Geometry Seminar
Tuesday, March 24, 2009
Room 317 WWH at 6:00 P.M.

Maximizing the number of colorings

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Let $P_G(q)$ denote the number of proper q-colorings of a graph G. This function, called the chromatic polynomial of G, was introduced by Birkhoff in 1912, who sought to attack the famous four-color problem by minimizing $P_G(4)$ over all planar graphs G. Since then, motivated by a variety of applications, much research was done on minimizing or maximizing $P_G(q)$ over various families of graphs.

In this work, we study an old problem of Linial and Wilf, to find the graphs with n vertices and m edges which maximize the number of q-colorings. We provide the first approach which enables one to solve this problem for many nontrivial ranges of parameters. Using our machinery, we show that for each $q \geq 4$ and sufficiently large $m < \kappa_q n2$ where $\kappa_q \approx 1/(q \log q)$, the extremal graphs are complete bipartite graphs minus the edges of a star, plus isolated vertices. Moreover, for $q = 3$, we establish the structure of optimal graphs for all large $m \leq n2/4$, confirming (in a stronger form) a conjecture of Lazebnik from 1989.

Joint work with Oleg Pikhurko and Benny Sudakov.

For more information please visit the seminar website at:
`http://www.math.nyu.edu/seminars/geometry_seminar.html`.