

# From SABR to Geodesics

A systematic approach for modeling volatility curves with applications to option market-making and pricing multi-asset equity derivatives

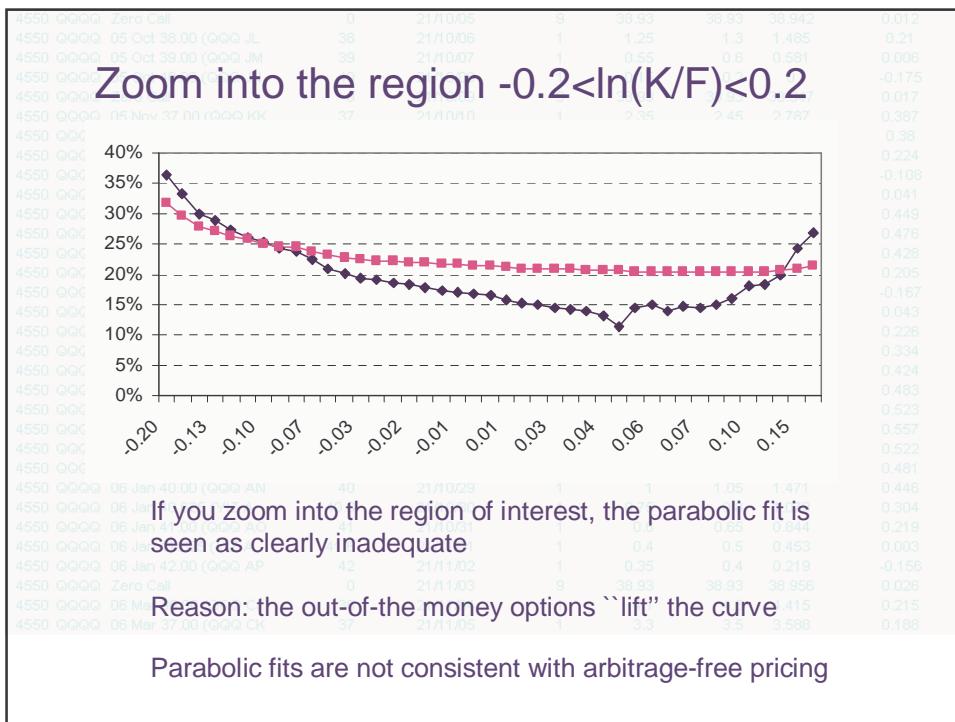
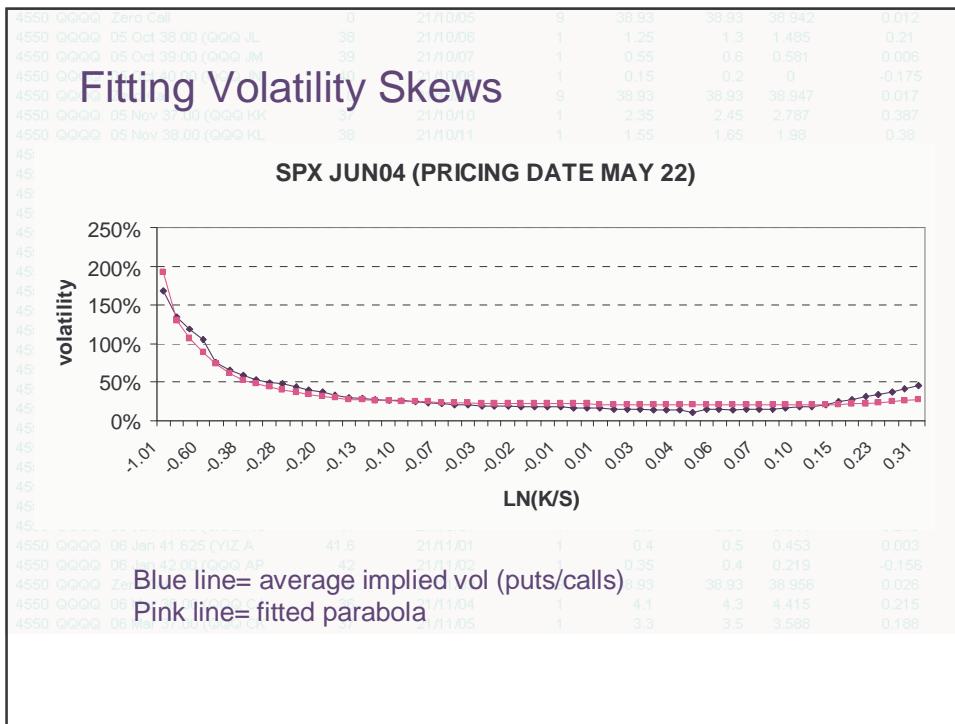
Marco Avellaneda  
Courant Institute, New York University

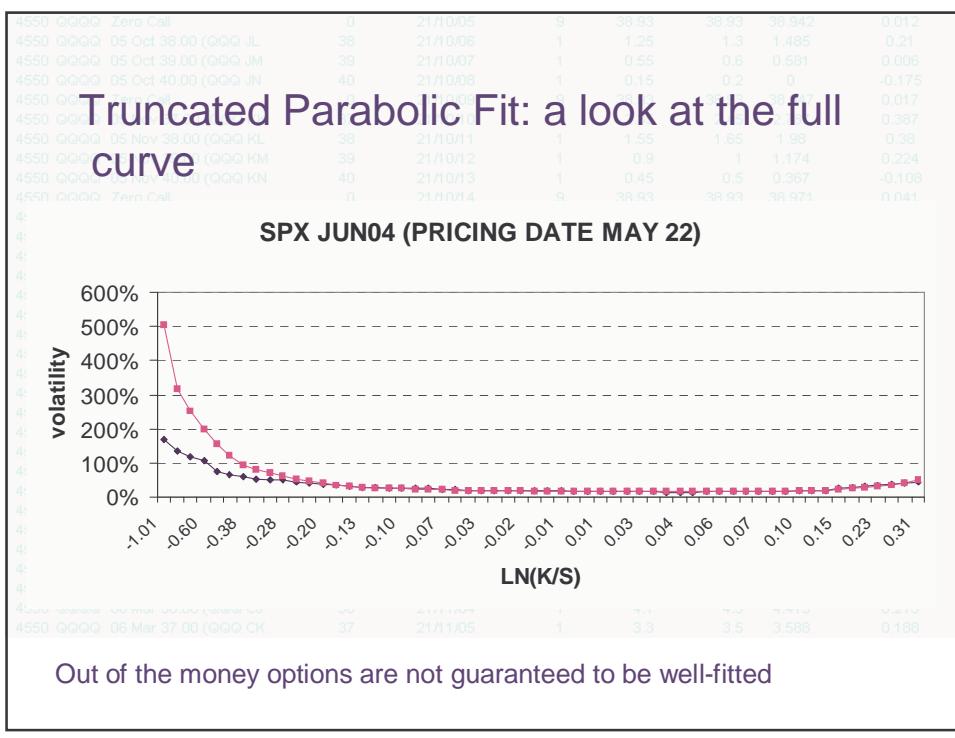
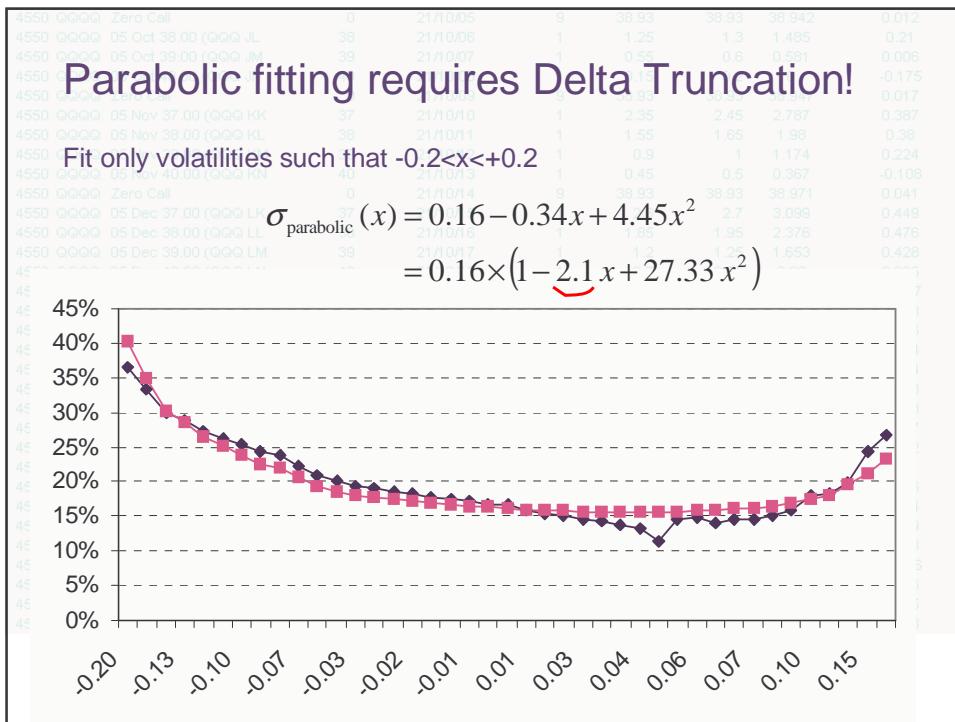
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## The Importance of Having a Listed Derivatives Historical Database

- Equity derivatives analysts can now access historical databases on listed options at relatively low cost. They can:
- Back-test models, especially calibration aspects
- Debunk myths about option models (there are plenty of them!)
- Back-test option strategies systematically, as is done for cash trading
- Test the stability of a skew and vol surface model with real data
- Learn more, by observation, get ideas...

Recommendation: IVY OptionMetrics





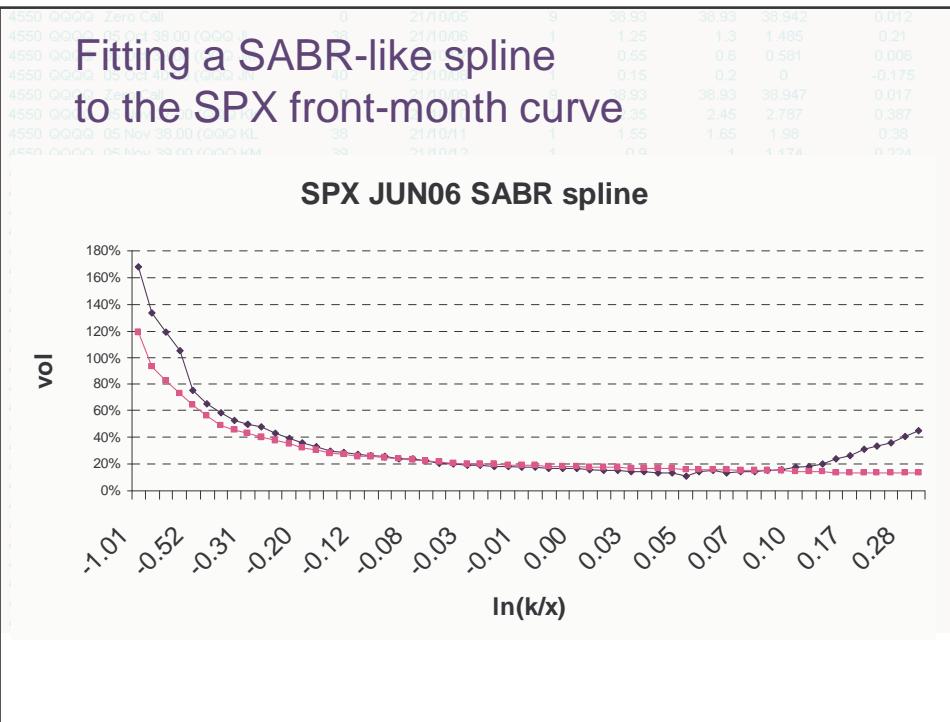
4550 QQQQ Zero Call	0	21/1/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/1/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/1/07	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Oct 40.00 (QQQ JK)	40	21/1/08	1	0.15	0.2	0	-0.175
4550 QQQQ 05 Nov 37.00 (QQQ KN)		21/1/09	1	2.05	2.45	2.787	0.307
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/11	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/1/12	1	0.9	1	1.174	0.224
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/1/13	1	0.45	0.5	0.367	-0.108
4550 QQQQ Zero Call	0	21/1/14	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 37.00 (QQQ LJ)	37	21/1/15	1	2.6	2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ LM)		21/1/16	1	1.55	1.65	1.98	0.376
4550 QQQQ 05 Dec 40.00 (QQQ LN)		21/1/17	1	0.9	0.93	1.174	0.224
4550 QQQQ 05 Dec 41.00 (QQQ LO)		21/1/18	1	0.45	0.5	0.367	-0.108
4550 QQQQ Zero Call		21/1/19	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Jan 38.00 (QQQ AJ)	3	21/1/21	1	3.7	3.8	3.975	0.226
4550 QQQQ 06 Jan 36.625 (YIZ A)	36.6	21/1/22	1	3.2	3.3	3.584	0.334
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/1/23	1	2.7	2.8	3.349	0.424
4550 QQQQ 06 Jan 37.625 (YIZ A)	37.6	21/1/24	1	2.45	2.5	2.958	0.483
4550 QQQQ 06 Jan 38.00 (QQQ AL)	38	21/1/25	1	2.15	2.25	2.723	0.523
4550 QQQQ 06 Jan 38.625 (YIZ A)	38.6	21/1/26	1	1.8	1.9	2.332	0.557
4550 QQQQ 06 Jan 39.00 (YIZ A)	39	21/1/27	1	1.5	1.6	2.097	0.522
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/1/28	1	1.2	1.3	1.706	0.481
4550 QQQQ 06 Jan 40.00 (QQQ AN)	40	21/1/29	1	1	1.15	1.471	0.446
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/1/30	1	0.8	0.9	1.079	0.304
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/1/31	1	0.6	0.65	0.844	0.219
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/1/32	1	0.4	0.5	0.453	0.003
4550 QQQQ Zero Call	0	21/1/33	9	38.93	38.93	38.956	0.026
4550 QQQQ 06 Mar 36.00 (QQQ CJ)	36	21/1/34	1	4.1	4.3	4.415	0.215
4550 QQQQ Formula is derived from a stochastic volatility model so it does not violate arbitrage conditions							0.188

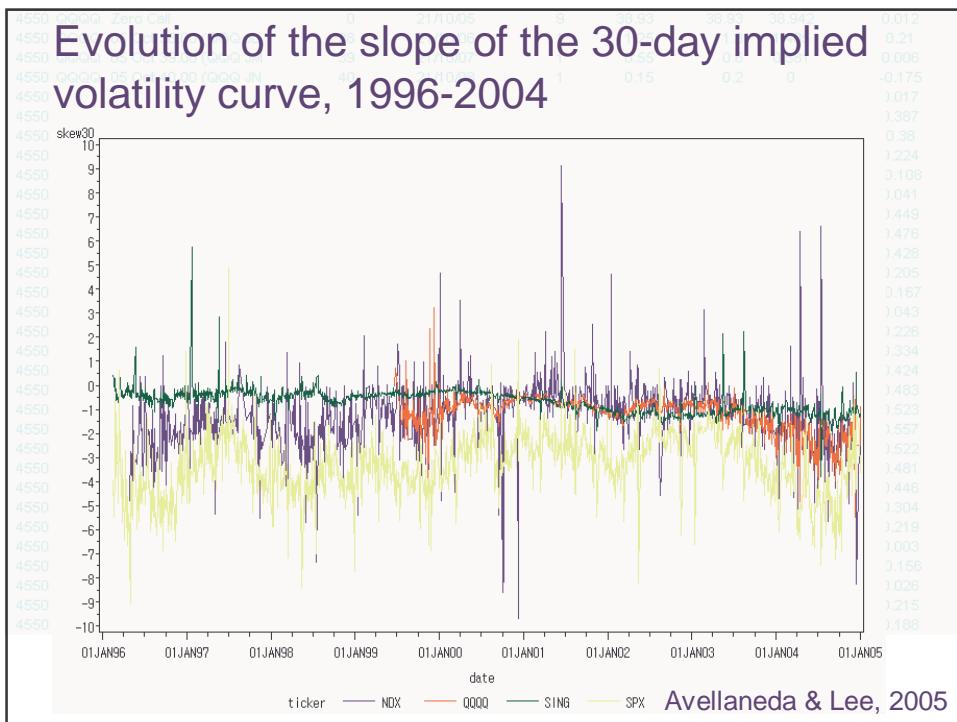
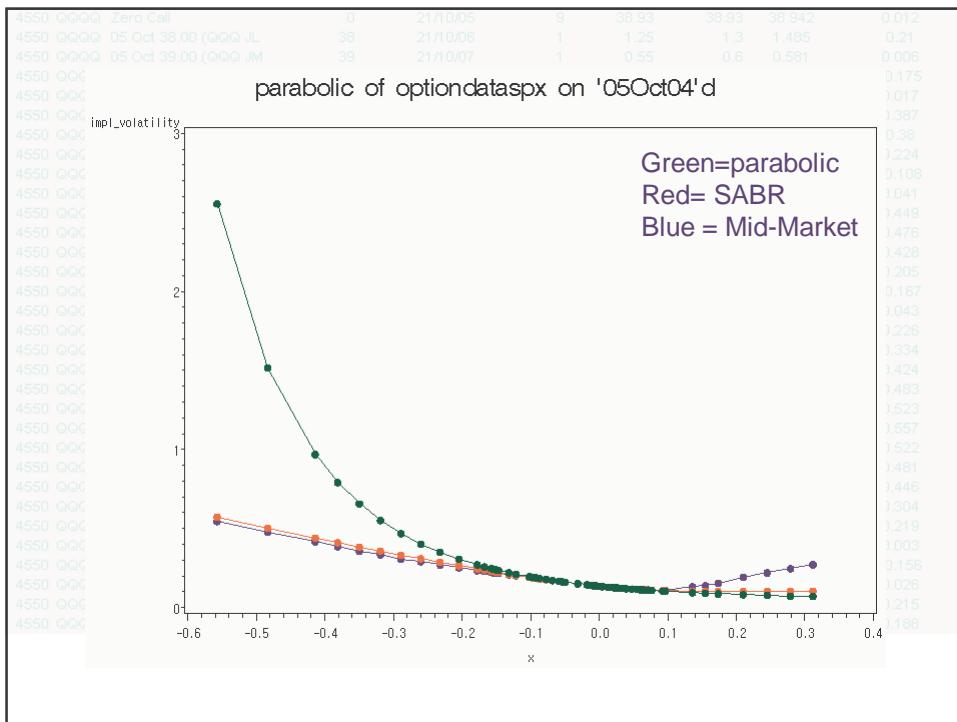
**Sigma, beta and kappa are adjustable parameters**

