BOUNDING THE EQUIVARIANT BETTI NUMBERS AND COMPUTING THE GENERALIZED EULER-POINCARÉ CHARACTERISTIC OF SYMMETRIC SEMI-ALGEBRAIC SETS

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ABSTRACT. Let R be a real closed field. We prove upper bounds on the equivariant Betti numbers of symmetric algebraic and semi-algebraic subsets of R^k. More precisely, we prove that if $S \subset \mathrm{R}^k$ is a semi-algebraic subset defined by a finite set of s symmetric polynomials of degree at most d, then the sum of the $\mathfrak{S}_k$-equivariant Betti numbers of S with coefficients in $\mathbb{Q}$ is bounded by $s^{5d}(kd)^{O(d)}$. Unlike the well known classical bounds due to Oleinik and Petrovskii, Thom and Milnor on the Betti numbers of (possibly non-symmetric) real algebraic varieties and semi-algebraic sets, the above bound is polynomial in k when the degrees of the defining polynomials are bounded by a constant. Moreover, our bounds are asymptotically tight. As an application we improve the best known bound on the Betti numbers of the projection of a compact semi-algebraic set improving for any fixed degree the best previously known bound for this problem due to Gabrielov, Vorobjov and Zell. As another application of our methods we obtain polynomial time (for fixed degrees) algorithms for computing the generalized Euler-Poincaré characteristic of semi-algebraic sets defined by symmetric polynomials. This is in contrast to the best complexity of the known algorithms for the same problem in the non-symmetric situation, which is singly exponential. This singly exponential complexity for the latter problem is unlikely to be improved because of hardness result (#**P**-hardness) coming from discrete complexity theory. (Joint work with Cordian Riener)