Geometry Seminar
Tuesday, March 23, 2009
Room 201 WWH at 6:00 P.M.

Unavoidable crossings in plane coverings.

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Two convex disks K and L in the plane are said to *cross each other* if the removal of their intersection causes each of the two disks to fall into disjoint components. Almost all major theorems concerning the covering density of a convex disk were proved for crossing-free coverings only. This includes the classical theorem of L. Fejes Tóth (1950) that uses the maximum area of a hexagon inscribed in the disk to give a significant lower bound for the covering density of the disk. From the early seventies, all attempts of generalizing this theorem were based on the common belief that crossings in a plane covering by congruent convex disks, being counterproductive for producing low density, are always avoidable.

A recently constructed example will be presented, showing that, in general, all such attempts must fail. Three perpendiculars drawn from the center of a regular hexagon to its three non-adjacent sides partition the hexagon into three congruent pentagons. Obviously, the plane can be tiled by such pentagons. But a slight modification produces a (non-tiling) pentagon with an unexpected property: every thinnest covering of the plane by congruent copies of the modified pentagon must contain crossing pairs. The example has no bearing on the validity of Fejes Tóth's bound in general, but it shows that any prospective proof must take into consideration the existence of unavoidable crossings.

For more information please visit the seminar website at:
`http://www.math.nyu.edu/seminars/geometry_seminar.html`.