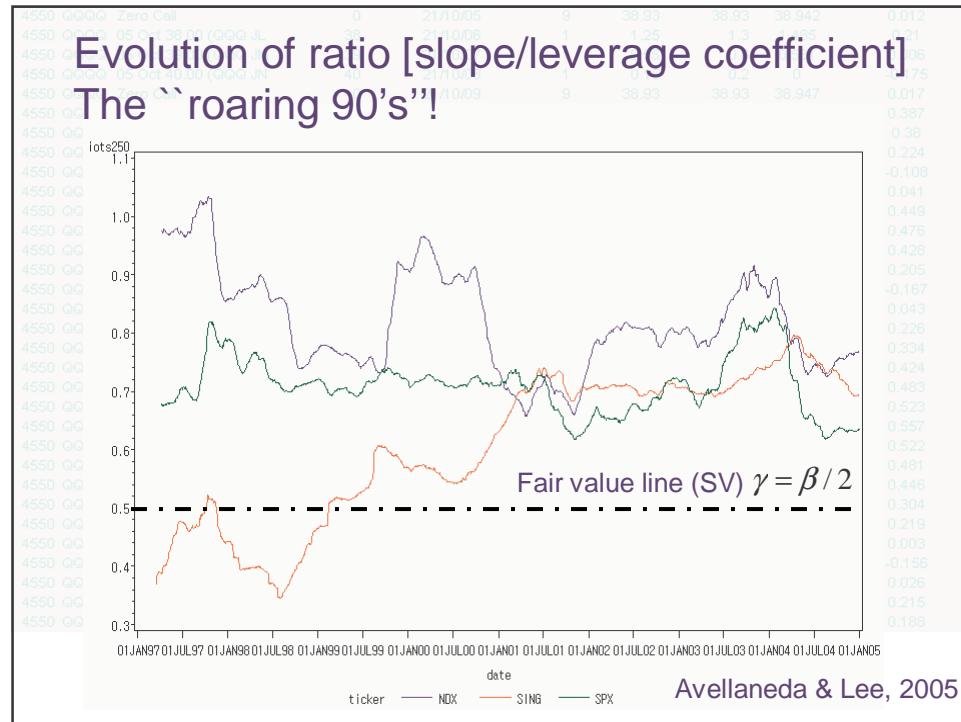
$x = \ln\left(\frac{K}{F_0}\right)$

$\gamma \equiv \text{slope } (x=0) = \frac{\beta}{2}$

Formula is derived from a **stochastic volatility model** so it does not violate arbitrage conditions







**Differential Geometry and  
Implied Volatility Modeling**

date	NDX	SING	SPX
01JAN97	1.0	0.4	0.7
01JAN97	1.1	0.5	0.7
01JAN98	0.9	0.4	0.7
01JUL98	0.8	0.4	0.7
01JAN99	0.8	0.4	0.7
01JUL99	0.8	0.5	0.7
01JAN00	0.8	0.4	0.7
01JUL00	0.8	0.5	0.7
01JAN01	0.8	0.5	0.7
01JUL01	0.8	0.5	0.7
01JAN02	0.8	0.5	0.7
01JUL02	0.8	0.5	0.7
01JAN03	0.8	0.5	0.7
01JUL03	0.8	0.5	0.7
01JAN04	0.8	0.5	0.7
01JUL04	0.8	0.5	0.7
01JAN05	0.8	0.5	0.7

4550	QQQQ	Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550	QQQQ	05 Oct 38.00 (QQQ JL)	36	21/10/06	1	1.25	1.3	1.485	0.21
4550	QQQQ	05 Nov 38.00 (QQQ LK)	37	21/10/07	1	0.55	0.6	0.581	0.006
4550	QQQQ	05 Nov 39.00 (QQQ KM)	38	21/10/08	1	1.55	1.6	1.744	0.224
4550	QQQQ	05 Nov 39.00 (QQQ KN)	39	21/10/09	1	0.9	0.95	0.947	0.008
4550	QQQQ	05 Nov 40.00 (QQQ LN)	40	21/10/10	1	0.45	0.5	0.52	-0.175
4550	QQQQ	Zero Call	0	21/10/09	9	38.93	38.93	38.947	0.017
4550	QQQQ	05 Nov 37.00 (QQQ LK)	37	21/10/10	1	2.35	2.45	2.787	0.387
4550	QQQQ	05 Nov 39.00 (QQQ LL)	38	21/10/11	1	1.55	1.65	1.95	0.38
4550	QQQQ	05 Nov 39.00 (QQQ KM)	39	21/10/12	1	0.9	0.95	0.947	0.008
4550	QQQQ	05 Nov 40.00 (QQQ KN)	40	21/10/13	1	0.45	0.5	0.52	-0.175
4550	QQQQ	Zero Call	0	21/10/10	9	38.93	38.93	38.947	0.017
4550	QQQQ	05 Dec 37.00 (QQQ LK)	36	21/10/11	1	2.6	2.7	2.958	0.449
4550	QQQQ	05 Dec 39.00 (QQQ LL)	37	21/10/12	1	1.85	1.95	2.248	0.476
4550	QQQQ	05 Dec 39.00 (QQQ LM)	38	21/10/13	1	1.2	1.25	1.653	0.428
4550	QQQQ	05 Dec 40.00 (QQQ LN)	39	21/10/14	1	0.7	0.75	0.83	0.205
4550	QQQQ	05 Dec 41.00 (QQQ LO)	41	21/10/15	1	0.35	0.4	0.208	-0.157
4550	QQQQ	Zero Call	0	21/10/20	9	38.93	38.93	38.973	0.043
4550	QQQQ	05 Jan 38.00 (QQQ AJ)	36	21/10/21	1	3.7	3.8	3.975	0.226
4550	QQQQ	05 Jan 36.625 (YIZ A)	37	21/10/22	1	2.3	2.3	2.584	0.334
4550	QQQQ	05 Jan 37.00 (QQQ AK)	37	21/10/23	1	2.45	2.5	2.958	0.424
4550	QQQQ	05 Jan 37.625 (YIZ A)	37.6	21/10/24	1	2.45	2.5	2.958	0.483
4550	QQQQ	05 Jan 38.00 (QQQ AL)	38	21/10/25	1	2.15	2.25	2.723	0.523
4550	QQQQ	05 Jan 38.625 (YIZ A)	38.6	21/10/26	1	1.75	1.8	2.332	0.557
4550	QQQQ	05 Jan 39.00 (QQQ AM)	39	21/10/27	1	1.55	1.6	2.0	0.522
4550	QQQQ	05 Jan 39.625 (YIZ A)	39.6	21/10/28	1	1.2	1.25	1.7	0.31
4550	QQQQ	05 Jan 40.00 (QQQ AO)	40	21/10/29	1	0.75	0.8	1.075	0.304
4550	QQQQ	05 Jan 40.625 (YIZ A)	40.6	21/10/30	1	0.75	0.8	1.075	0.304
4550	QQQQ	05 Jan 41.00 (QQQ AO)	41	21/10/31	1	0.6	0.65	0.844	0.219
4550	QQQQ	05 Jan 41.625 (YIZ A)	41.6	21/10/01	1	0.4	0.5	0.453	0.003
4550	QQQQ	$E\{F(x(T)) x(t)=x\} = \int_{y \in R^n} F(y) \pi(x, t; y, T) d^n y$				38.93	38.956	0.026	
4550	QQQQ	05 Mar 38.00 (QQQ CJ)	36	21/11/04	1	4.1	4.3	4.415	0.215
4550	QQQQ	05 Mar 37.00 (QQQ CK)	37	21/11/05	1	3.3	3.5	3.588	0.188

CIR-type setting,  $\sigma^2 = 0.025$

X= state variables  $x_i(t) = (X_1(t), \dots, X_n(t))$

W= m-dim Brownian motion

$$dx_i = \sum_{k=1}^m \sigma_j^k dW_k + b_i dt, \quad i=1,2,3,\dots,n$$

Diffusion kernel

$$\pi(x, t; y, T) = \text{Prob.}\{x(T) = y | x(t) = x\}$$

4550	QQQQ	Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550	QQQQ	05 Oct 38.00 (QQQ JL)	36	21/10/06	1	1.25	1.3	1.485	0.21
4550	QQQQ	05 Nov 38.00 (QQQ LK)	37	21/10/07	1	0.55	0.6	0.581	0.006
4550	QQQQ	05 Nov 39.00 (QQQ KM)	38	21/10/08	1	1.55	1.6	1.744	0.224
4550	QQQQ	05 Nov 39.00 (QQQ KN)	39	21/10/09	1	0.9	1	1.174	0.224
4550	QQQQ	05 Nov 40.00 (QQQ LN)	40	21/10/10	1	0.45	0.5	0.367	-0.108
4550	QQQQ	Zero Call	0	21/10/10	9	38.93	38.93	38.971	0.041
4550	QQQQ	05 Dec 37.00 (QQQ LK)	37	21/10/11	1	2.6	2.7	3.099	0.449
4550	QQQQ	05 Dec 39.00 (QQQ LM)	38	21/10/12	1	1.2	1.25	1.653	0.428
4550	QQQQ	05 Dec 40.00 (QQQ LN)	39	21/10/13	1	0.7	0.75	0.93	0.205
4550	QQQQ	05 Dec 41.00 (QQQ LO)	41	21/10/14	1	0.35	0.4	0.208	-0.157
4550	QQQQ	Zero Call	0	21/10/20	9	38.93	38.93	38.973	0.043
4550	QQQQ	05 Jan 36.00 (QQQ AJ)	36	21/10/21	1	2.9	3	3.76	0.226
4550	QQQQ	05 Jan 36.625 (YIZ A)	36.6	21/10/22	1	3.2	3.3	3.584	0.334
4550	QQQQ	05 Jan 37.00 (QQQ AK)	37	21/10/23	1	2.9	2.95	3.349	0.424
4550	QQQQ	05 Jan 37.625 (YIZ A)	37.6	21/10/24	1	2.45	2.5	2.958	0.483
4550	QQQQ	05 Jan 38.00 (QQQ AL)	38	21/10/25	1	2.15	2.25	2.723	0.523
4550	QQQQ	05 Jan 38.625 (YIZ A)	38.6	21/10/26	1	1.75	1.8	2.332	0.557
4550	QQQQ	05 Jan 39.00 (QQQ AM)	39	21/10/27	1	1.2	1.25	1.706	0.481
4550	QQQQ	05 Jan 39.625 (YIZ A)	39.6	21/10/28	1	0.6	1.05	1.471	0.446
4550	QQQQ	05 Jan 40.00 (QQQ AN)	40	21/10/29	1	1	1.05	1.471	0.446
4550	QQQQ	05 Jan 40.625 (YIZ A)	40.6	21/10/30	1	0.75	0.8	1.079	0.304
4550	QQQQ	05 Jan 41.00 (QQQ AO)	41	21/10/31	1	0.6	0.65	0.844	0.219
4550	QQQQ	05 Jan 41.625 (YIZ A)	41.6	21/11/01	1	0.4	0.5	0.453	0.003
4550	QQQQ	05 Mar 38.00 (QQQ CJ)	36	21/11/02	1	3.5	3.5	3.588	0.188
4550	QQQQ	05 Mar 37.00 (QQQ CK)	37	21/11/03	1	4.1	4.3	4.415	0.215
4550	QQQQ	05 Mar 37.625 (YIZ A)	37.6	21/11/04	1	3.3	3.5	3.588	0.188

Covariance matrix of state variables

$$a_{ij} = \sum_{k=1}^m \sigma_i^k \sigma_j^k$$

volatility of S&P=0.15

t=1 yr. corresponds to

tau=0.0225<<1

Dimensionless time

``typical variance'' of x

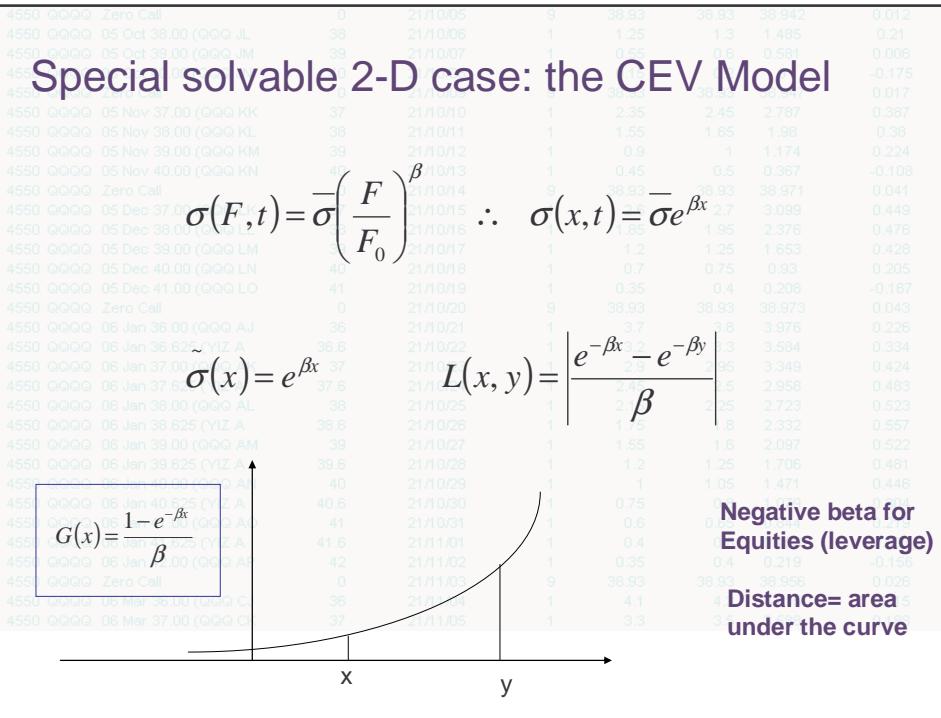
Varadhan Asymptotics for the Diffusion Kernel						
4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.942
4550 QQQQ 05 Oct 39.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/1/007	1	0.55	0.6	0.581
4550 QQQQ 05 Nov 39.00 (QQQ KN)	40	21/1/008	1	0.15	0.2	0
4550 QQQQ 05 Nov 39.00 (QQQ KK)	37	21/1/009	9	38.93	38.93	38.947
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/010	1	2.35	2.45	2.787
4550 QQQQ 05 Nov 39.00 (QQQ LL)	38	21/1/011	1	1.55	1.65	1.99
4550 QQQQ 05 Nov 39.00 (QQQ KM)	38	21/1/012	1	0.5	0.6	0.224
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/1/013	1	0.45	0.5	0.367
4550 QQQQ Zero Call	0	21/1/014	1	38.93	38.93	38.97
4550 QQQQ 05 Dec 37.00 (QQQ LL)	38	21/1/015	1	1.85	1.95	2.376
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/1/017	1	1.2	1.25	1.653
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/1/018	1	0.7	0.75	0.93
4550 QQQQ 05 Dec 41.00 (QQQ LN)	41	21/1/019	1	0.35	0.4	0.208
4550 QQQQ Zero Call	0	21/1/020	1	2.3	2.35	3.343
4550 QQQQ 06 Jan 38.00 (QQQ AJ)	36	21/1/021	1	3.7	3.8	3.976
4550 QQQQ 06 Jan 38.625 (YIZ A)	36.6	21/1/022	1	3.2	3.3	3.584
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/1/023	1	2.3	2.35	3.343
4550 QQQQ 06 Jan 37.625 (YIZ A)	37.6	21/1/024	1	2.45	2.5	2.958
4550 QQQQ 06 Jan 38.00 (QQQ AL)	38	21/1/025	1	2.15	2.25	2.723
4550 QQQQ 06 Jan 38.625 (YIZ A)	38.6	21/1/026	1	1.75	1.8	2.332
4550 QQQQ 06 Jan 39.00 (QQQ AM)	39	21/1/027	1	1.55	1.6	2.097
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/1/028	1	1.2	1.25	1.706
4550 QQQQ 06 Jan 40.00 (QQQ AN)	40	21/1/029	1	1	1.05	1.471
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/1/030	1	0.75	0.8	1.079
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/1/031	1	0.6	0.65	0.844
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/1/032	1	0.4	0.5	0.453
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/1/033	1	0.35	0.4	0.219
4550 QQQQ Zero Call	0	21/1/034	9	38.93	38.93	38.956
4550 QQQQ 06 Mar 38.00 (QQQ CJ)	36	21/1/035	1	4.1	4.3	4.415
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/1/036	1	3.3	3.3	3.588
Dimensionless Riemann tensor						

