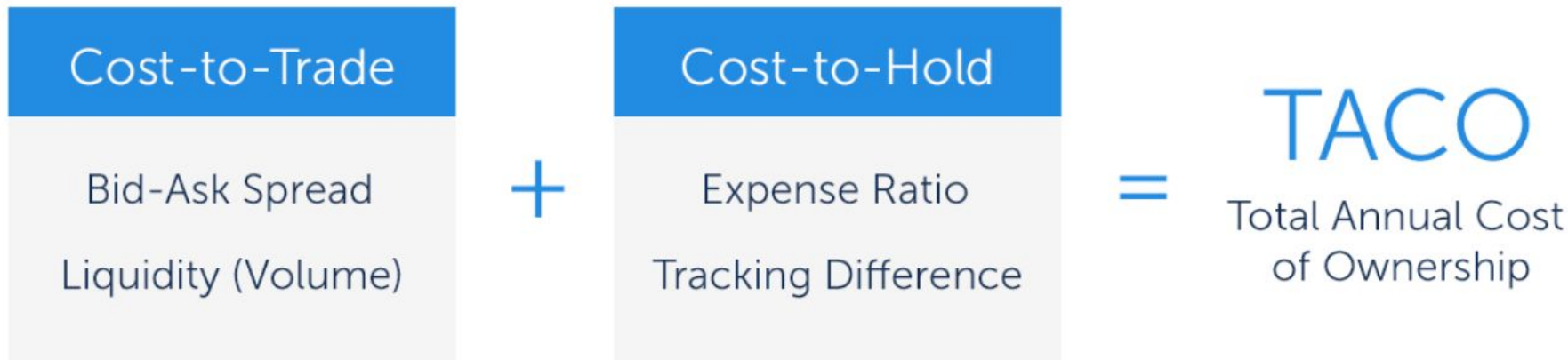
## Average Annual Tax Offset by Strategy

70% stock, 2000-2013



# Choice of alternate tickers is important

Alternate security should have 1) high correlation to primary security, 2) comparable fees, and 3) sufficient liquidity.





# Setting threshold for harvesting a loss

- 1) Expected benefit of harvest should be greater than transaction costs
- 2) Consider opportunity cost of harvesting now vs harvesting later
  - Cannot harvest in the asset class for 30 days because of wash sale rule

# Setting threshold for harvesting a loss

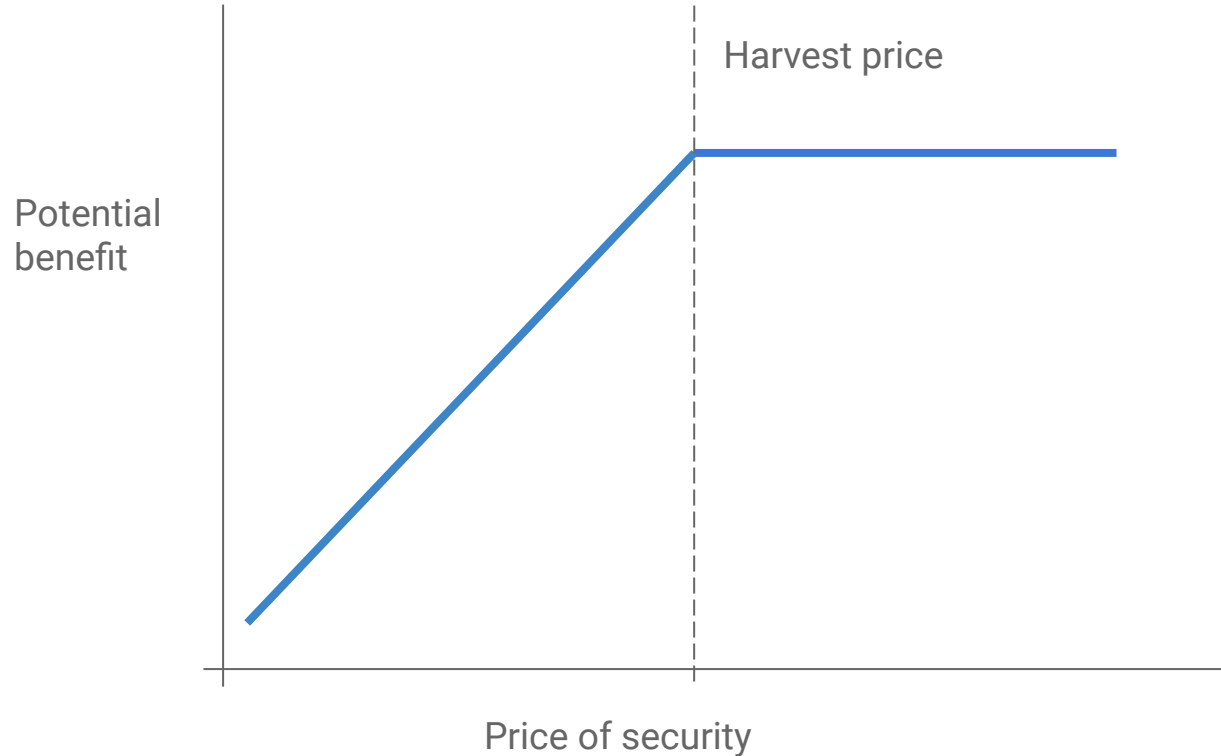
- 1) Expected benefit of harvest should be greater than transaction costs
  - **Very low volatility assets are not TLH'ed**
    - **Short-term treasuries**
    - **Short-term TIPS**
- 2) Consider opportunity cost of harvesting now vs harvesting later
  - Cannot harvest in the asset class for 30 days because of wash sale rule

