Geometry Seminar
Tuesday, September 1, 2009
Room 202 WWH at 6:00 P.M.

The Van Der Waerden conjecture for the mixed volume, its proof and algorithmic applications

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Let $\mathbf{K} = (K_1, \ldots, K_n)$ be a n-tuple of convex compact subsets in the Euclidean space $\mathbf{R}^n$, and let $V()$ be the Euclidean volume in $\mathbf{R}^n$. The Minkowski polynomial $V_{\mathbf{K}}$ is defined as $V_{\mathbf{K}}(x_1, \ldots, x_n) = V(\lambda_1 K_1 + \ + \lambda_n K_n)$ and the mixed volume $V(K_1, \ldots, K_n)$ as

$$V(K_1, \ldots, K_n) = \frac{\partial^n}{\partial\lambda_1 \ldots \partial\lambda_n} V_{\mathbf{K}}(\lambda_1 K_1 + \ldots + \lambda_n K_n).$$

If the convex sets K_i are the boxes(coordinate zonotopes), *i.e.,* $K_i = \{(t_1, \ldots, t_n) : 0 \leq t_j \leq A(i,j), 1 \leq j \leq n\}$, then the mixed volume $V(K_1, \ldots, K_n) = Per(A)$.

In other words, the permanent of a nonnegative matrix is a particular case of the mixed volume. In the first part of the talk, I will state the mixed volume analogue of the Van Der Waerden conjecture for the permanent(i.e. the mixed volume analogue of the Falikman-Egorychev inequality) and will sketch its proof.

This new inequality is in the heart of a new randomized poly-time algorithm, which computes the mixed volume of well-presented convex compact sets with multiplicative error e^n. Because of the famous Barany-Furedi lower bound, even such rate is not achievable by a poly-time deterministic oracle algorithm. I will describe the algorithm and will compare it with the previous approaches.

For more information please visit the seminar website at:
`http://www.math.nyu.edu/seminars/geometry_seminar.html`.