4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	0.012
4550 QQQQ 05 Oct 39.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/1/007	1	0.55	0.6	0.581
4550 QQQQ 05 Nov 39.00 (QQQ KN)	40	21/1/008	1	0.15	0.2	0
4550 QQQQ 05 Nov 39.00 (QQQ KK)	37	21/1/009	9	38.93	38.93	38.947
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/010	1	2.35	2.45	2.887
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/1/011	1	1.2	1.25	1.653
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/1/012	1	0.7	0.75	0.93
4550 QQQQ Zero Call	0	21/1/013	1	3.5	3.6	4.041
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/1/014	1	2.6	2.7	3.099
4550 QQQQ 05 Dec 39.00 (QQQ LL)	38	21/1/015	1	1.85	1.95	2.376
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/1/016	1	1.2	1.25	1.653
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/1/017	1	0.75	0.8	1.079
4550 QQQQ 05 Dec 41.00 (QQQ LN)	41	21/1/018	1	0.6	0.65	0.844
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/1/019	1	0.4	0.5	0.453
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/1/020	1	0.35	0.4	0.219
4550 QQQQ Heuristically: Diffusion Kernels ``resemble'' Gaussian Kernels with $ x-y $ replaced by $L(x,y)$						
4550 QQQQ 06 Jan 38.00 (QQQ AJ)	36	21/1/021	1	3.7	3.8	3.976
4550 QQQQ 06 Jan 38.625 (YIZ A)	36.6	21/1/022	1	3.2	3.3	3.584
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/1/023	1	2.9	2.95	3.349
4550 QQQQ 06 Jan 37.625 (YIZ A)	37.6	21/1/024	1	2.45	2.5	2.958
4550 QQQQ 06 Jan 38.00 (QQQ AL)	38	21/1/025	1	2.15	2.25	2.723
4550 QQQQ 06 Jan 38.625 (YIZ A)	38.6	21/1/026	1	1.75	1.8	2.332
4550 QQQQ 06 Jan 39.00 (QQQ AM)	39	21/1/027	1	1.55	1.6	2.097
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/1/028	1	1.2	1.25	1.706
4550 QQQQ 06 Jan 40.00 (QQQ AN)	40	21/1/029	1	1	1.05	1.471
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/1/030	1	0.75	0.8	1.079
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/1/031	1	0.6	0.65	0.844
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/1/032	1	0.4	0.5	0.453
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/1/033	1	0.35	0.4	0.219
4550 QQQQ We shall use this approximation to compute option prices and implied volatilities assuming tau is small						
4550 QQQQ 06 Mar 38.00 (QQQ CJ)	36	21/1/034	1	4.1	4.3	4.415
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/1/035	1	3.3	3.3	3.588

4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 36.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/1/007	1	0.95	0.9	0.501	0.006
4550 QQQQ 05 Nov 36.00 (QQQ KK)	37	21/1/010	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/011	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/1/012	1	0.9	1	1.174	0.224
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/1/014	9	38.93	38.93	38.971	0.041
4550 QQQQ Zero Call	0	21/1/015	1	2	2.7	3.099	0.449
4550 QQQQ 05 Dec 37.00 (QQQ LL)	37	21/1/016	1	1.85	1.95	2.376	0.476
4550 QQQQ 05 Dec 39.00 (QQQ LM)	38	21/1/017	1	1.2	1.25	1.653	0.428
4550 QQQQ 05 Dec 40.00 (QQQ LN)	39	21/1/018	1	0.7	0.75	0.93	0.205
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/1/019	1	0.35	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/1/020	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Jan 36.00 (QQQ AJ)	36	21/1/021	1	3.7	3.8	3.976	0.226
4550 QQQQ 06 Jan 36.625 (YIZ A)	37	21/1/022	1	3	3	3.584	0.334
4550 QQQQ 06 Jan 37.00 (QQQ AK)	38	21/1/023	1	2.5	2.5	3.349	0.424
4550 QQQQ 06 Jan 37.5	39	21/1/024	1	1.5	1.5	2.958	0.483
4550 QQQQ 06 Jan 38.00 (YIZ AL)	40	21/1/025	1	1.25	1.25	2.723	0.523
4550 QQQQ 06 Jan 38.625 (YIZ A)	41	21/1/026	1	1.2	1.25	2.332	0.557
4550 QQQQ 06 Jan 39.00 (QQQ AM)	36	21/1/027	1	1.2	1.25	2.097	0.522
4550 QQQQ 06 Jan 39.625 (YIZ A)	37	21/1/028	1	1.05	1.05	1.706	0.481
4550 QQQQ 06 Jan 40.00 (QQQ AN)	40	21/1/029	1	1	1.05	1.471	0.446
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/1/030	1	0.75	0.8	1.079	0.304
4550 QQQQ 06 Jan 41.00 (YIZ A)	41	21/1/031	1	0.5	0.5	0.844	0.219
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/1/031	1	0.4	0.5	0.453	0.003
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/1/032	1	0.35	0.4	0.219	-0.156
4550 QQQQ Zero Call	0	21/1/033	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Mar 36.00 (QQQ CL)	36	21/1/034	1	1	1	1	1
4550 QQQQ 06 Mar 37.00 (QQQ CL)	37	21/1/035	1	1	1	1	1

$$L(x, y) = \left| \int_x^y \frac{du}{\sigma(u)} \right| = |G(y) - G(x)|$$

1-dimensional distances are always 'trivial'



4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ IM)	39	21/1/007	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Nov 38.00 (QQQ KM)	38	21/1/012	1	0.15	0.2	0	-0.175
4550 QQQQ Zero Call	0	21/1/009	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/1/010	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/011	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/1/012	1	0.9	1	1.174	0.224
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/1/013	1	0.45	0.5	0.387	-0.108
4550 QQQQ Zero Call	0	21/1/015	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/1/016	1	1.85	1.95	2.376	0.476
4550 QQQQ 05 Dec 39.00 (QQQ LL)	38	21/1/017	1	1.2	1.25	1.653	0.428
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/1/018	1	0.7	0.75	0.93	0.205
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/1/019	1	0.35	0.4	0.453	0.049
4550 QQQQ Zero Call	0	21/1/021	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Jan 38.00 (QQQ AJ)	36	21/1/022	1	3.7	3.8	3.976	0.226
4550 QQQQ 05 Jan 36.625 (YIZ A)	36.6	21/1/022	1	3.2	3.3	3.584	0.334
4550 QQQQ 05 Jan 37.00 (QQQ AK)	37	21/1/023	1	2.9	2.95	3.349	0.424
4550 QQQQ 05 Jan 37.625 (YIZ A)	37.6	21/1/024	1	2.45	2.5	2.958	0.483
4550 QQQQ 05 Jan 38.00 (QQQ AM)	38	21/1/025	1	1.15	1.25	2.725	0.523
4550 QQQQ 05 Jan 38.625 (YIZ A)	38.6	21/1/027	1	1.55	1.6	2.097	0.522
4550 QQQQ 05 Jan 39.00 (QQQ IM)	39	21/1/027	1	1.2	1.25	1.706	0.481
4550 QQQQ 05 Jan 39.625 (YIZ A)	39.6	21/1/028	1	1	1.05	1.471	0.446
4550 QQQQ 05 Jan 40.00 (QQQ AN)	40	21/1/029	1	0.75	0.8	1.079	0.304
4550 QQQQ 05 Jan 40.625 (YIZ A)	40.6	21/1/030	1	0.6	0.65	0.844	0.219
4550 QQQQ 05 Jan 41.00 (QQQ AO)	41	21/1/031	1	0.4	0.5	0.453	0.003
4550 QQQQ 05 Jan 41.625 (YIZ A)	41.6	21/1/031	1	0.35	0.4	0.453	0.003
4550 QQQQ 05 Jan 42.00 (QQQ AP)	42	21/1/032	1	0.35	0.4	0.453	0.003
4550 QQQQ Zero Call	0	21/1/033	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Mar 38.00 (QQQ CJ)	36	21/1/04	1	4.1	4.2	4.415	0.315
4550 QQQQ 05 Mar 37.00 (QQQ CK)	35	21/1/05	1	3.3	3.5	3.588	0.338

4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ IM)	39	21/1/007	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Nov 38.00 (QQQ KM)	38	21/1/012	1	0.15	0.2	0	-0.175
4550 QQQQ 05 Nov 39.00 (QQQ KN)	40	21/1/013	1	0.45	0.5	0.387	-0.108
4550 QQQQ Zero Call	0	21/1/014	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Dec 38.00 (QQQ AJ)	37	21/1/015	1	1.15	1.25	1.706	0.481
4550 QQQQ 05 Dec 39.00 (QQQ IM)	39	21/1/015	1	1.55	1.65	2.097	0.522
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/1/016	1	1	1.05	1.471	0.446
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/1/017	1	0.75	0.8	1.079	0.304
4550 QQQQ Zero Call	0	21/1/020	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Jan 38.00 (QQQ AJ)	36	21/1/021	1	3.7	3.8	3.976	0.226
4550 QQQQ 05 Jan 39.625 (YIZ A)	39.6	21/1/022	1	3.2	3.3	3.584	0.334
4550 QQQQ 05 Jan 40.625 (YIZ A)	40.6	21/1/023	1	2.3	2.45	2.958	0.483
4550 QQQQ 05 Jan 41.625 (YIZ A)	41.6	21/1/024	1	2.45	2.5	2.958	0.483
4550 QQQQ 05 Feb 38.00 (QQQ AL)	38	21/1/025	1	2.15	2.25	2.723	0.523
4550 QQQQ 05 Feb 39.625 (YIZ A)	38.6	21/1/026	1	2.3	2.35	2.332	0.357
4550 QQQQ 05 Mar 38.00 (QQQ AM)	39	21/1/027	1	1.15	1.25	2.097	0.522
4550 QQQQ 05 Mar 39.625 (YIZ A)	39.6	21/1/028	1	1.2	1.25	1.706	0.481
4550 QQQQ 05 Mar 40.625 (YIZ A)	40	21/1/029	1	1.05	1.05	1.471	0.446
4550 QQQQ 05 Mar 41.625 (YIZ A)	41	21/1/030	1	0.6	0.65	0.844	0.219
4550 QQQQ Zero Call	0	21/1/031	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Apr 38.00 (QQQ AL)	37	21/1/032	1	0.35	0.4	0.219	-0.56
4550 QQQQ 05 Apr 39.625 (YIZ A)	38.6	21/1/033	1	4.1	4.3	4.415	0.315
4550 QQQQ 05 Apr 40.625 (YIZ A)	40.6	21/1/034	1	3.3	3.5	3.588	0.338

## Equivalent Model with Independent Brownian Motions (SABR)

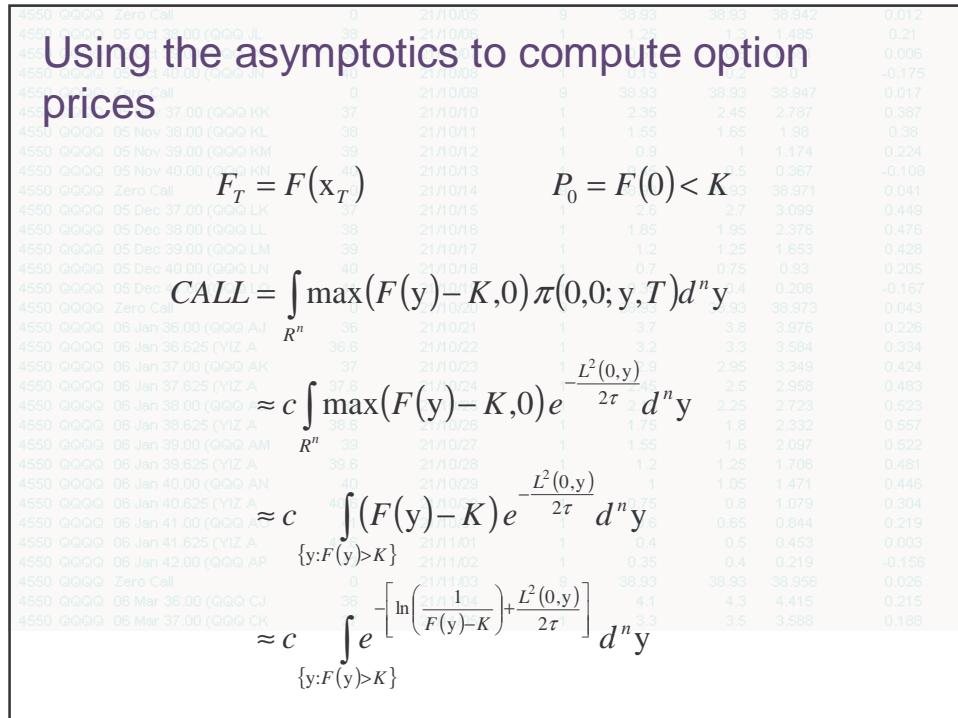
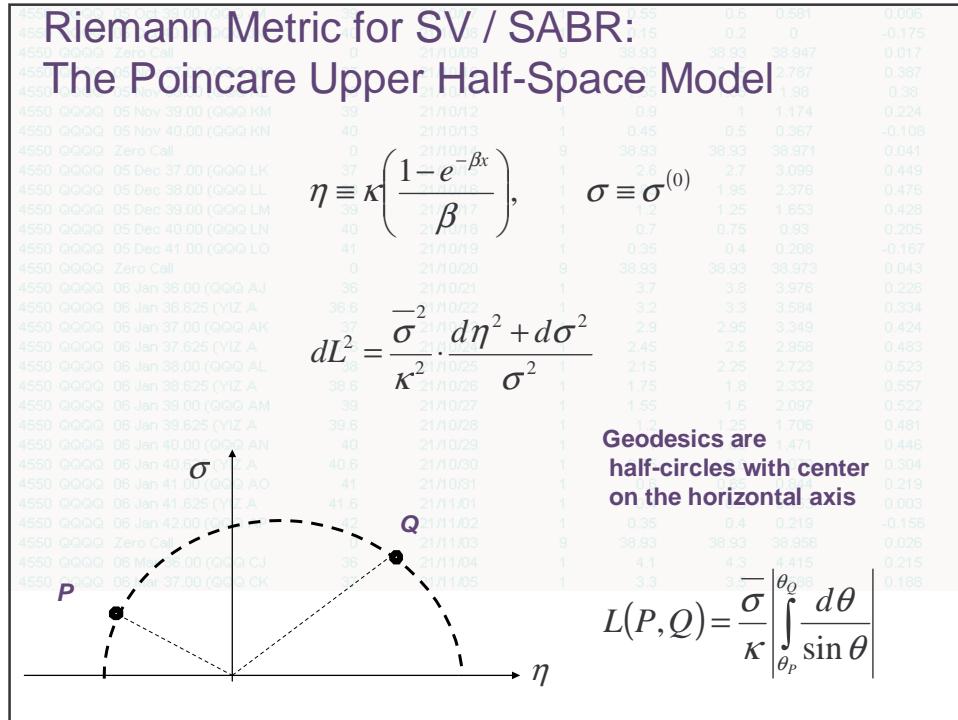
$$\sigma_t = \sigma_t^{(0)} \exp(\beta x_t) \quad x_t = \ln\left(\frac{F_t}{F_0}\right)$$