# Setting harvest triggers using options theory

Harvesting a loss is similar to writing a 30-day put struck at the harvest price on the security

- We 'lose' when the value of the security drops below the harvest price (ie: we could have harvested at a better price)

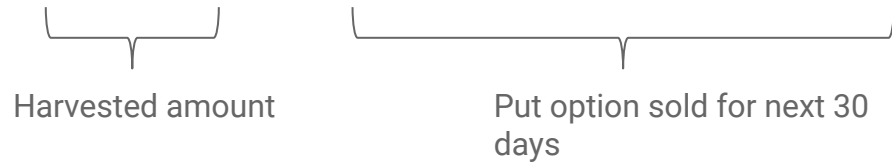
# Benefit of harvest over holdout period



# Harvest when value harvested > option value

## Harvest loss when:

Basis - Price > American put ( $K = \text{price}$ ,  $t = 30$ )

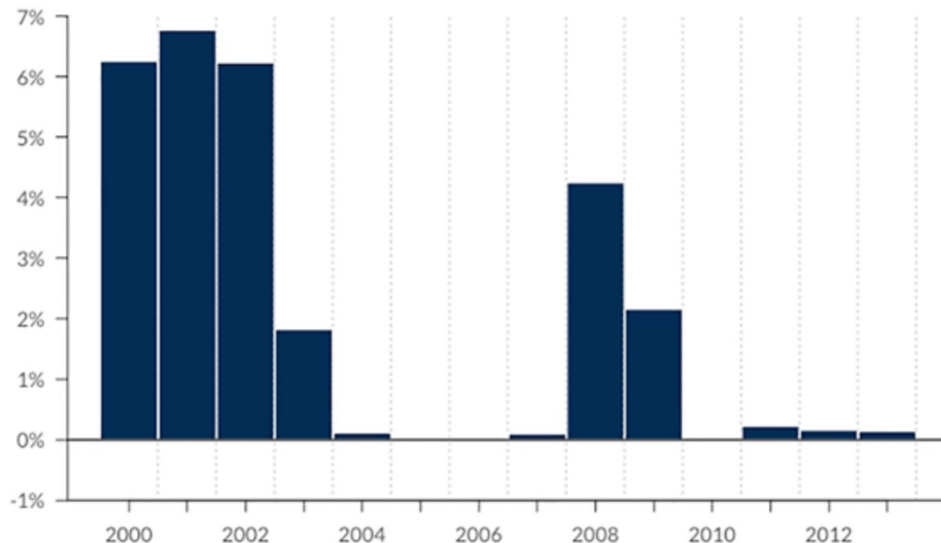


Volatility of security is biggest driver of loss amount needed to harvest

# Tax offsets each year

## Annual Tax Offsets with Tax Loss Harvesting+

*as percent of portfolio value at the start of each year*

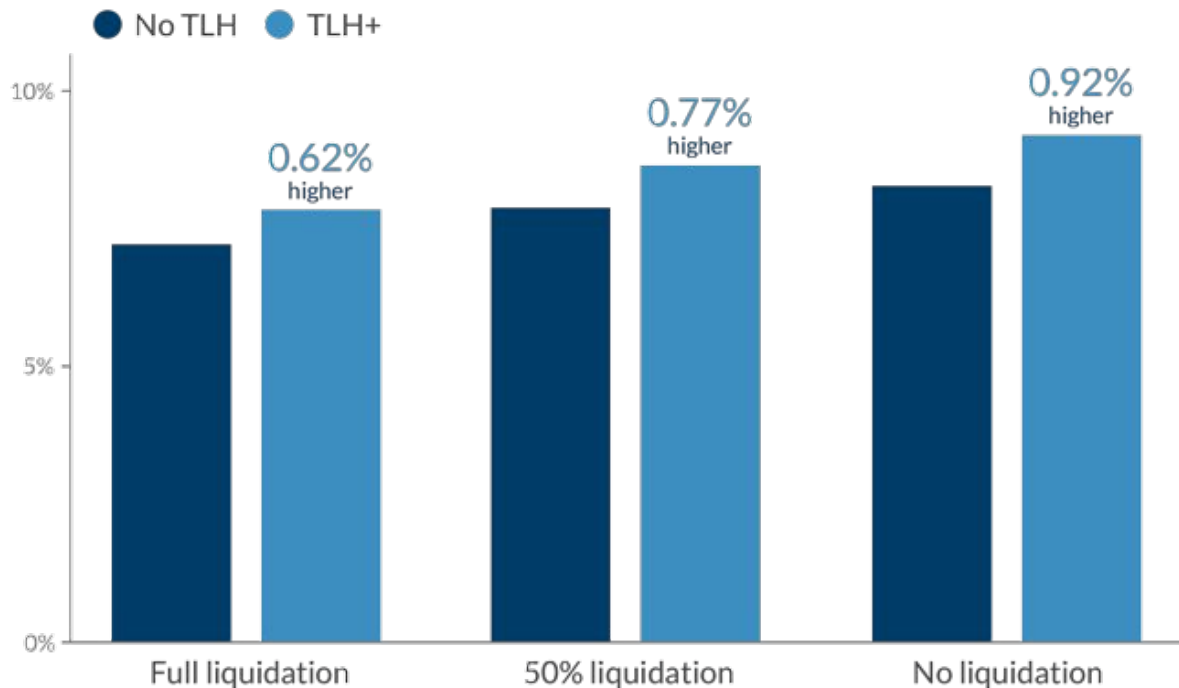


This backtest assumes an initial deposit of \$50K, a constant 70% stock allocation, and deposits twice a month that start at \$750 and increase annually by 5%. Performance is net of Betterment's fees (0.25% when balance is below \$100k, and 0.15% thereafter). Tax is assumed at maximum California rates (federal: 39.6% on income, 23.8% on LTCG; state: 12.3%). All harvested losses are assumed to offset like gains outside the portfolio. Assumes no liquidation. Not actual client performance.



- Provide positive tax offsets in 11 out of 13 years
