Structural Slippage of Leveraged ETFs

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1 Introduction

Leveraged Exchange Traded Funds (LETFs) have gained a lot of popularity since their creation around 2006. According to tracker IndexUniverse, there are 275 leveraged ETFs with nearly $33 billion dollars in assets. Leveraged ETFs reference an underlying index or ETF, and offer a multiple of the daily returns of the underlying index minus hedging costs and management fees. Leveraged ETFs belong to the class of actively managed funds, and the daily rebalancing of a LETF requires the fund manager to systematically change the amount of index exposure. In order to achieve the investment objective of the fund, the managers of LETFs usually increase or reduce their exposure to the underlying index by using total return swaps (TRS) with the appropriate leverage ratio[1]. This daily rebalancing may open the possibility of front-running and other market frictions. It is not understood how the actively managed strategies of LETFs affect returns compared to the benchmark. The purpose of this paper is to get a better understanding of this issue.

It is difficult to accurately measure market impact[2] caused by daily rebalancing. Here, one can compare the daily returns of the leveraged fund to β times the daily returns of an unleveraged ETF tracking the same underlying index. Fees and management costs are somewhat important when determining the profitability of these funds, but are generally much smaller than the effects we seek to capture here.

All indications show that LETFs have negative overnight expected returns when compared to the benchmark. As we shall argue, this is most likely due to "inefficiencies" associated with hedging by the manager. Note that this is not connected with convexity effects for LETFs reported in other papers [e.g., Madhavan, Avellaneda, Zhang]. Rather,

[1]From here-on we refer to the leverage factor of a LETF as β. This factor can be either 2 or 3 in the case of a bullish LETF, or -1, -2, or -3 in the case of a bearish LETF. In addition, the manager of a LETF can also use futures and other derivatives to increase or to reduce his exposure. For the sake of simplicity, throughout the paper we assume that the exposure is modified using only TRS. In any case, LETFs require daily rebalancing.

[2]We use the term market impact here to mean the effect that a market participant has when it buys or sells an asset.

this is due to market impact caused by active management as studied by Tang and Xu [1]. Essentially, the size and the direction of daily rebalancing is based on public information. This information could be used by market participants in ways which cause further slippage of LETF returns from their expected returns.

Initially, it was our goal to formulate a mathematical model to capture the inefficiencies produced by LETFs. We hypothesized that the loss in expected return for LETFs is proportional to the volatility of the underlying index, and inversely proportional to the average trading volume of the underlying index. Yet, this model proved too mechanical for describing the inefficiencies the market produces. We tried correlating volatility to impact, and found no significant relation. Likewise with the other variables in the model. So that classical mechanical impact models[3] are too crude and cannot always explain market impact. But, we find that periods that are very volatile usually give rise to large underperformance. It is on these empirical results that the paper focuses.

Another very important factor in determining the economics of shorting LETFs are borrowing costs. Most LETFs are hard to borrow with often very limited availability of stocks and high borrow rates. In fact, the typical fee for borrowing[4] is on the order of 200-600 basis points, whereas management fees range from 75-95 basis points. We believe that the main reason for such large borrow fees is due to the daily market impact incurred by LETFs which induce arbitrageurs to short these funds and to hedge exposure[5].

2 Managing LETFs; Buy High, Sell Low

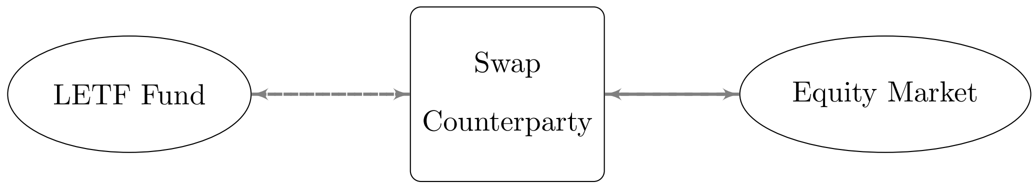
We develop a framework in order to better understand how LETF managers rebalance their daily exposure to the underlying index. For this purpose, consider a hypothetical market with an index or ETF, and a LETF with leverage ratio β. We use discrete time

[3] In the sense of [2].

[4] In the sense that fee = fed funds - short rate.

[5] For a popularized discussion of this topic refer to [3].

with Δt equal to 1 day, and denote the price of the LETF at date t by L_t. Due to the product design, the LETF is supposed to have a return which is equal to β times the return of the underlying index over any one day. The manager achieves his desired exposure by using total return swaps. The relevant diagram for this transaction is depicted below. Note that all costs of hedging are ultimately transferred to the fund.