“Parametric leverage”  
SV for tails

$$\frac{d\sigma_t}{\sigma_t} = \frac{d\sigma_t^{(0)}}{\sigma_t^{(0)}} + \beta dx_t$$

“CEV” with stochastic  
independent volatility  
is equivalent to SV model  
with correlated volatility,  
from the Riemann viewpoint

$$\left\{ \begin{array}{l} dx_t = e^{\beta x_t} \sigma_t^{(0)} dW_t \\ \frac{d\sigma_t^{(0)}}{\sigma_t^{(0)}} = \kappa dZ_t \\ E(dW_t dZ_t) = 0 \end{array} \right.$$



## Steepest-descent approximation for computing implied volatilities

4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Nov 39.00 (QQQ KL)	39	21/10/11	1	1.25	1.3	1.485	0.006
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/10/12	1	0.9	1	1.174	0.38
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/10/13	1	0.45	0.5	0.367	-0.108
4550 QQQQ Zero Call	0	21/10/14	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/10/15	1	2.6	2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ LL)	39	21/10/16	$\min_{y:F(y)>K} \left[ \ln\left(\frac{1}{F(y)-K}\right) + \frac{L^2(0,y)}{2\tau} \right]$	0.45	0.5	0.449	0.476
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/10/17	$d^n y \approx e^{-\min_{y:F(y)>K} \left[ \ln\left(\frac{1}{F(y)-K}\right) + \frac{L^2(0,y)}{2\tau} \right]}$	0.75	0.75	0.75	0.428
4550 QQQQ 05 Dec 40.00 (QQQ LO)	41	21/10/19	1	0.35	0.4	0.208	-0.167
4550 QQQQ $\{y:F(y)>K\}$	0	21/10/20	9	38.93	38.93	38.973	0.043
4550 QQQQ 05 Jan 38.00 (QQQ AJ)	36	21/10/21	1	3.7	3.8	3.976	0.226
4550 QQQQ 05 Jan 38.625 (YIZ A)	36.6	21/10/22	1	3.2	3.3	3.584	0.334
4550 QQQQ 05 Jan 37.00 (QQQ AK)	37	21/10/23	1	2.9	2.95	3.349	0.424
4550 QQQQ 05 Jan 37.625 (YIZ A)	37.6	21/10/24	1	2.45	2.5	2.958	0.483
4550 QQQQ 05 Jan 38.00 (QQQ AL)	38	21/10/25	1	2.15	2.25	2.723	0.523
4550 QQQQ 05 Jan 38.625 (YIZ A)	38.6	21/10/26	1	1.75	1.8	2.332	0.567
$\min_{y:F(y)>K} \left[ \ln\left(\frac{1}{F(y)-K}\right) + \frac{L^2(0,y)}{2\tau} \right] = \frac{1}{\tau} \min_{y:F(y)>K} \left[ \tau \ln\left(\frac{1}{F(y)-K}\right) + \frac{L^2(0,y)}{2} \right]$							
4550 QQQQ 05 Jan 38.625 (YIZ A)	39	21/10/27	1	1.55	1.5	1.95	0.22
4550 QQQQ 05 Jan 39.00 (QQQ AN)	39	21/10/28	1	1.25	1.25	1.71	0.511
4550 QQQQ 05 Jan 39.625 (YIZ A)	40.6	21/10/30	0.75	0.75	0.75	0.04	
4550 QQQQ 05 Jan 41.00 (QQQ AO)	41	21/10/31	1	0.6	0.65	0.844	0.219
4550 QQQQ 05 Jan 41.625 (YIZ A)	41.6	21/11/01	1	0.4	0.5	0.453	0.003
4550 QQQQ 05 Jan 42.00 (QQQ AP)	42	21/11/02	1	0.35	0.4	0.219	-0.156
4550 QQQQ Zero Call	0	21/11/03	9	38.93	38.93	38.956	0.026
4550 QQQQ 06 Mar 36.00 (QQQ CJ)	36	21/11/04	1	4.1	4.3	4.415	0.215
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/11/05	1	3.3	3.5	3.588	0.108
$\approx \frac{1}{2\tau} \min \{L^2(0,y)   y:F(y)>K\}, \quad \tau \ll 1$							

## Equate formulas for OTM calls with Black-Scholes ...

4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Nov 39.00 (QQQ KL)	39	21/10/07	1	0.75	0.8	0.581	0.006
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/10/11	1	1.25	1.3	1.485	-0.175
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/10/13	1	0.45	0.5	0.367	-0.108
4550 QQQQ Zero Call	0	21/10/14	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/10/15	1	2.6	2.7	3.099	0.449
$L^*(K) = \min \{L^*(y)   y:F(y)>K\}$							
Minimum distance from 0 to the region $\{F(y)>0\}$							
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/10/17	1	1.25	1.25	1.657	0.128
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/10/18	1	0.75	0.75	0.75	0.04
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/10/19	1	0.35	0.4	0.208	-0.157
4550 QQQQ Zero Call	0	21/10/20	9	38.93	38.93	38.973	0.043
4550 QQQQ 05 Jan 36.00 (QQQ AJ)	36	21/10/21	1	3.7	3.8	3.976	0.226
4550 QQQQ 05 Jan 36.625 (YIZ A)	36.6	21/10/22	1	3.2	3.3	3.584	0.334
4550 QQQQ 05 Jan 37.00 (QQQ AL)	37	21/10/23	1	2.95	3.05	3.349	0.424
4550 QQQQ 05 Jan 37.625 (YIZ A)	37.6	21/10/24	1	2.5	2.5	2.958	0.483
4550 QQQQ 05 Jan 38.00 (QQQ AN)	38	21/10/25	1	2.15	2.25	2.723	0.523
4550 QQQQ 05 Jan 38.625 (YIZ A)	38.6	21/10/26	1	1.75	1.8	2.332	0.557
4550 QQQQ 05 Jan 39.00 (QQQ AM)	39	21/10/27	1	1.55	1.6	2.097	0.522
4550 QQQQ 05 Jan 39.625 (YIZ A)	39.6	21/10/28	1	1.2	1.25	1.706	0.481
4550 QQQQ 05 Jan 40.00 (QQQ AN)	40	21/10/29	1	1	1.05	1.471	0.446
4550 QQQQ 05 Jan 40.625 (YIZ A)	40.6	21/10/30	1	0.75	0.8	1.079	0.304
$\ln CALL \approx \frac{\ln(K/F_0)^2}{2\sigma_{imp}^2(K)\tau} = \frac{\ln(K/F_0)^2}{2\sigma_{imp}^2(K)\tau}$							
Small-tau asymptotics (model)							
4550 QQQQ Zero Call	0	21/10/31	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Mar 36.00 (QQQ CJ)	36	21/11/04	1	4.1	4.3	4.415	0.215
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/11/05	1	3.3	3.5	3.588	0.108
Small-tau asymptotics (Black-Scholes)							

4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/10/07	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Oct 40.00 (QQQ JK)	40	21/10/08	1	0.12	0.12	0	-0.175
4550 QQQQ Zero Call	0	21/10/09	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 38.00 (QQQ JL)	38	21/10/10	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/10/12	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/10/13	1	0.45	0.5	0.387	-0.108
4550 QQQQ Zero Call	0	21/10/14	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/10/15	1	2.6	2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ LN)	39	21/10/16	1	1.85	1.95	2.376	0.476
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/10/17	1	0.75	0.93	0.205	
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/10/18	1	0.35	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/10/20	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Jan 38.00 (QQQ AJ)	36	21/10/21	1	3.7	3.8	3.976	0.226
4550 QQQQ 06 Jan 38.625 (YIZ A)	36.6	21/10/22	1	3.2	3.3	3.584	0.334
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/10/23	1	2.9	2.95	3.349	0.424
4550 QQQQ 06 Jan 37.625 (YIZ A)	37.6	21/10/24	1	2.45	2.5	2.958	0.483
4550 QQQQ 06 Jan 38.00 (QQQ AL)	38	21/10/25	1	0.35	0.4	0.225	0.523
4550 QQQQ 06 Jan 38.625 (YIZ A)	38.6	21/10/27	1	1.55	1.6	2.332	0.557
4550 QQQQ 06 Jan 39.00 (QQQ AM)	39	21/10/28	1	1.2	1.25	1.706	0.481
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/10/29	1	1	1.05	1.471	0.446
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/10/30	1	0.75	0.8	1.079	0.304
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/10/31	1	0.6	0.65	0.844	0.219
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/10/01	1	0.4	0.5	0.453	0.003
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/10/02	1	0.35	0.4	0.219	-0.156
4550 QQQQ Zero Call	0	21/10/03	9	38.93	38.93	38.956	0.026
4550 QQQQ 06 Mar 36.00 (QQQ CK)	36	21/10/04	1	4.1	4.3	4.415	0.215
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/10/05	1	3.3	3.5	3.588	0.188

4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/10/07	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Oct 40.00 (QQQ JK)	40	21/10/08	1	0.15	0.12	0	-0.175
4550 QQQQ Zero Call	0	21/10/09	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 37.00 (QQQ KL)	37	21/10/10	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/10/11	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/10/12	1	0.9	1	1.174	0.224
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/10/13	1	0.45	0.5	0.387	-0.108
4550 QQQQ Zero Call	0	21/10/14	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/10/15	1	2.6	2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ LN)	39	21/10/16	1	1.85	1.95	2.376	0.476
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/10/17	1	0.75	0.75	0.93	0.205
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/10/18	1	0.35	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/10/19	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Jan 38.00 (QQQ AJ)	36	21/10/20	1	3.7	3.8	3.976	0.226
4550 QQQQ 06 Jan 38.625 (YIZ A)	36.6	21/10/22	1	3.2	3.3	3.584	0.334
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/10/23	1	2.9	2.95	3.349	0.424
4550 QQQQ 06 Jan 37.625 (YIZ A)	37.6	21/10/24	1	2.45	2.5	2.958	0.483
4550 QQQQ 06 Jan 38.00 (QQQ AL)	38	21/10/25	1	2.15	2.25	2.723	0.523
4550 QQQQ 06 Jan 38.625 (YIZ A)	38.6	21/10/26	1	1.75	1.8	2.332	0.557
4550 QQQQ 06 Jan 39.00 (QQQ AM)	39	21/10/27	1	1.55	1.6	2.097	0.522
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/10/28	1	1.2	1.25	1.706	0.481
4550 QQQQ 06 Jan 40.00 (QQQ AN)	40	21/10/29	1	1	1.05	1.471	0.446
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/10/30	1	0.75	0.8	1.079	0.304
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/10/31	1	0.6	0.65	0.844	0.219
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/11/01	1	0.4	0.5	0.453	0.003
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/11/02	1	0.35	0.4	0.219	-0.156
4550 QQQQ Zero Call	0	21/11/03	9	38.93	38.93	38.956	0.026
4550 QQQQ 06 Mar 36.00 (QQQ CJ)	36	21/11/04	1	4.1	4.3	4.415	0.215
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/11/05	1	3.3	3.5	3.588	0.188

Berestycki, Busca and Florent, 2001

## Example 2: Constant Elasticity of Variance

4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/10/07	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/10/10	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/10/11	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ LM)	39	21/10/12	1	0.9	1	1.174	0.224
4550 QQQQ 05 Nov 37.00 (YIZ A)	37	21/10/13	1	0.5	0.5	0.367	-0.108
4550 QQQQ Zero Call	0	21/10/14	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/10/15	1	2.6	2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ LL)	38	21/10/16	1	1.85	1.95	2.376	0.476
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/10/17	1	1.2	1.25	1.653	0.428
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/10/18	1	0.7	0.75	0.93	0.205
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/10/19	1	0.35	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/10/20	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Jan 36.625 (YIZ A)	36	21/10/21	1	3.2	3.3	3.584	0.334
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/10/22	1	2.9	2.95	3.349	0.424
4550 QQQQ 06 Jan 37.625 (YIZ A)	37.6	21/10/24	1	2.45	2.5	2.958	0.483
4550 QQQQ 06 Jan 38.00	0				25	2.723	0.523
4550 QQQQ 06 Jan 38.625					1.8	2.332	0.557
4550 QQQQ 06 Jan 39.00	1				1.6	2.097	0.522
4550 QQQQ 06 Jan 39.625					25	1.706	0.481
4550 QQQQ 06 Jan 40.00	2				1.05	1.471	0.446
4550 QQQQ 06 Jan 40.625					0.8	1.079	0.304
4550 QQQQ 06 Jan 41.00	3				.65	0.844	0.219
4550 QQQQ 06 Jan 41.625					0.5	0.453	0.003
4550 QQQQ 06 Jan 42.00	4				0.4	0.219	-0.156
4550 QQQQ Zero Call	0				.93	38.956	0.026
4550 QQQQ 06 Mar 36.00					4.3	4.415	0.215
4550 QQQQ 06 Mar 37.00	37				3.5	3.588	0.108

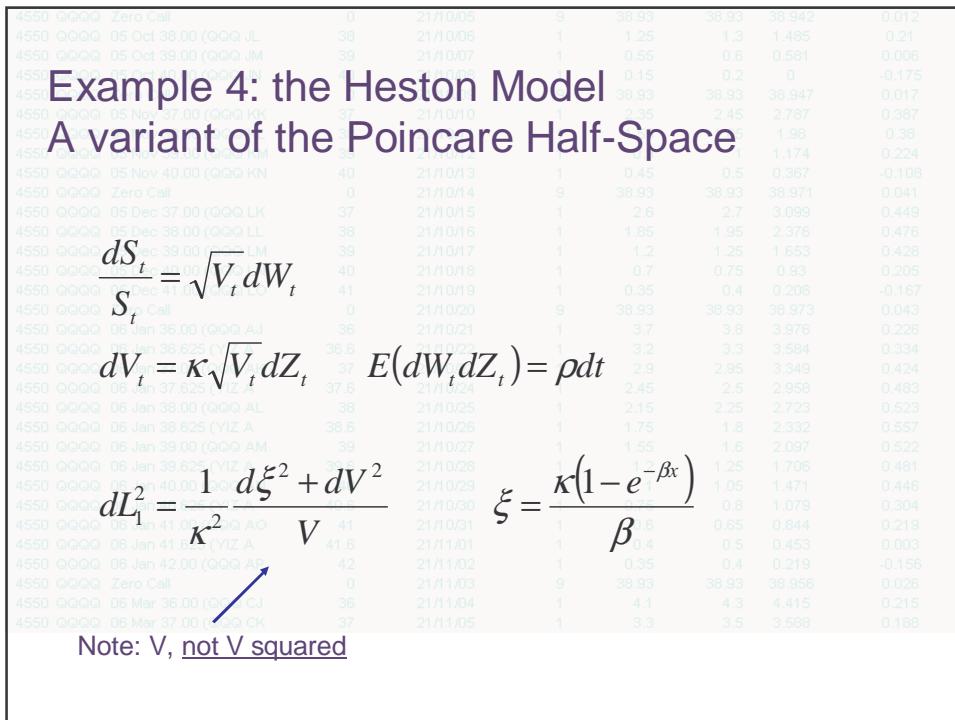
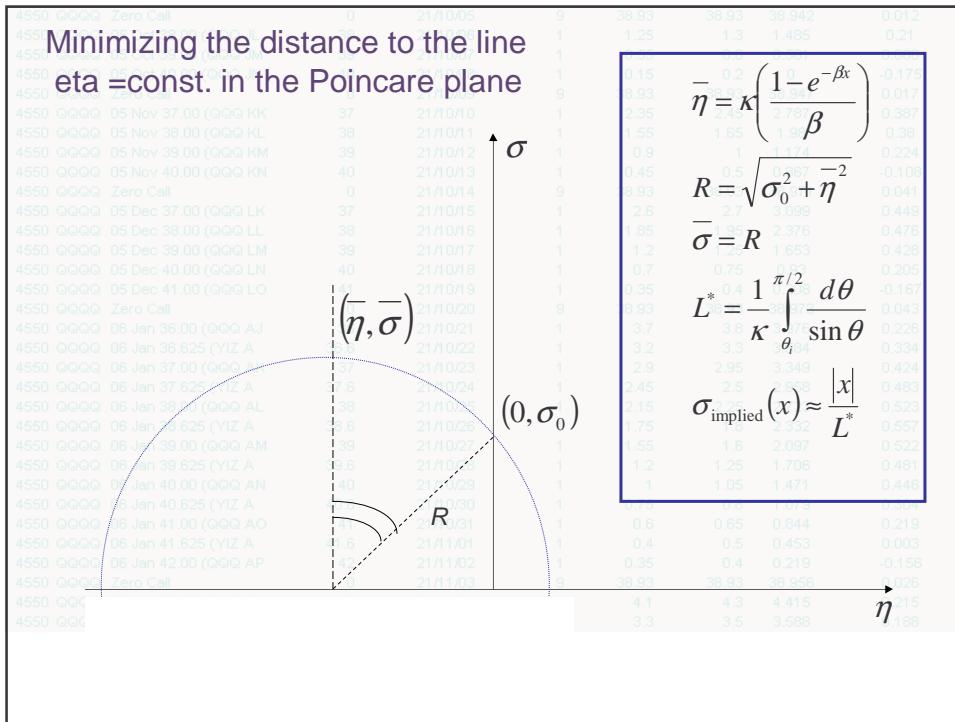
$\sigma_{\text{imp}}(x) \approx \sigma_0 \left| \frac{\beta x}{1 - e^{-\beta x}} \right|$  Implied volatility

volatility(%)