- No negative tax offsets over the period, even though the portfolio was regularly rebalanced.
- The mean annual tax offset was 1.94%.

# TLH is backtested to demonstrate after-tax alpha over 30 years



- Average rate taxpayer
- Average savings schedule

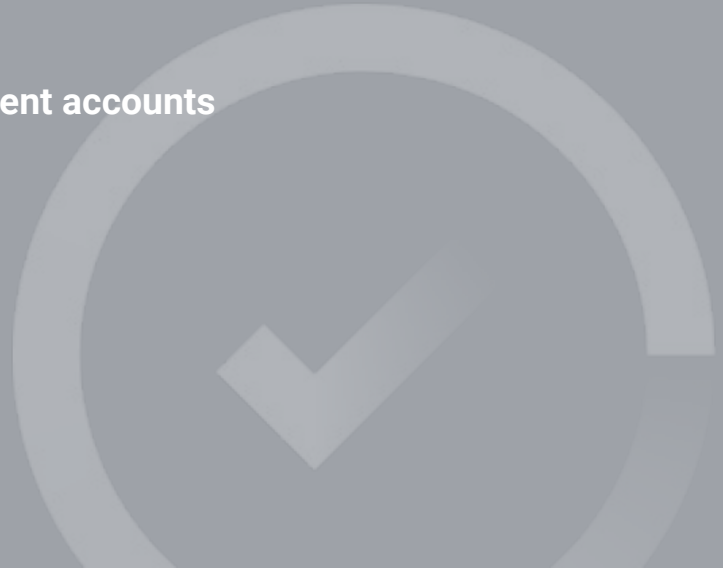
# TLH innovations

- Stay in secondary ticker after harvest (ie, don't switch back)
- Decision to harvest considers opportunity cost future wash sales -- conditional on volatility of asset
- Rebalance with each harvest
- Considering spousals accounts to avoid wash sales
- Different secondary ticker for IRA accounts to avoid permanent wash sale rule
  - Generally, a “washed” loss is postponed until the replacement is sold, but if the replacement is purchased in an IRA/401(k) account, the loss is permanently disallowed.
- Continuous, intraday monitoring



# Asset location

Putting tax-inefficient investments (bonds) in tax-efficient accounts (IRAs)



# Setting up the problem

Certain assets are tax inefficient -- bonds, intl stocks (no QDI)

Accounts have different tax treatment

- Roth IRA/401k -- Growth is tax-free, no taxes on withdrawal
- Traditional IRA/401k -- Growth is tax-free, taxed as ordinary income on withdrawal
- Taxable account -- dividends taxed each year, withdrawals taxed at capital gains rate

Allocate assets to each account to maximize after-tax expected return

s.t.