We give two examples to demonstrate this process. Suppose that the LETF has $\beta = 2$, an underlying index value of \$100, and one billion dollars (denoted \$1B) of AUM. Consider the following scenario:

Table 1: $\beta = 2$, Initial Index Value = \$100, Initial AUM = \$1B

Day	Index Value(\$)	AUM(\$M)[1]	Exposure Needed(\$M)	Exposure Before Adjustment(\$M)	TRS Adjustments(\$M)
0	100	1000	2000	—	—
1	90	800	1600	1800	-200
2	99	960	1920	1760	+160

[1] \$M denotes dollars in millions. For simplicity, we assume the overnight interest rate on AUM is zero.
[2] A negative sign denotes selling, and a positive sign denotes buying.

- The manager begins (Day 0) long \$2B TRS hedged by the swap counterparty.
- On Day 1, the index drops by 10 percent to \$90, so the value of AUM drops to \$1B$\times(1 - 2 \times .1)$ = \$800M. On Day 1 the manager needs an exposure of $2 \times$ \$800M=\$1600M.
- The notional value of the TRS from Day 0 has become \$2000M$\times(1-.1)$ = \$1800M giving an exposure of \$1800M before adjustment. Thus, the swap counterparty must **sell** (go short stock synthetically) \$1800M-\$1600M=\$200M of TRS.
- On Day 3, the index rises by 10 percent to \$99, so the value of AUM grows to \$800M$\times(1 + 2 \times .1)$ = \$960M, which means the manager needs an exposure

of $2 \times$ \$960M=\$1920M. The notional value of the TRS from Day 1 has become \$1600M $\times(1 + .1)$ = \$1760M giving an exposure before adjustment of \$1760M.

- Hence, to achieve the desired exposure of \$1920M the swap counterparty must **buy** (go long stock synthetically) \$1920M-\$1760M=\$160M of TRS.

For the sake of completeness, we also consider the inverse LETF and suppose that now the LETF has $\beta = -2$. As before, we have an initial index value of \$100, and \$1B of AUM.

Table 2: $\beta = -2$, Initial Index Value = \$100, Initial AUM = \$1B

Day	Index Value(\$)	AUM(\$M)[1]	Exposure Needed(\$M)	Exposure Before Adjustment(\$M)	TRS Adjustments(\$M)
0	100	1000	-2000	—	—
1	90	1200	-2400	-1800	-600
2	99	960	-1920	-2640	+720

[1] \$M denotes dollars in millions. For simplicity, we assume the overnight interest rate on AUM is zero.
[2] A negative sign denotes selling, and a positive sign denotes buying.

- The manager begins (Day 0) short \$2B TRS hedged by the swap counterparty.
- On Day 1, the index drops by 10 percent to \$90, so the value of AUM grows to \$1000M$\times(1 + 2 \times .1)$ = \$1200M, which means the manager needs an exposure of $-2 \times$ \$1200M= -\$2400M.
- The notional value of the TRS from Day 0 has become $-$\$2000M$\times(1 - .1)$ = $-$\$1800M giving an exposure of \$1800M short. Thus, the swap counterparty must **sell** (go short stock synthetically) \$2400M$-$\$1800M=\$600M of TRS.
- On Day 3, the index rises by 10 percent to \$99, so the value of AUM drops to \$1200M$\times(1 - 2 \times .1)$ = \$960M, which means the manager needs an exposure of $-2 \times$ \$960M=-\$1920M. The notional value of the TRS from Day 1 has become $-$\$2400M $\times(1 + .1)$ = $-$\$2640M giving an exposure of \$2640M short.
- Hence, to achieve the desired exposure of \$1920M short the swap counterparty must **buy** (go long stock synthetically) \$2640M-\$1920M=\$720M of TRS.

These examples illustrate a fundamental feature of LETFs; the fund always rebalances in the same direction as the underlying index regardless of whether it is a bullish or a bearish leveraged fund[6]. In other words, in this active management strategy, the fund always buys high and sells low.