$x = \ln(K/F)$

## Example 3: Stochastic Volatility / SABR

4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/10/07	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/10/10	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/10/11	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/10/12	1	0.9	1	1.174	0.224
4550 QQQQ 06 Jan 40.00 (QQQ KN)	40	21/10/13	1	0.45	0.5	0.367	-0.175
4550 QQQQ Zero Call	0	21/10/14	9	38.93	38.93	38.971	0.041
4550 QQQQ 06 Dec 37.00 (QQQ LK)	37	21/10/15	1	2.6	2.7	3.099	0.449
4550 QQQQ 06 Dec 39.00 (QQQ LL)	38	21/10/16	1	1.85	1.95	2.376	0.476
4550 QQQQ 06 Dec 39.00 (QQQ LN)	39	21/10/17	1	1.2	1.25	1.653	0.428
4550 QQQQ 06 Dec 40.00 (QQQ LO)	40	21/10/18	1	0.75	0.75	0.93	0.205
4550 QQQQ 06 Dec 41.00 (QQQ LO)	41	21/10/19	1	0.4	0.4	0.208	-0.157
4550 QQQQ Zero Call	0				ln \left( \frac{\kappa  x }{\sigma_0 \beta} + \sqrt{1 + \kappa^2 \left( \frac{1 - e^{-\beta x}}{\sigma_0 \beta} \right)^2} \right)	38.93	38.973
4550 QQQQ 06 Jan 36.00 (YIZ A)	36	21/10/23	1	2.9	2.95	3.349	0.424
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/10/23	1	2.15	2.25	2.723	0.523
4550 QQQQ 06 Jan 38.00 (QQQ AL)	38	21/10/25	1	1.75	1.8	2.332	0.557
4550 QQQQ 06 Jan 38.625 (YIZ A)	38.6	21/10/26	1	1.55	1.6	2.097	0.522
4550 QQQQ 06 Jan 39.00 (QQQ AM)	39	21/10/27	1	1.25	1.25	1.706	0.481
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/10/28	1	1	1.05	1.471	0.446
4550 QQQQ 06 Jan 40.00 (QQQ AO)	40	21/10/29	1	0.75	0.8	1.079	0.304
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/10/30	1	0.6	0.65	0.844	0.219
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/10/31	1	0.4	0.5	0.453	0.003
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/11/01	1	0.35	0.4	0.219	-0.156
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/11/02	1	0.3	0.35	0.219	0.026
4550 QQQQ Zero Call	0	21/11/03	9	38.93	38.93	38.956	0.026
4550 QQQQ 06 Mar 36.00 (QQQ CK)	36	21/11/04	1	4.1	4.3	4.415	0.215
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/11/05	1	3.3	3.5	3.588	0.108



Closed-form solution for geodesics							
4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 38.00 (QQQ KM)	39	21/1/007	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Oct 38.00 (QQQ CK)	40	21/1/008	1	0.2	0	-0.175	
4550 QQQQ Zero Call	0	21/1/009	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/1/010	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/011	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/1/012	1	0.7	1	1.174	0.224
4550 QQQQ 05 Nov 39.00 (QQQ CK)	40	21/1/013	1	0.5	0.5	0.387	-0.108
4550 QQQQ 05 Dec 37.00 (QQQ KK)	37	21/1/014	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 39.00 (QQQ KL)	38	21/1/015	1	2.8	2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ KM)	39	21/1/016	1	1.85	1.95	2.376	0.476
4550 QQQQ 05 Dec 39.00 (QQQ CK)	40	21/1/017	1	1.2	1.25	1.653	0.428
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/1/018	1	0.7	0.75	0.93	0.205
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/1/019	1	0.35	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/1/020	9	38.93	38.93	38.973	0.043
4550 QQQQ 05 Jan 38.00 (QQQ AJ)	36	21/1/021	1	3.7	3.8	3.976	0.226
4550 QQQQ 05 Jan 38.625 (YIZ A)	36.6	21/1/022	1	3.2	3.3	3.584	0.334
4550 QQQQ 05 Jan 39.00 (QQQ AL)	37	21/1/023	1	2.9	2.85	3.349	0.424
4550 QQQQ 05 Jan 39.00 (YIZ AL)	38	21/1/024	1	2.15	2.15	2.55	0.483
4550 QQQQ 05 Jan 38.625 (YIZ A)	38.6	21/1/025	1	1.75	1.75	2.03	0.557
4550 QQQQ 05 Jan 39.00 (QQQ AL)	39	21/1/026	1	1.55	1.55	1.8	0.522
4550 QQQQ 05 Jan 40.00 (QQQ AN)	40	21/1/027	1	1	1.05	1.471	0.446
4550 QQQQ 05 Jan 40.625 (YIZ A)	40.6	21/1/028	1	0.75	0.75	1.079	0.304
4550 QQQQ 05 Jan 41.00 (QQQ AO)	41	21/1/029	1	0.4	0.4	0.844	0.219
4550 QQQQ 05 Jan 41.625 (YIZ A)	41.6	21/1/030	1	0.25	0.25	0.453	0.003
4550 QQQQ 05 Jan 41.625 (YIZ AP)	42	21/1/031	1	0.2	0.2	0.219	-0.156
4550 QQQQ Zero Call	0	21/1/032	9	38.93	38.93	38.956	0.026
4550 QQQQ 05 Mar 38.00 (QQQ AP)	37	21/1/033	1	4	4.3	4.415	0.215
4550 QQQQ 05 Mar 37.00 (QQQ CK)	37	21/1/034	1	3.5	3.5	3.588	0.188

$\xi(\theta) = \frac{R^2}{2} (\theta - \sin \theta \cos \theta) + \xi(0)$

$V(\theta) = R^2 \sin^2 \theta \quad 0 \leq \theta \leq \pi$

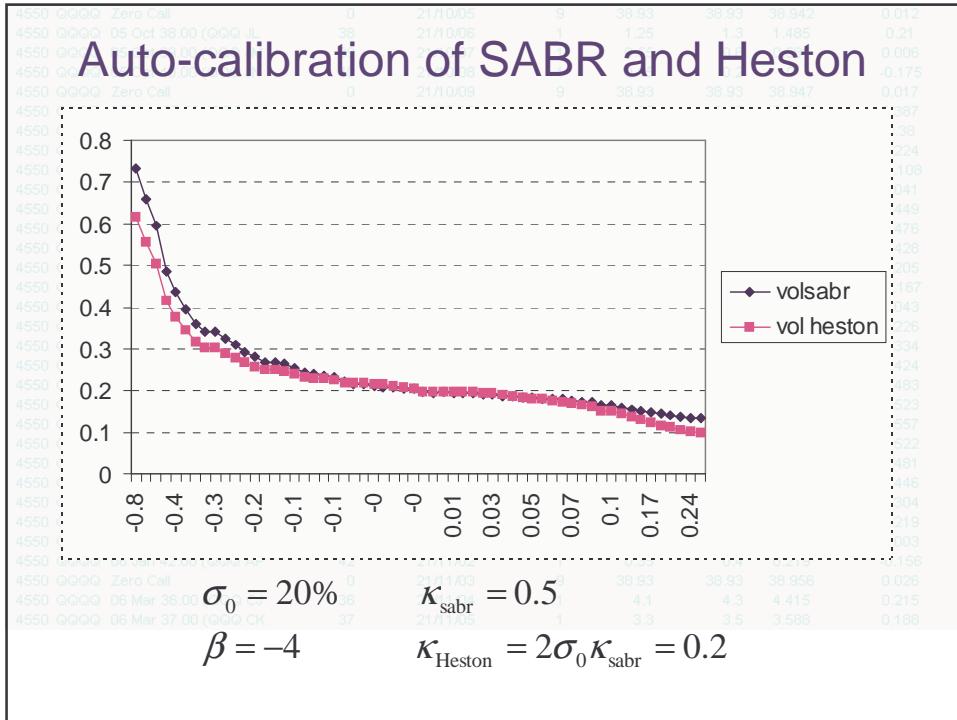
$dL = \frac{2R^2}{K} \sin \theta d\theta$

Implied volatility curve for Heston model is obtained as an algebraic system							
4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 38.00 (QQQ KM)	39	21/1/007	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Oct 38.00 (QQQ CK)	40	21/1/008	1	0.2	0	-0.175	
4550 QQQQ Zero Call	0	21/1/009	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/1/010	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KN)	38	21/1/011	1	1.55	1.65	1.98	0.38
4550 QQQQ Zero Call	0	21/1/012	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/1/013	1	2.6	2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ LL)	38	21/1/014	1	1.85	1.95	2.376	0.476
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/1/015	1	1.2	1.25	1.653	0.428
4550 QQQQ 05 Dec 39.00 (QQQ LN)	40	21/1/016	1	0.75	0.75	0.844	0.205
4550 QQQQ 05 Dec 39.00 (QQQ AP)	41	21/1/017	1	0.4	0.4	0.453	0.003
4550 QQQQ 05 Jan 38.625 (YIZ A)	36.6	21/1/018	1	0.25	0.25	0.219	-0.156
4550 QQQQ 05 Jan 39.00 (QQQ AL)	37	21/1/019	1	0.2	0.2	0.215	0.215
4550 QQQQ 05 Jan 39.00 (YIZ AL)	38	21/1/020	1	0.15	0.15	0.188	0.188
4550 QQQQ 05 Jan 39.00 (QQQ CK)	39	21/1/021	1	0.1	0.1	0.188	0.188
4550 QQQQ 05 Jan 38.625 (YIZ A)	38.6	21/1/022	1	0.1	0.1	0.188	0.188
4550 QQQQ 05 Jan 39.00 (QQQ CK)	40	21/1/023	1	0.1	0.1	0.188	0.188
4550 QQQQ 05 Jan 39.00 (YIZ CK)	41	21/1/024	1	0.1	0.1	0.188	0.188
4550 QQQQ 05 Jan 39.00 (QQQ AP)	42	21/1/025	1	0.1	0.1	0.188	0.188
4550 QQQQ Zero Call	0	21/1/026	9	38.93	38.93	38.973	0.043
4550 QQQQ 05 Mar 38.00 (QQQ CK)	36	21/1/027	1	0.1	0.1	0.188	0.188
4550 QQQQ 05 Mar 37.00 (QQQ CK)	37	21/1/028	1	0.1	0.1	0.188	0.188

$\xi = \frac{\sigma_0^2}{\sin^2 \theta_{\text{init}}} \left( \frac{\pi}{2} - \theta_{\text{init}} + \sin \theta_{\text{init}} \cos \theta_{\text{init}} \right)$

$\sigma(\xi) = \frac{\kappa |\xi| \sin^2 \theta_{\text{init}}}{2 \sigma_0^2 |\cos \theta_{\text{init}}|}$

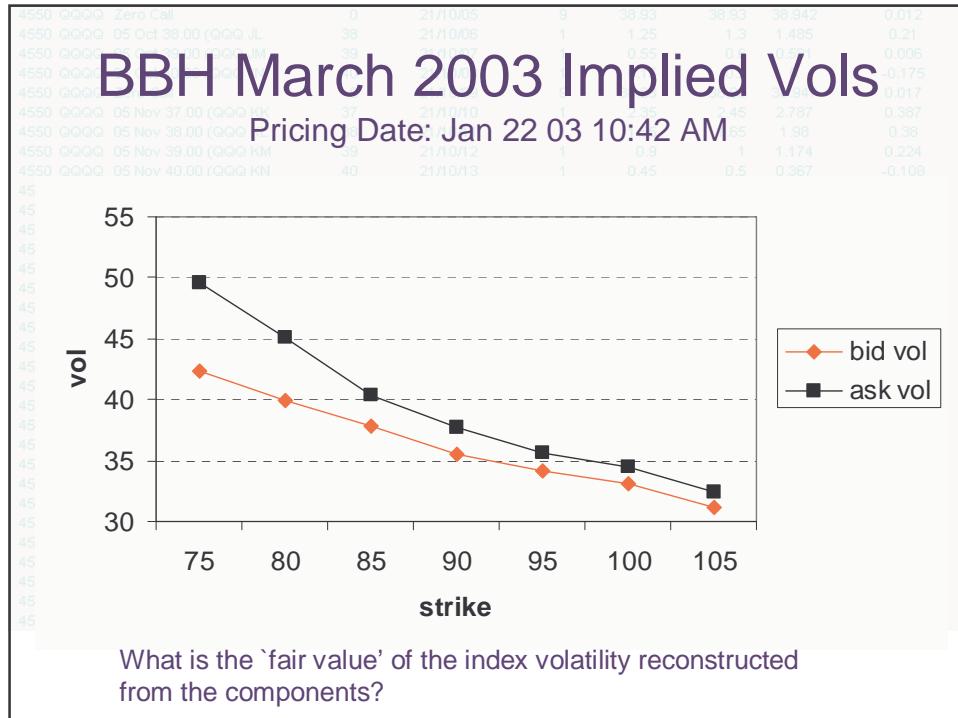
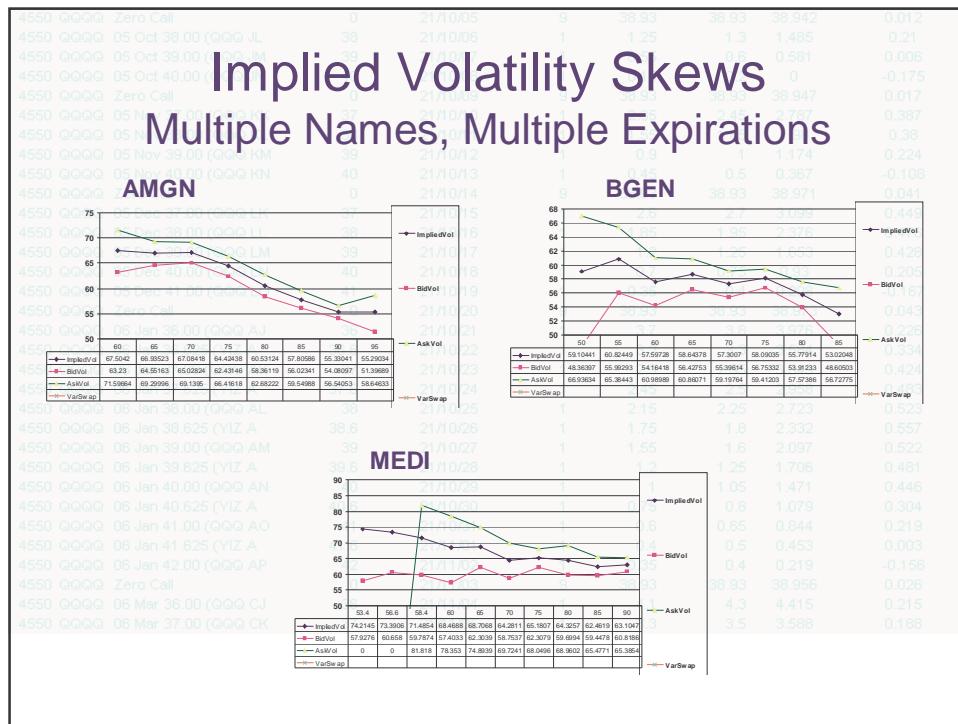
Given  $\xi$ , solve for  $\theta_{\text{init}}$ , and substitute in the second equation