- Sum of weights across accounts equal target portfolio weights
- Account balances are preserved

# Tax treatment of security types

	<b>Dividends (taxed annually)</b>	<b>Capital Gains (taxed when sold)</b>
Ordinary Rate	Most bonds Non-QDI stocks (foreign)	Any security held for a year or less (STCG)
Preferential Rate	QDI stocks (domestic and some foreign)	Any security held for more than a year (LTCG)
No Tax	Municipal bonds	Any security transferred upon death or donated to charity

# Asset Location

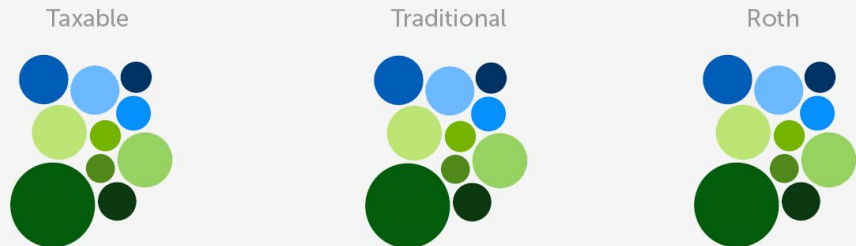
## Not asset located:

- Self-contained diversified portfolio in each account -- Roth, Traditional and taxable accounts

Aggregate Asset Allocation (70% Stocks)



Asset Allocation By Account



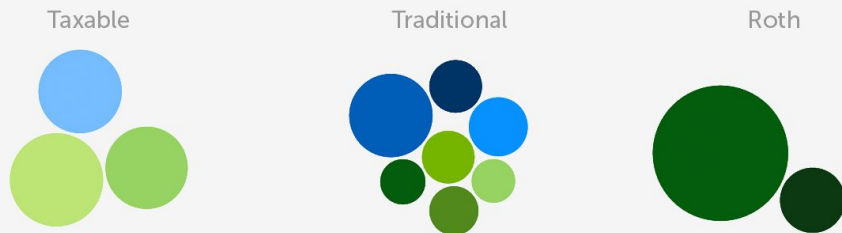
## Asset located:

- Maintain overall level of diversification
- Locate tax-inefficient assets in tax-advantaged accounts

Aggregate Asset Allocation (70% Stocks)

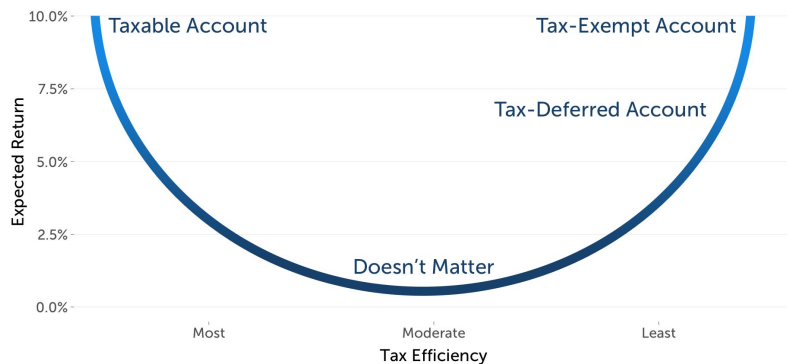


Asset Allocation By Account



# Asset location can be optimized

- Can be defined as linear programming problem
- Accounts are continuously monitored
- Go beyond rules of thumb:
  - Consider growth rate of assets
    - Dividend yield *and* growth rate matter
  - Consider tax drag *and* liquidation taxes
  - QDI ratios



Source: The Kitces Report, March/April 2014.

# Asset location as a LP problem

## Setup Tax Coordinated Portfolios LP:

$$\max_x \sum_j \sum_i r_{ji} x_{ji} - \gamma \left( \sum_i (y_i^+ + y_i^-) \right)$$

where

$r$  is the *after-tax* return for i-asset in j-th account

- $x_{ji} > 0$  - position in i-asset in j-th account
- $y_i^+$  and  $y_i^-$  parametrize positive and negative drift at the asset level (slack variables)
- $\gamma > 0$  - penalty factors

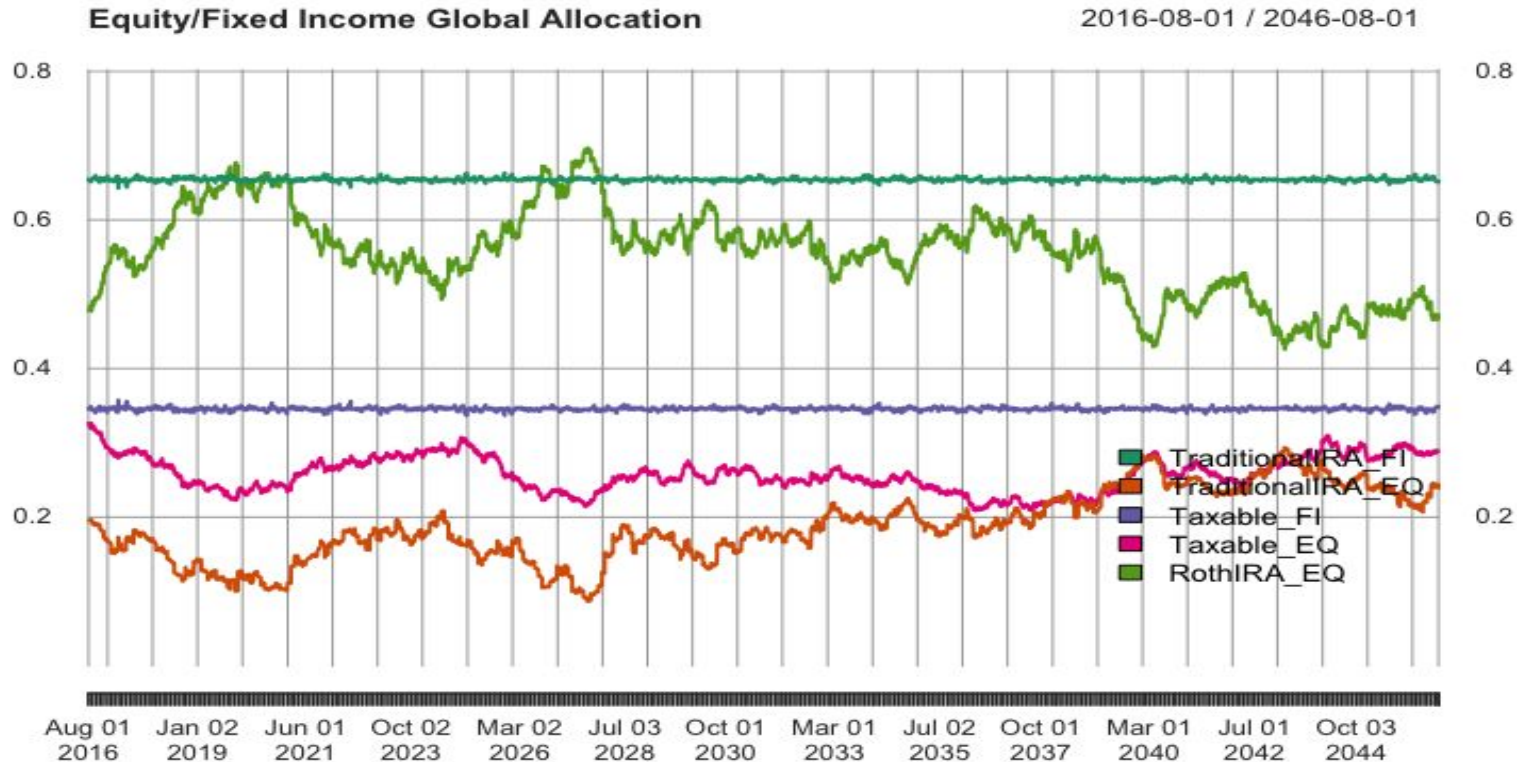
Subject to the following constraints:

$\sum_i x_{ji} = b_j$  , where  $b_j$  is balance in j-th account - balance preservation constraint

$\sum_j \sum_i x_{ji} = \sum_j b_j$  , where  $b_j$  is balance in j-th account - total balance preservation constraint

$\sum_j \sum_i x_{ji} + \sum_i (y_i^+ + y_i^-) = w_i \sum_j b_j$  , where  $w_i$  is i-th asset weight in the target asset allocation - global asset allocation constraint