4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/1/007	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Oct 40.00 (QQQ JN)	40	21/1/008	1	0.15	0.2	0	-0.175
4550 QQQQ Zero Call	0	21/1/009	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/1/010	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/011	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/1/012	1	0.9	1	1.174	0.224
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/1/013	1	0.45	0.5	0.387	-0.108
4550 QQQQ Zero Call	0	21/1/014	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/1/015	1	2.6	2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ LL)	38	21/1/016	1	1.85	1.95	2.376	0.476
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/1/017	1	1.2	1.25	1.653	0.428
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/1/018	1	0.7	0.75	0.93	0.205
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/1/019	1	0.35	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/1/020	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Jan 36.00 (QQQ LJ)	36	21/1/021	1	3.7	3.8	3.976	0.226
4550 QQQQ 06 Jan 36.625 (YIZ A)	36.6	21/1/022	1	3.7	3.8	3.976	0.226
4550 QQQQ 06 Jan 37.00 (QQQ LJ)	37	21/1/023	1	3.7	3.8	3.976	0.226
4550 QQQQ 06 Jan 37.625 (YIZ A)	37.6	21/1/024	1	2.45	2.5	2.958	0.483
4550 QQQQ 06 Jan 38.00 (QQQ AL)	38	21/1/025	1	2.15	2.25	2.723	0.523
4550 QQQQ 06 Jan 38.625 (YIZ A)	38.6	21/1/026	1	1.75	1.8	2.332	0.557
4550 QQQQ 06 Jan 39.00 (QQQ AM)	39	21/1/027	1	1.55	1.6	2.097	0.522
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/1/028	1	1.2	1.25	1.706	0.481
4550 QQQQ 06 Jan 40.00 (QQQ AN)	40	21/1/029	1	1	1.05	1.471	0.448
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/1/030	1	0.75	0.8	1.079	0.304
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/1/031	1	0.6	0.65	0.844	0.219
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/1/031	1	0.4	0.5	0.453	0.003
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/1/032	1	0.35	0.4	0.219	-0.156
4550 QQQQ Zero Call	0	21/1/033	9	38.93	38.93	38.956	0.026
4550 QQQQ 06 Mar 36.00 (QQQ CJ)	36	21/1/034	1	4.1	4.3	4.415	0.215
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/1/035	1	3.3	3.5	3.588	0.188

4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012				
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21				
4550 QQQQ 05 Nov 38.00 (QQQ JL)	39	21/10/07	1	0.75	0.8	0.941	0.006				
4550 QQQQ 05 Nov 38.00 (QQQ JL)	40	21/10/08	1	0.75	0.8	0.941	0				-0.175
4550 QQQQ Zero Call	0	21/10/09	9	38.93	38.93	38.947	0.017				
4550 QQQQ 05 Dec 38.00 (QQQ KK)	37	21/10/10	1	2.35	2.45	2.787	0.387				
4550 QQQQ 05 Nov 38.00 (QQQ KL)	38	21/10/11	1	1.55	1.65	1.98	0.38				
4550 QQQQ 05 Nov 38.00 (QQQ KM)	39	21/10/12	1	0.9	1	1.174	0.224				
Derive index volatility skew from <b>single-stock skews</b> and <b>correlation matrix</b>											
4550 QQQQ Zero Call	0	21/10/14	9	38.93	38.93	38.971	0.041				
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/10/15	1	2.6	2.7	3.099	0.449				
4550 QQQQ 05 Dec 39.00 (QQQ LL)	38	21/10/16	1	1.85	1.95	2.376	0.476				
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/10/17	1	1.2	1.25	1.653	0.428				
4550 QQQQ 05 Dec 40.00 (QQQ LH)	40	21/10/18	1	0.7	0.75	0.93	0.205				
4550 QQQQ 05 Jan 38.00 (YIZ AL)	38	21/10/19	1	0.35	0.4	0.208	-0.167				
4550 QQQQ Zero Call	0	21/10/20	9	38.93	38.93	38.973	0.043				
4550 QQQQ 05 Jan 38.00 (YIZ AL)	38	21/10/21	1	3.7	3.8	3.976	0.226				
4550 QQQQ 05 Jan 36.625 (YIZ A)	37	21/10/22	1	3.2	3.3	3.584	0.334				
4550 QQQQ 05 Jan 37.00 (QQQ AK)	37	21/10/23	1	2.9				N equations for			0.424
4550 QQQQ 05 Jan 37.625 (YIZ A)	37.6	21/10/24	1	2.45	2.5	2.958	0.483	the index components			
4550 QQQQ 05 Jan 38.00 (QQQ AL)	38	21/10/25	1	2.15							
4550 QQQQ 05 Jan 38.625 (YIZ A)	38.6	21/10/26	1	1.75	1.8	2.332	0.557				
4550 QQQQ 05 Jan 39.00 (QQQ AM)	39	21/10/27	1	1.55	1.6	2.097	0.522				
4550 QQQQ 05 Jan 39.625 (YIZ A)	39.6	21/10/28	1	1.2	1.25	1.706	0.481				
4550 QQQQ 05 Jan 40.00 (QQQ AN)	40	21/10/29	1	1.05	1.05	1.471	0.446				
4550 QQQQ 05 Jan 40.625 (YIZ A)	40.6	21/10/30	1	0.8	0.8	1.079	0.304				
4550 QQQQ 05 Jan 41.00 (QQQ AP)	41	21/10/01	1	0.65	0.65	0.844	0.219				
4550 QQQQ Zero Call	0	21/10/03	9	38.93	38.93	38.956	0.026				
4550 QQQQ 05 Mar 38.00 (QQQ CJ)	38	21/10/04	1	4.1	4.3	4.415	0.215				
4550 QQQQ 05 Mar 37.00 (QQQ CK)	37	21/10/05	1	3.3	3.5	3.588	0.188				

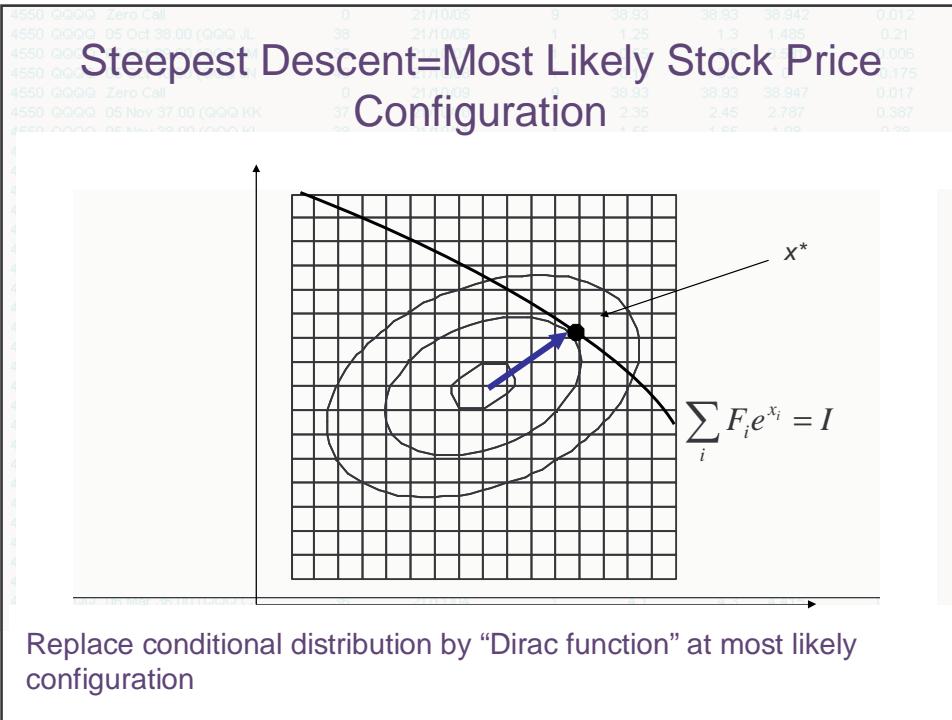
4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012				
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21				
4550 QQQQ 05 Nov 38.00 (QQQ JL)	39	21/10/07	1	0.75	0.8	0.941	0.006				
4550 QQQQ 05 Nov 38.00 (QQQ JL)	40	21/10/08	1	0.75	0.8	0.941	0.017				
4550 QQQQ Zero Call	0	21/10/09	9	38.93	38.93	38.947	0.017				
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/10/10	1	2.45	2.45	2.787	0.387				
4550 QQQQ 05 Nov 39.00 (QQQ KL)	39	21/10/12	1	0.9	1	1.174	0.224				
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/10/13	1	0.45	0.5	0.357	-0.108				
4550 QQQQ Zero Call	0	21/10/14	9	38.93	38.93	38.971	0.041				
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/10/15	1	2.6	2.7	3.099	0.449				
4550 QQQQ 05 Dec 38.00 (QQQ LL)	38	21/10/16	1	1.85	1.95	2.376	0.476				
4550 QQQQ 05 Dec 39.00 (YIZ A)	39	21/10/17	1	1.55	1.6	2.097	0.522				
4550 QQQQ 05 Dec 39.625 (YIZ A)	39.6	21/10/18	1	1.2	1.25	1.706	0.481				
4550 QQQQ 05 Dec 40.00 (QQQ KN)	40	21/10/19	1	1.05	1.05	1.471	0.446				
4550 QQQQ 05 Dec 40.625 (YIZ A)	40.6	21/10/20	1	0.8	0.8	1.079	0.304				
4550 QQQQ Zero Call	0	21/10/21	9	38.93	38.93	38.956	0.026				
<b>BBH: ETF of 20 Biotechnology Stocks</b> ( Components of IBH)											
Ticker	Dec 35	Shares	ATM ImVol	12	Ticker	Dec 35	Shares	ATM ImVol	12	ATM ImVol	46
ABI	05 Dec 41.00 (QQQ LO)	18	41	21/10/55	GILD	1	1.35	0.8	0.208	0.44	46
AFFX	6 Jan 38.00 (QQQ AJ)	4	36	21/10/64	HGSI	1	1.37	3.8	3.976	3.684	84
ALKS	6 Jan 37.00 (QQQ AK)	4	37	21/10/106	ICOS	1	1.29	2.4	3.349	3.958	64
AMGN	6 Jan 38.00 (QQQ AL)	46	38	21/10/40	IDPH	1	1.15	12	2.723	2.723	72
BGEN	6 Jan 39.00 (QQQ AM)	13	39	21/10/41	MEDI	1	1.55	15	2.097	0.52	82
CHIR	6 Jan 40.00 (QQQ AN)	16	40	21/10/37	MLNM	1	1.15	12	1.070	1.471	92
CRA	6 Jan 41.00 (QQQ AO)	41	41	21/10/55	QLTI	1	0.6	0.5	0.644	0.453	64
DNA	6 Jan 42.00 (QQQ AP)	42	42	21/10/53.5	SEPR	9	0.35	6.8271	0.219	4.415	84
ENZN	6 Mar 38.00 (QQQ C)	3	36	21/10/81	SHPGY	13	0.1	3.588	0.026	4.415	47
GENZ		14	56	BBH		-	-	32			



4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 39.00 (QQQ JM)	39	21/10/07	1	0.55	0.6	0.581	0.006
4550 QQQQ Zero Call	0	21/10/08	1	0.15	0.2	0	-0.175
4550 QQQQ 05 Nov 38.00 (QQQ KN)	38	21/10/09	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/10/12	1	0.8	1	1.174	0.224
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/10/13	1	0.45	0.5	0.387	-0.108
4550 QQQQ Zero Call	0	21/10/14	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/10/15	1	2.6	2.7	3.099	0.449
4550 QQQQ 05 Dec 38.00 (QQQ LL)	38	21/10/16	1	1.85	1.95	2.376	0.476
4550 QQQQ 05 Dec 39.00 (QQQ LP)	39	21/10/17	1	1.2	1.25	1.653	0.428
4550 QQQQ 05 Dec 40.00 (QQQ LP)	40	21/10/18	1	0.7	0.75	0.93	0.205
4550 QQQQ 05 Dec 41.00 (QQQ LC)	41	21/10/19	1	0.35	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/10/20	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Jan 38.00 (QQQ AJ)	36	21/10/21	1	3.7	3.8	3.976	0.226
4550 QQQQ 06 Jan 36.625 (YIZ A)	36.6	21/10/22	1	3.2	3.3	3.584	0.334
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/10/23	1	2.9	2.95	3.349	0.424
4550 QQQQ 06 Jan 37.625 (YIZ A)	38	21/10/24	1	2.45	2.5	2.958	0.483
4550 QQQQ 06 Jan 38.00 (QQQ AJ)	38	21/10/25	1	2.25	2.25	2.723	0.523
4550 QQQQ 06 Jan 38.625 (YIZ A)	38.6	21/10/26	1	1.8	1.8	2.332	0.557
4550 QQQQ 06 Jan 39.00 (QQQ AP)	39	21/10/27	1	1.5	1.6	2.097	0.522
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/10/28	1	1.2	1.25	1.706	0.481
4550 QQQQ 06 Jan 40.00 (QQQ AN)	40	21/10/29	1	1	1.05	1.471	0.446
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/10/30	1	0.75	0.8	1.079	0.304
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/10/31	1	0.6	0.65	0.844	0.219
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/10/32	1	0.4	0.5	0.453	0.003
4550 QQQQ 06 Jan 42.00 (QQQ AO)	42	21/10/33	1	0.25	0.3	0.386	-0.156
4550 QQQQ 06 Mar 37.00 (GIGQ CK)	37	21/11/05	1	3.3	3.5	3.588	0.188

If correlations are constant, the metric is ``flat'': it is Euclidean  
metric after making the change of variables  $x \rightarrow y$ .

Geodesics are straight lines in the y-coordinates



4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Nov 38.00 (QQQ KM)	39	21/10/07	1	0.55	0.55	0.551	0.006
4550 QQQQ 05 Nov 38.00 (QQQ KL)	38	21/10/08	1	1.25	1.3	1.485	-0.175
4550 QQQQ Zero Call	0	21/10/09	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/10/10	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KM)	38	21/10/11	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/10/12	1	0.8	1	1.174	0.224
4550 QQQQ 05 Nov 40.00 (QQQ KN)	40	21/10/13	1	0.45	0.5	0.367	-0.108
4550 QQQQ Zero Call	0	21/10/14	$\bar{x}$				
4550 QQQQ 05 Dec 37.00 (QQQ LK)	37	21/10/15	$x_i^*$	$x_j^*$	38.93	38.971	0.041
4550 QQQQ 05 Dec 39.00 (QQQ LL)	38	21/10/16	$\sum_{ij=1}^n (\rho^{-1})_{ij} \int_0^{x_i^*} \frac{du}{\sigma_i(u,0)} \int_0^{x_j^*} \frac{du}{\sigma_j(u,0)}$		2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/10/17			1.95	2.376	0.476
4550 QQQQ 05 Dec 40.00 (QQQ LN)	40	21/10/18			1.25	1.653	0.428
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/10/19			0.75	0.93	0.205
4550 QQQQ Zero Call	0	21/10/20	9	38.93	38.93	38.973	0.043
4550 QQQQ 06 Jan 38.00 (QQQ AJ)	36	21/10/21	1	3.7	3.8	3.976	0.226
4550 QQQQ 06 Jan 36.625 (YIZ A)	36.6	21/10/22	$\bar{x}$	3.2	3.3	3.584	0.334
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/10/23	1	2.9	2.95	3.349	0.424
4550 QQQQ 06 Jan 37.625 (YIZ A)	37.6	21/10/24	$\bar{x}$	2.5	2.58	2.958	0.483
4550 QQQQ 06 Jan 38.00 (QQQ AL)	38	21/10/25	$x_i^*$	$x_j^*$	2.25	2.723	0.523
4550 QQQQ 06 Jan 38.625 (YIZ A)	38.6	21/10/26	$\sum_{ij=1}^n (\rho^{-1})_{ij} \sigma_{impl,i}(x_i^*) \sigma_{impl,j}(x_j^*)$		1.8	2.332	0.557
4550 QQQQ 06 Jan 39.00 (QQQ AM)	39	21/10/27			1.6	2.097	0.522
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/10/28			1.25	1.706	0.481
4550 QQQQ 06 Jan 40.00 (QQQ AN)	40	21/10/29			1.05	1.471	0.446
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/10/30			0.8	1.079	0.304
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/10/31			0.65	0.844	0.219
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/10/32			0.5	0.453	0.003
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/10/33			0.35	0.4	-0.156
4550 QQQQ Zero Call	0	21/10/34	$\bar{x}$				
4550 QQQQ 06 Mar 38.00 (QQQ CK)	38	21/10/35	$\int_0^{x_i^*} \frac{du}{\sigma_i(u,0)}$	$\sum_{j=1}^n \rho_{ij} p_j(x_j^*) \sigma_j(x_j^*, 0)$	i = 1, 2, ..., n	4.1	4.415
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/10/36			3.5	3.588	0.188