# Allocations become more tax efficient over time



# How is this different/better?

- Need to consider annual taxes and liquidation taxes
- Need to consider QDI
- Need to consider growth of assets given different liquidation taxes
- Solves LP problem on every rebalance/cash flow.
  - Constantly working to more optimally locate assets in accounts

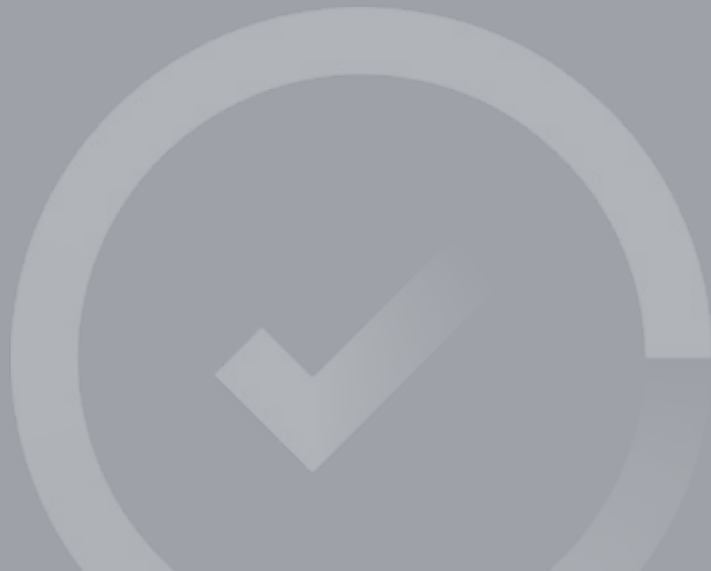


# Asset location implementation is tested for tax alpha using tax lot monte carlo simulations

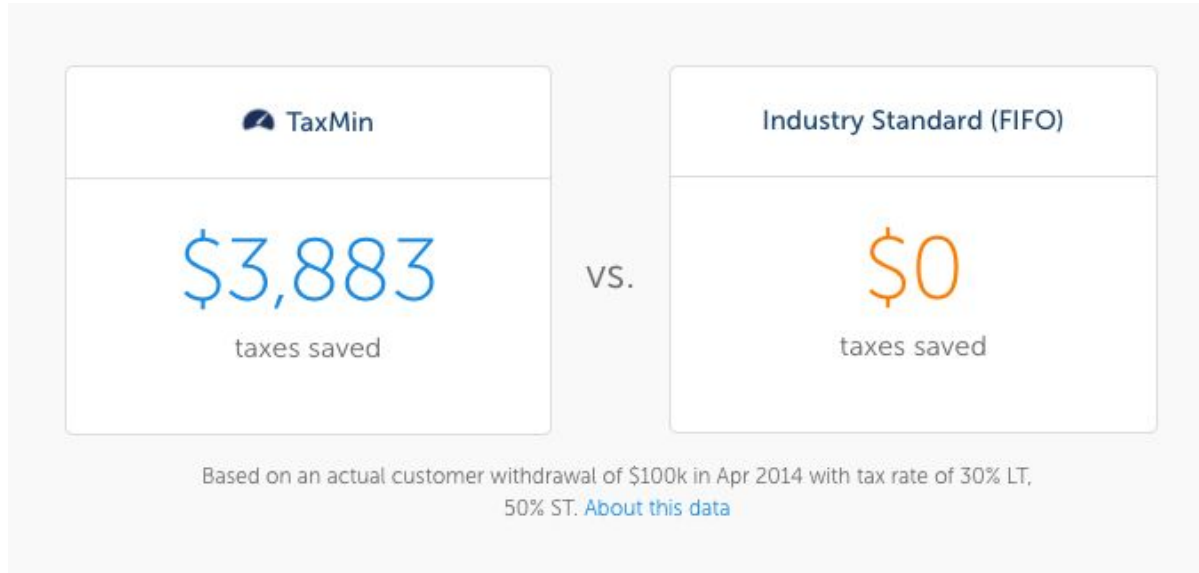
As a result, investors can get higher take-home returns from TCP than they do traditionally

Asset Allocation	Additional Tax Alpha with TCP (Annualized)
50% Stocks	0.82%
70% Stocks	0.48%
90% Stocks	0.27%

# Tax lot management



# Intelligent lot sorting



- Automatically manage sales at the lot-level intelligently
- Sell lots with losses first, then least gains