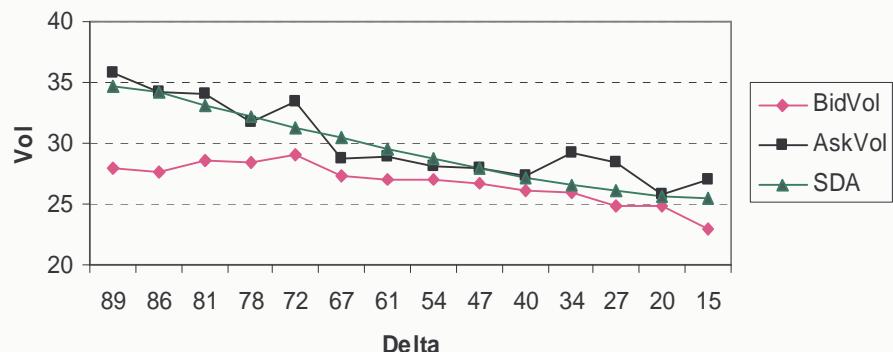
4550 QQQQ Zero Call	0	21/10/05	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/10/06	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Nov 38.00 (QQQ KM)	39	21/10/07	1	0.55	0.55	0.551	0.006
4550 QQQQ 05 Nov 38.00 (QQQ KL)	38	21/10/08	1	1.25	1.3	1.485	-0.175
4550 QQQQ 05 Nov 39.00 (QQQ KK)	37	21/10/09	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KM)	38	21/10/10	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 40.00 (QQQ KN)	39	21/10/11	1	0.8	1	1.174	0.224
4550 QQQQ Zero Call	0	21/10/12	$\bar{x}$				
4550 QQQQ 05 Dec 37.00 (QQQ CK)	37	21/10/13	$x_i^*$	$x_j^*$	0.5	0.367	-0.108
4550 QQQQ 05 Dec 39.00 (QQQ CK)	39	21/10/14	$\bar{x}$		2.25	2.723	0.523
4550 QQQQ 05 Dec 40.00 (QQQ CK)	40	21/10/15	$\sum_{ij=1}^n (\rho^{-1})_{ij} \beta_i \beta_j$		1.8	2.332	0.557
4550 QQQQ 05 Dec 41.00 (QQQ CK)	41	21/10/16	$\bar{x}$		1.6	2.097	0.522
4550 QQQQ Zero Call	0	21/10/17	$\bar{x}$		3.8	3.976	0.226
4550 QQQQ 06 Jan 38.00 (QQQ AJ)	36	21/10/18	$\bar{x}$		3.3	3.584	0.334
4550 QQQQ 06 Jan 36.625 (YIZ A)	36.6	21/10/19	$\bar{x}$		2.95	3.349	0.424
4550 QQQQ 06 Jan 37.00 (QQQ AK)	37	21/10/20	$\bar{x}$		2.5	2.958	0.483
4550 QQQQ 06 Jan 39.00 (QQQ AM)	39	21/10/21	$\bar{x}$		2.25	2.723	0.523
4550 QQQQ 06 Jan 39.625 (YIZ A)	39.6	21/10/22	$\bar{x}$		1.8	2.332	0.557
4550 QQQQ 06 Jan 40.00 (QQQ AN)	40	21/10/23	$\bar{x}$		1.6	2.097	0.522
4550 QQQQ 06 Jan 40.625 (YIZ A)	40.6	21/10/24	$\bar{x}$		1.05	1.471	0.446
4550 QQQQ 06 Jan 41.00 (QQQ AO)	41	21/10/25	$\bar{x}$		0.8	1.079	0.304
4550 QQQQ 06 Jan 41.625 (YIZ A)	41.6	21/10/26	$\bar{x}$		0.65	0.844	0.219
4550 QQQQ 06 Jan 42.00 (QQQ AP)	42	21/10/27	$\bar{x}$		0.5	0.453	0.003
4550 QQQQ Zero Call	0	21/10/28	$\bar{x}$		3.5	3.588	0.188
4550 QQQQ 06 Mar 36.00 (QQQ CK)	36	21/10/29	$\sum_{ij=1}^n (\rho^{-1})_{ij} \beta_i \beta_j$		3.8	3.976	0.226
4550 QQQQ 06 Mar 37.00 (QQQ CK)	37	21/10/30	$\sum_{ij=1}^n \rho_{ij} p_i p_j \sigma_{imp,i}(\bar{x}) \sigma_{imp,j}(\bar{x})$		3.3	3.588	0.188
4550 QQQQ Zero Call	0	21/10/31	$\bar{x}$				

Performs well in the range -0.2 < x < +0.2

## DJX: Dow Jones Industrial Average

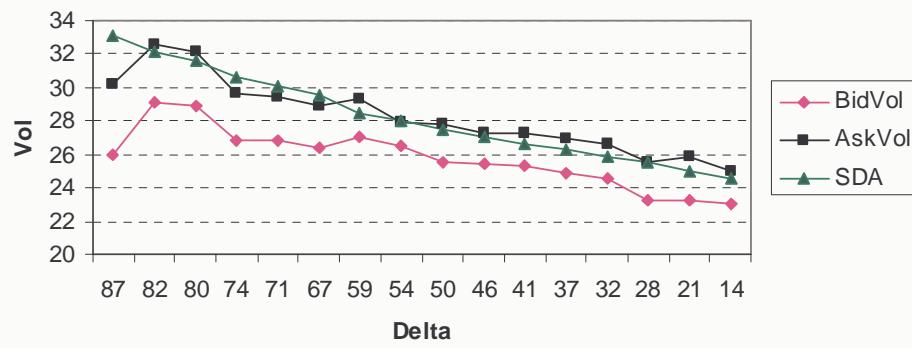
T=1 month

**DJX Nov 02 Pricing Date: 10/25/02**

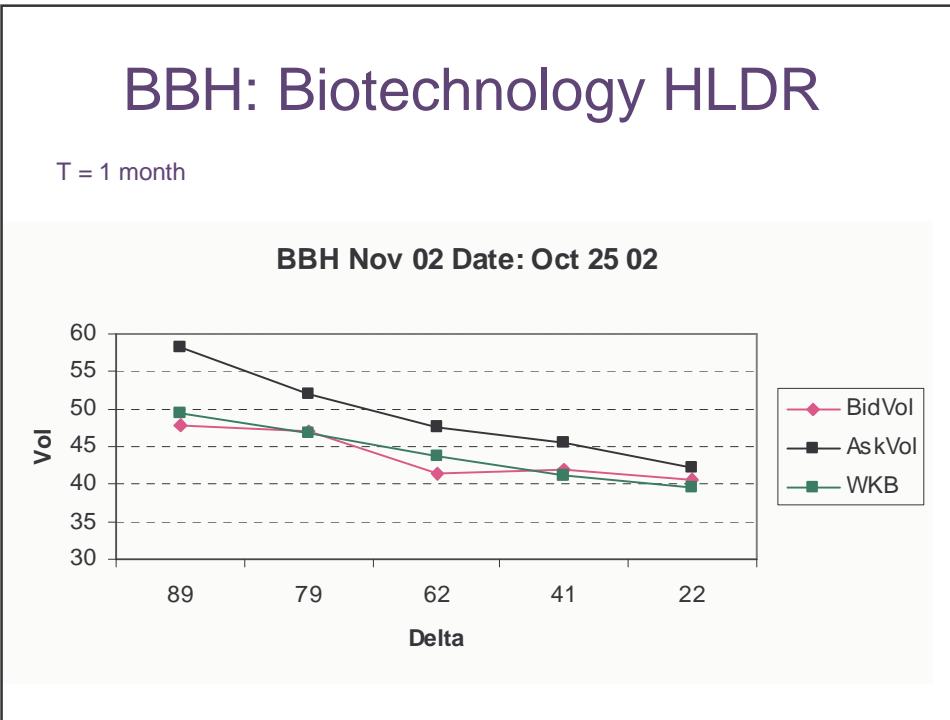


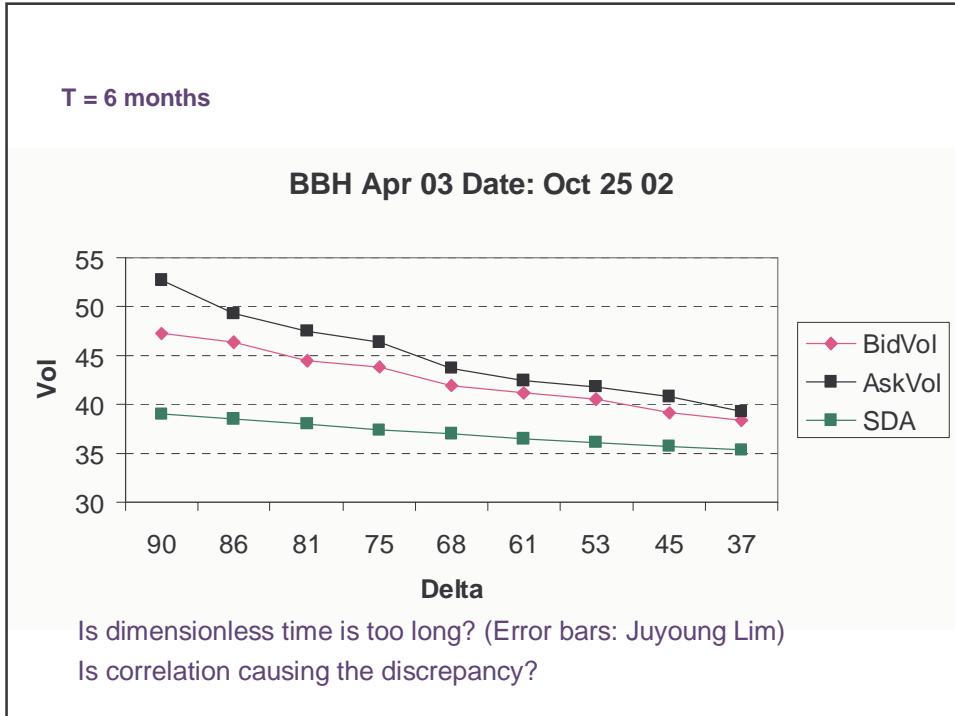
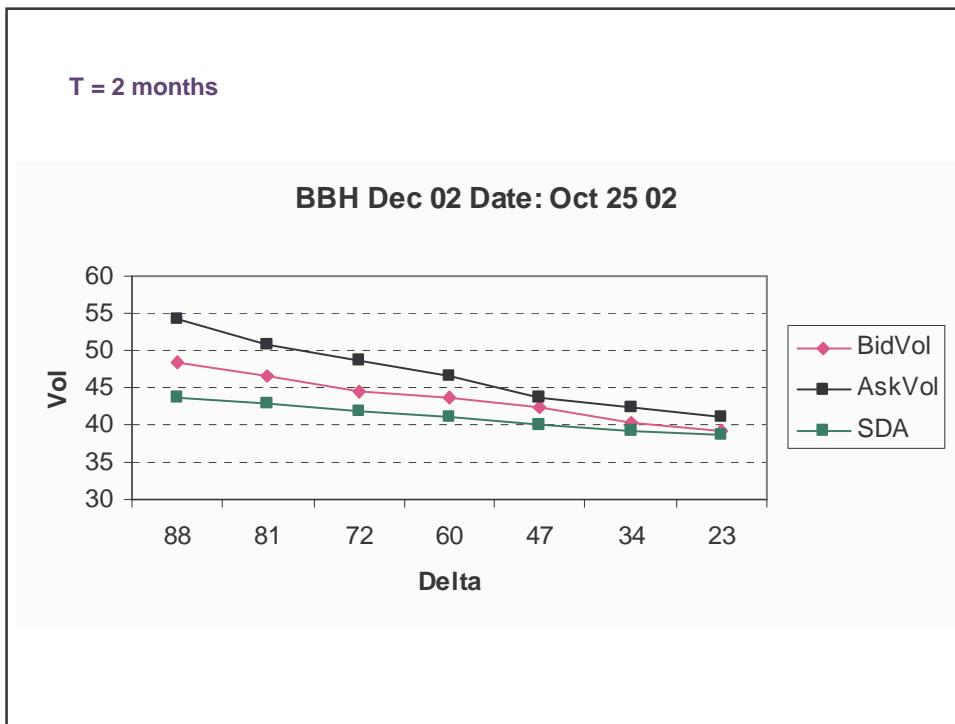
T= 2 months

**DJX Dec 02 Pricing Date: 10/25/02**



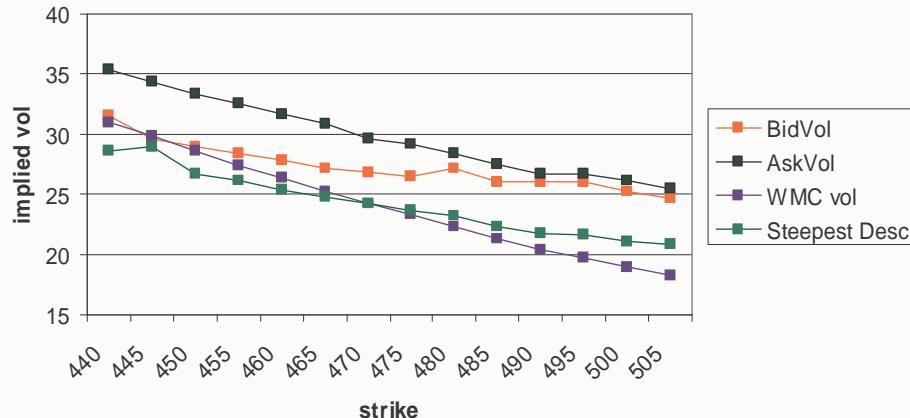






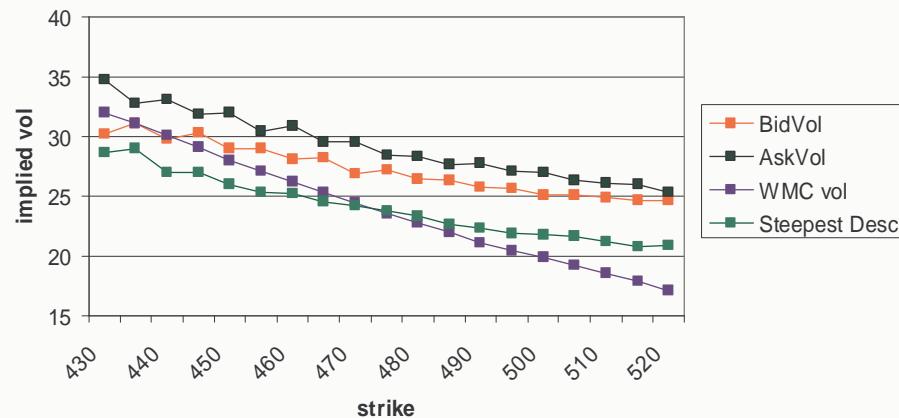
## S&P 100 Index Options (Quote date: Aug 20, 2002)

Expiration: Sep 02



## S&P 100 Index Options (Quote date: Aug 20, 2002)

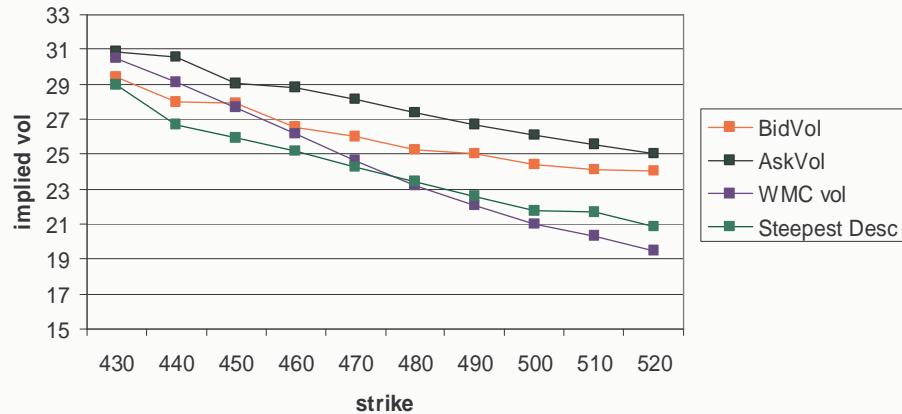
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## S&P 100 Index Options

(Quote date: Aug 20, 2002)