# Lot order preference

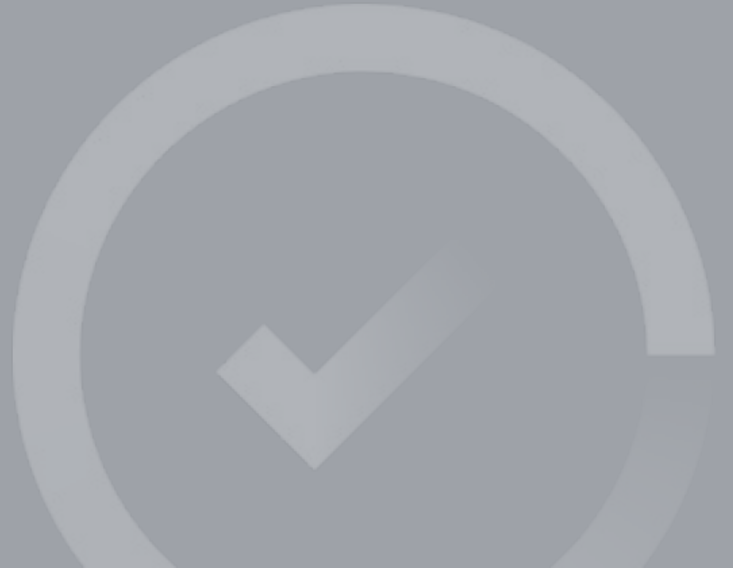
1. Short-term losses
2. Long-term losses
3. Long-term gains
4. Short-term gains \*
  - STCG are blocked except for withdrawals
  - Do not incur STCG when rebalancing

Lot sorting used whenever there is a sell -- rebalancing, withdrawal, fees

# Again, losses are useful

- Net against capital gains
- Reduce taxable income up to \$3,000
- Carry excess losses forward into subsequent tax years

# Managing tax rate uncertainty



# Portfolio construction with tax uncertainty

- **Black-Litterman**
  - Add relative after-tax performance as a view
  - Can specify level of confidence
- **Robust optimization**
  - Draw from distribution of BL posterior returns, many times
  - Optimize
  - Average across sets of weights

# Introduce after-tax outperformance as view in Black-Litterman framework

- BL view matrix  $Q$  holds expected relative after-tax performance
- Uncertainty of view matrix allows us to control likelihood of tax regime



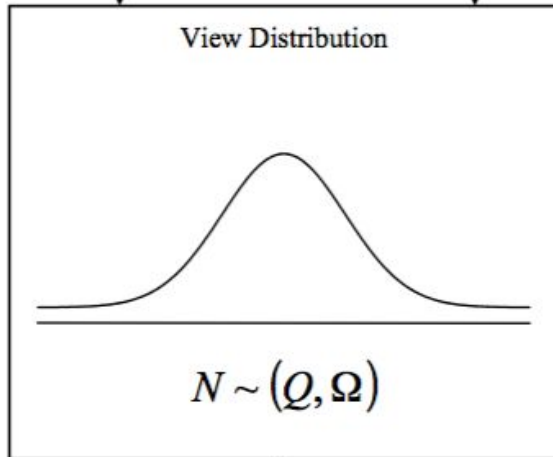
# Incorporating Views

Relative tax  
efficiency

Confidence level  
(0% to 100%)



Source: [Idzorek\(2004\)](#)



$\Omega$

is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view ( $K \times K$  matrix);

$\Pi$

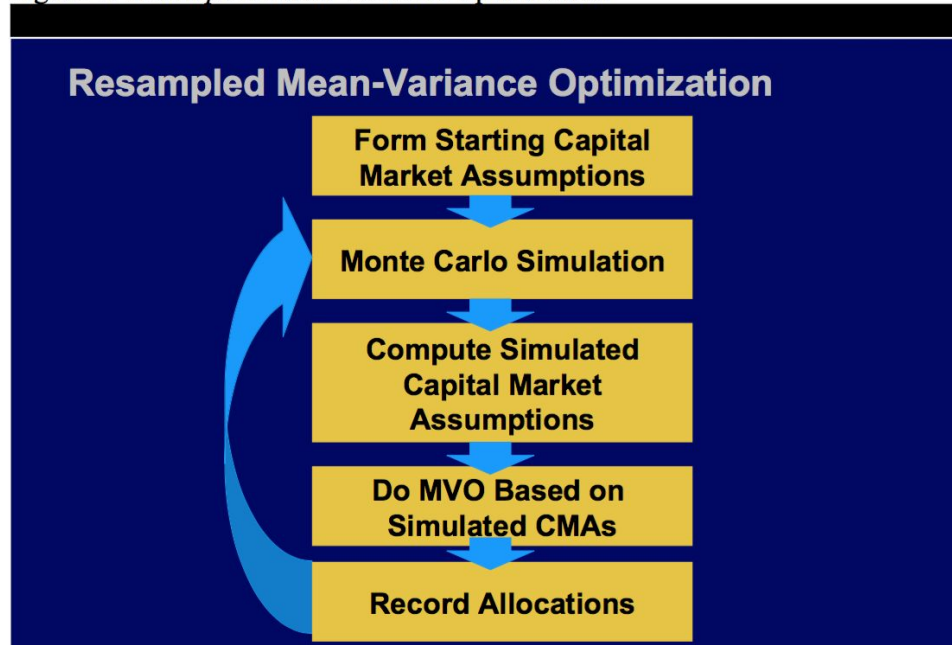
is the Implied Equilibrium Return Vector ( $N \times 1$  column vector); and,