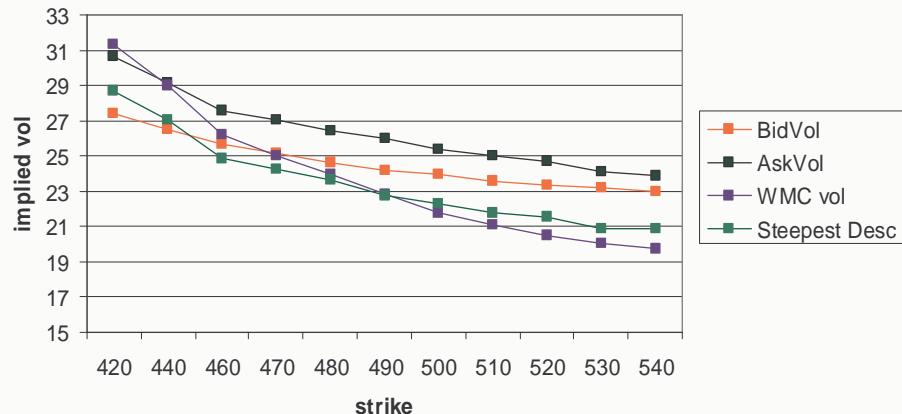
Expiration: Nov 02



## S&P 100 Index Options

(Quote date: Aug 20, 2002)

Expiration: Dec 02



## Implied Correlation: a single correlation coefficient consistent with index vol

$$(\sigma_I^{\text{impl}})^2 = \sum_{i=1}^N p_i^2 (\sigma_i^{\text{impl}})^2 + \rho \sum_{i \neq j}^N p_i p_j \sigma_i^{\text{impl}} \sigma_j^{\text{impl}}$$

$$\therefore \bar{\rho} = \frac{(\sigma_I^{\text{impl}})^2 - \sum_{i=1}^N p_i (\sigma_i^{\text{impl}})^2}{\sum_{i \neq j}^N p_i p_j \sigma_i^{\text{impl}} \sigma_j^{\text{impl}}} = \frac{(\sigma_I^{\text{impl}})^2 - \sum_{i=1}^N p_i^2 (\sigma_i^{\text{impl}})^2}{\left( \sum_{i=1}^N p_i \sigma_i^{\text{impl}} \right)^2 - \sum_{i=1}^N p_i^2 (\sigma_i^{\text{impl}})^2}$$

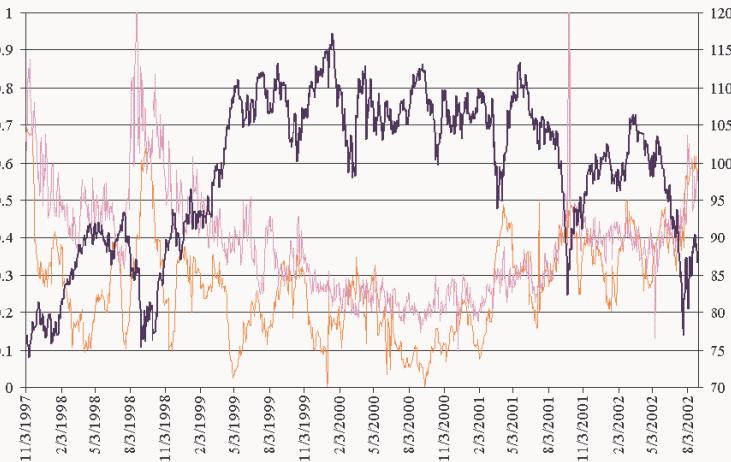
Approximate formula:

$$\bar{\rho} \approx \left( \frac{\sigma_I^{\text{impl}}}{\sum_{i=1}^N p_i \sigma_i^{\text{impl}}} \right)^2$$

Implied correlation can be defined for different strikes, using SDA

## Dow Jones Index

— Historical Correlation — 3 month Implied Correlation — Index Price



## Dow Jones Index: Correlation Skew





## A model for ``Correlation skew'': Stochastic Volatility Systems

$$\begin{aligned} \frac{dS_i}{S_i} &= \sigma_i dW_i & \frac{d\sigma_i}{\sigma_i} &= \kappa_i dZ_i \\ E(dW_i dW_j) &= \rho_{ij} dt & E(dW_i dZ_j) &= r_{ij} dt \end{aligned}$$

$$\bar{x} = \frac{dI}{I}, \quad x_i = \frac{dS_i}{S_i}, \quad y_i = \frac{d\sigma_i}{\sigma_i}$$

Look for most likely configuration of stocks and vols  
 $(x_1, \dots, x_n, y_1, \dots, y_n)$  corresponding to a given index  
 displacement  $x$

## Most likely configuration for Stochastic Volatility Systems

$$x_i^* = \bar{\beta}_i \bar{x} \quad \beta_i = \frac{\sigma_i \rho_{il}}{\sigma_I}$$

Most likely configuration for stocks moves and volatility moves, given the index move

$$y_i^* = \bar{\gamma}_i \bar{x} \quad \gamma_i = \frac{\kappa_i r_l}{\sigma_I}$$

$$\sigma_{I,\text{loc}}^2(\bar{x}, t) \equiv \sum_{ij=1}^n p_i p_j \sigma_i(0, t) \sigma_j(0, t) e^{\bar{\gamma}_i \bar{x}} e^{\bar{\gamma}_j \bar{x}} \rho_{ij}$$

SDA

## Method I: Dupire & Most Likely Configuration for Stock Moves

N-dimensional Equity market

$$\sigma_1(x_1, t)$$

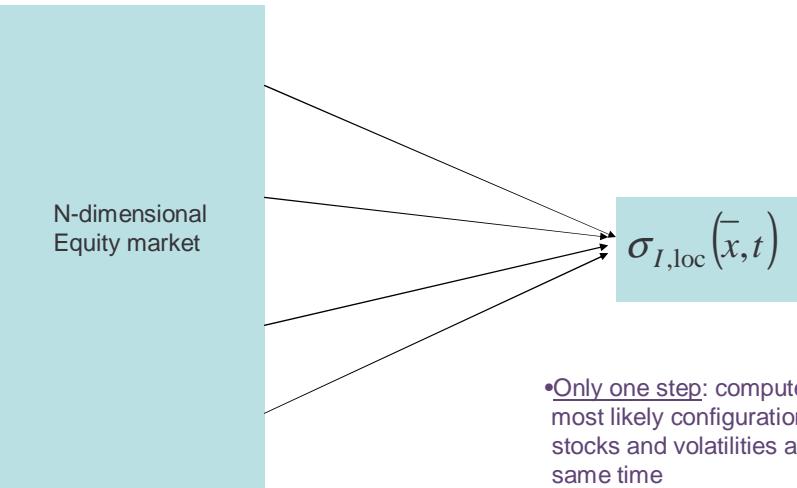
$$\sigma_i(x_i, t)$$

$$\sigma_{I,\text{loc}}(\bar{x}, t)$$

$$\sigma_n(x_n, t)$$

- Step 1: Local volatility for each stock consistent with options market
- Step 2: Find most likely configuration for stocks

## Method II: Stochastic Volatility System and joint MLC for Stocks and Volatilities



## Methods I and II are not ‘equivalent’

Dupire local vol. for single names

$$\sigma_{i,\text{loc}}(x_i, t) \approx \sigma_i(0, t) e^{\varpi_i x_i} \quad \varpi_i = \frac{\kappa_i r_{ii}}{\sigma_i}$$

Index vol., Method I

$$\sigma_{I,\text{loc}}^2(\bar{x}, t) = \sum_{ij} p_i p_j \sigma_i(0, t) \sigma_j(0, t) \rho_{ij} e^{\varpi_i \beta_i \bar{x}} e^{\varpi_j \beta_j \bar{x}}$$

Index vol., Method II

$$\sigma_{I,\text{loc}}^2(\bar{x}, t) = \sum_{ij} p_i p_j \sigma_i(0, t) \sigma_j(0, t) \rho_{ij} e^{\gamma_i \bar{x}} e^{\gamma_j \bar{x}}$$

## Stochastic Volatility Systems give rise to Index-dependent correlations

$$\sigma_{I,\text{loc}}^2(\bar{x}, t) \approx \sum_{ij} p_i p_j \sigma_i(0, t) \sigma_j(0, t) \rho_{ij} e^{\gamma_i \bar{x}} e^{\gamma_j \bar{x}} \quad \text{Method II}$$

$$\begin{aligned} & \approx \sum_{ij} p_i p_j \cancel{\sigma_i(0, t) e^{\beta_i \varpi_i \bar{x}}} \sigma_j(0, t) e^{\beta_j \varpi_j \bar{x}} \rho_{ij} e^{\gamma_i \bar{x}} e^{\gamma_j \bar{x}} e^{-\beta_i \varpi_i \bar{x}} e^{-\beta_j \varpi_j \bar{x}} \\ & \approx \sum_{ij} p_i p_j \sigma_{i,\text{loc}}(\beta_i \bar{x}, t) \sigma_{j,\text{loc}}(\beta_j \bar{x}, t) \rho_{ij}(\bar{x}) \end{aligned}$$

$$\rho_{ij}(\bar{x}) \equiv \rho_{ij} e^{(\gamma_i + \gamma_j - \beta_i \varpi_i - \beta_j \varpi_j) \bar{x}}$$

## Equivalence holds only under additional assumptions on stock-volatility correlations

$$\varpi_i \beta_i = \frac{\kappa_i r_{ii}}{\sigma_i} \frac{\sigma_i \rho_{il}}{\sigma_I} = \frac{\kappa_i r_{ii} \rho_{il}}{\sigma_I} \quad \text{Method I}$$

$$\gamma_i = \frac{\kappa_i r_{il}}{\sigma_I} \quad \text{Method II}$$

$$r_{il} = r_{ii} \rho_{il}$$

$$r_{ij} = r_{ii} \rho_{ij}$$

Conditions under which both methods give equivalent valuations

## Open (and very doable) problems

4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 38.00 (QQQ JM)	39	21/1/007	1	0.55	0.6	0.581	0.006
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/011	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ LO)	39	21/1/012	1	0.9	1	1.174	0.224
4550 QQQQ 05 Nov 39.00 (QQQ LM)	38	21/1/009	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/1/010	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/011	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/1/012	1	0.9	1	1.174	0.224
4550 QQQQ Zero Call	0	21/1/009	9	38.93	38.93	38.971	0.041
4550 QQQQ 05 Dec 39.00 (QQQ LM)	37	21/1/013	1	2.6	2.7	3.099	0.449
4550 QQQQ 05 Dec 39.00 (QQQ LL)	38	21/1/016	1	1.55	1.95	2.376	0.476
4550 QQQQ 0 If we use a Local Volatility model (e.g. BGM with square-root volatility), the answer is identical to the previous formula	0	21/1/016	1	0.75	0.93	0.205	
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/1/017	1	0.4	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/1/020	9	38.93	38.93	38.973	0.043
4550 QQQQ 05 Jan 38.00 (QQQ AJ)	36	21/1/021	1	3.7	3.8	3.976	0.226
4550 QQQQ 05 Jan 38.625 (YIZ A)	36.6	21/1/022	1	3.2	3.3	3.584	0.334
4550 QQQQ 0 The ``full'' SABR multi-asset model gives rise to a complicated Riemannian metric	0	21/1/024	1	2.45	2.5	2.958	0.483
4550 QQQQ 05 Jan 38.625 (YIZ AL)	36	21/1/025	1	2.15	2.25	2.723	0.523
4550 QQQQ 05 Jan 38.00 (YIZ A)	38.6	21/1/026	1	1.75	1.8	2.332	0.557
4550 QQQQ 05 Jan 39.00 (QQQ AM)	39	21/1/027	1	1.55	1.6	2.097	0.522
4550 QQQQ 05 Jan 39.625 (YIZ Y)	39.6	21/1/028	1	1.2	1.25	1.706	0.481
4550 QQQQ 05 Jan 40.00 (QQQ AN)	40	21/1/029	1	1.05	1.471	0.446	
4550 QQQQ 05 Jan 40.625 (YIZ A)	40.6	21/1/030	1	0.75	0.8	1.079	0.304
4550 QQQQ 05 Jan 41.00 (QQQ AO)	41	21/1/031	1	0.6	0.65	0.844	0.219
4550 QQQQ 05 Jan 41.625 (YIZ A)	41.6	21/1/031	1	0.4	0.5	0.453	0.003
4550 QQQQ 0 Credit default models for pricing CDOs are amenable to the same approach, especially copula-type models. I am not aware of any solutions	0	21/1/035	1	3.3	3.5	3.588	0.188

## Epilogue: Structural Credit Model

4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.942	0.012
4550 QQQQ 05 Oct 38.00 (QQQ JL)	38	21/1/006	1	1.25	1.3	1.485	0.21
4550 QQQQ 05 Oct 38.00 (QQQ JM)	39	21/1/007	1	0.55	0.6	0.581	0.006
4550 QQQQ Zero Call	0	21/1/005	9	38.93	38.93	38.947	0.017
4550 QQQQ 05 Nov 37.00 (QQQ KK)	37	21/1/010	1	2.35	2.45	2.787	0.387
4550 QQQQ 05 Nov 39.00 (QQQ KL)	38	21/1/011	1	1.55	1.65	1.98	0.38
4550 QQQQ 05 Nov 39.00 (QQQ KM)	39	21/1/012	1	0.9	1	1.174	0.224
4550 QQQQ 05 Nov 40.00 (QQQ LN)	40	21/1/013	1	1.2	1.25	1.653	0.428
4550 QQQQ 05 Dec 39.00 (QQQ LM)	39	21/1/017	1	0.75	1.25	1.653	0.428
4550 QQQQ 05 Dec 40.00 (QQQ LO)	40	21/1/018	1	0.35	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/1/020	9	38.93	38.93	38.973	0.043
4550 QQQQ 05 Jan 38.00 (QQQ AJ)	36	21/1/021	1	3.7	3.8	3.976	0.226
4550 QQQQ 05 Jan 38.625 (YIZ A)	36.6	21/1/022	1	3.2	3.3	3.584	0.334
4550 QQQQ 05 Jan 39.625 (YIZ A)	39.6	21/1/023	1	1.75	1.8	2.332	0.557
4550 QQQQ 05 Jan 40.00 (QQQ AM)	40	21/1/024	1	1.55	1.6	2.097	0.522
4550 QQQQ Firm i defaults before time T if $x_i(T) < \alpha_i$	0	21/1/024	1	1.2	1.25	1.706	0.481
4550 QQQQ 05 Dec 41.00 (QQQ LO)	41	21/1/025	1	0.35	0.4	0.208	-0.167
4550 QQQQ Zero Call	0	21/1/020	9	38.93	38.93	38.973	0.043
4550 QQQQ 05 Jan 38.00 (QQQ AJ)	36	21/1/021	1	3.7	3.8	3.976	0.226
4550 QQQQ 05 Jan 38.625 (YIZ A)	38.6	21/1/026	1	1.75	1.8	2.332	0.557
4550 QQQQ 05 Jan 39.00 (QQQ AM)	39	21/1/027	1	1.55	1.6	2.097	0.522
4550 QQQQ Equal weighted CDO : loss of m dollars if $\sum_{i \in I} \alpha_i > m$	0	21/1/028	1	0.95	0.95	3.349	0.424
4550 QQQQ 05 Jan 37.625 (YIZ A)	36.6	21/1/024	1	2.45	2.5	2.958	0.483
4550 QQQQ 05 Jan 38.00 (QQQ AL)	38	21/1/025	1	2.15	2.25	2.723	0.523
4550 QQQQ 05 Jan 38.625 (YIZ A)	38.6	21/1/026	1	1.75	1.8	2.332	0.557
4550 QQQQ 05 Jan 39.00 (QQQ AM)	39	21/1/027	1	1.55	1.6	2.097	0.522
4550 QQQQ $\mathbf{x}(T) \in \Omega_m = \bigcup_{\substack{I \subseteq \{1, \dots, n\} \\ \text{card}(I) \geq m}} \{\mathbf{x} : x_i < \alpha_i\}$	0	21/1/028	1	1.2	1.25	1.706	0.481
4550 QQQQ 05 Jan 40.00 (QQQ AN)	40	21/1/029	1	1	1.05	1.471	0.446
4550 QQQQ 05 Jan 40.625 (YIZ A)	40.6	21/1/030	1	0.75	0.8	1.079	0.304
4550 QQQQ 05 Jan 41.00 (QQQ AO)	41	21/1/031	1	0.6	0.65	0.844	0.219
4550 QQQQ 05 Jan 41.625 (YIZ A)	41.6	21/1/031	1	0.4	0.5	0.453	0.003
4550 QQQQ Solve	0	21/1/032	1	0.35	0.4	0.219	-0.156
4550 QQQQ Zero Call	0	21/1/033	9	38.93	38.93	38.956	0.026
4550 QQQQ 05 Mar 38.00 (QQQ CJ)	36	21/1/034	1	4.1	4.3	4.415	0.215
4550 QQQQ 05 Mar 37.00 (QQQ CJ)	37	21/1/035	1	3.3	3.5	3.588	0.188

$$\inf \{L(0, \mathbf{x}) : \mathbf{x} \in \Omega_m\}$$