$Q$

is the View Vector ( $K \times 1$  column vector).

# Robust optimization

Figure 7: *Resampled* Mean-Variance Optimization



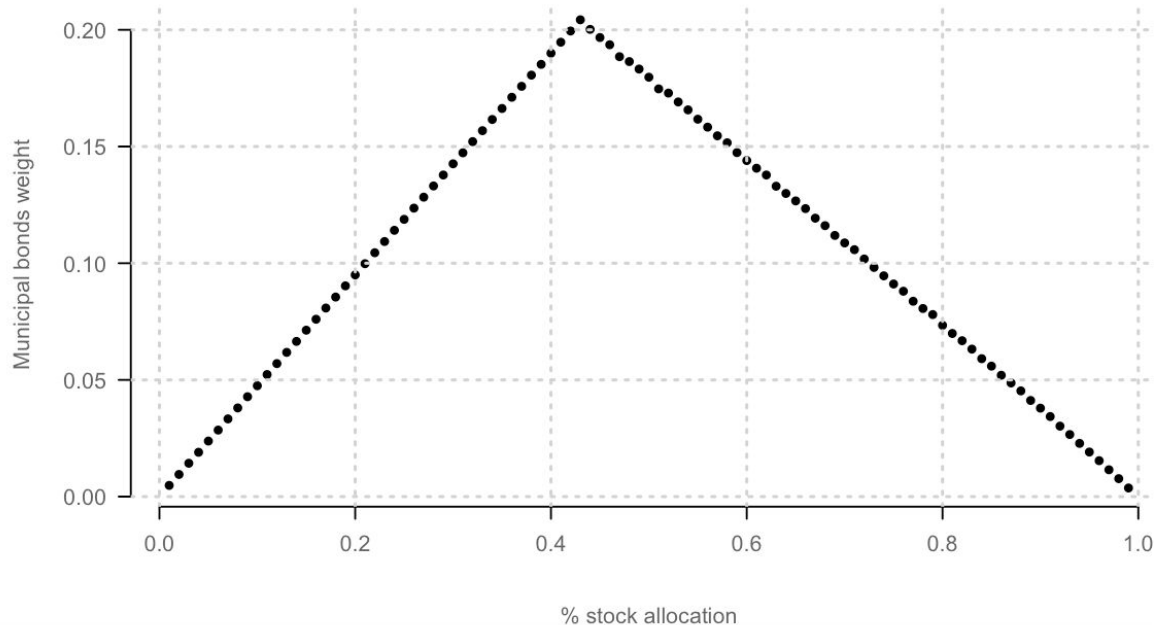
We employ parametric Monte Carlo and simulate many scenarios

Assume returns are generated by a multivariate normal distribution with mean vector given by our Black-Litterman posterior expected returns estimates and covariance

In the end, we get 100 to 500 estimated frontiers (sets of allocations) and we average the portfolio weights for each asset across these simulations.

# Most meaningful difference: taxable portfolio loads on muni bonds

Increase in municipal bond allocation in taxable portfolio



# Thanks!

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[Important disclosures](#)

# Estimating Posterior Moments

New Combined Return Distribution

Source: [Idzorek\(2004\)](#)

$$E[R] = \left[ (\tau \Sigma)^{-1} + P' \Omega^{-1} P \right]^{-1} \left[ (\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q \right]$$

$$N \sim \left( E[R], \left[ (\tau \Sigma)^{-1} + (P' \Omega^{-1} P) \right]^{-1} \right)$$

Adjustment added to  
covariance matrix

- $E[R]$  is the new (posterior) Combined Return Vector ( $N \times 1$  column vector);
- $\tau$  is a scalar;
- $\Sigma$  is the covariance matrix of excess returns ( $N \times N$  matrix);
- $P$  is a matrix that identifies the assets involved in the views ( $K \times N$  matrix or  $1 \times N$  row vector in the special case of 1 view);