We believe that when the swap counterparty rebalances the TRS it will incur an impact[7] which is passed on to the fund along with the swap dealer's fees[8]. We derive a formula for calculating the adjustment needed to rebalance a leveraged fund. In general, denote by E_t the AUM of the LETF at time t. Also, let S_t denote the price of the underlying index at time t. The profit and loss (P&L) corresponding to trading one share of the underlying index is given by:

$$\Delta S_t - rS_t\Delta t,$$

where r denotes the daily risk-free interest rate, and the time-step Δt is one day. Now, let n_t[9] denote the number of shares of the underlying index held as a hedge at time t. The corresponding P&L of the total number of shares is thus given by:

$$n_t(\Delta S_t - rS_t\Delta t).$$

Maintaining a constant leverage β is equivalent to verifying the following equation holds at the beginning of each time period:

$$\beta E_t = n_t S_t \implies E_t = \frac{n_t S_t}{\beta}.$$

This equation relates the AUM and the number of shares which should be held at time t. The P&L of the fund comes from the change in value of the TRS position and the

[6]Bullish LETFs are also referred to as ultra LETFs, and correspondingly, $\beta = 2$ or 3 for these funds. On the other hand, bearish LETFs are also referred to as ultra-short LETFs and their leverage factor is either -1, -2, or -3.

[7]From here on we refer to the cost incurred by daily rebalancing as slippage.

[8]These fees will in general be much smaller than the resulting slippage.

[9]We use the convention that $n_t > 0$ if we are long the underlying index, and $n_t < 0$ if we are short the underlying index.

interest accumulated on AUM[10]:

$$\Delta E_t = n_t(\Delta S_t - rS_t\Delta t) + E_t r\Delta t. \tag{1}$$

Expanding equation 1, and using the fact that $E_t = \frac{n_t S_t}{\beta}$ ultimately gives[11]:

$$\frac{\Delta n_t}{n_t} = (\beta - 1)\left(1 - \frac{(1 + r\Delta t)S_t}{S_{t+1}}\right). \tag{2}$$

These equations confirm our initial observation that maintaining a constant leverage obliges the LETF manager to buy high and sell low. In particular, both the long and the short leveraged ETFs always rebalance in the same direction as the market.

3 A Model for Slippage

Due to systematic adjustment of the swap notional, we argue that LETFs should have negative expected returns with respect to the underlying benchmark. The market knows the exact direction and the exact size of the hedge; which is somewhat analogous to a market participant announcing ahead of time a large market order.

We began our analysis with the expectation that the change in value of the leveraged fund due to temporary market impact should be given by:

$$\Delta E_t = \underbrace{n_t(\Delta S_t - S_t r\Delta t) + E_t r\Delta t}_{\text{change in AUM without impact}} - \underbrace{E_t(\mu_t + \epsilon_t)}_{\text{additional change caused by daily rebalancing}}. \tag{3}$$

In equation 3, μ_t is a positive number, and ϵ_t is a mean-zero noise term. We represent slippage at time t by $(\mu_t + \epsilon_t)$. If the manager does not rebalance daily to maintain constant leverage, we would expect the change in AUM of the fund to read as in equation 1. Since this is not the case, we expect the change in AUM to also reflect the impact

[10]Usually held in cash.

[11]See Appendix A1 for full derivation.

the manager produces by rebalancing, hence the true change in AUM should be as in equation 3. Dividing through by E_t in equation 3, and rearranging terms gives:

$$\underbrace{\frac{\Delta E_t}{E_t}}_{\text{actual return of fund}} - \underbrace{(\beta \frac{\Delta S_t}{S_t} + r\Delta t(1-\beta))}_{\text{expected return of fund}} = -\mu_t - \epsilon_t. \tag{4}$$

So that the difference between the actual return of the fund and the expected return of the fund is given by the daily slippage term $-\mu_t - \epsilon_t$.

4 Empirical Results for Slippage

We give very strong empirical evidence for the existence of slippage for LETFs. There are several ways to capture slippage. If we short both the bullish LETF and the bearish LETF we should be able to capture some portion of the resulting slippage,[12] while remaining market-neutral. For those LETFs for which an underlying index exists, we can also capture slippage by shorting the bear LETF and shorting $|\beta|$ of the underlying index, or by shorting the bull LETF and going long $|\beta|$ of the underling index.

Our analysis covers the period June 26, 2009 until July 8, 2011. We analyse 21 pair-trades among bullish LETFs and their bearish counterpart, and for those cases where the LETFs have a physical underlying index, we also consider the slippage produced when we go long $\pm\beta$[13] times the underlying index and short the corresponding LETF. For each day during this period and for each pair-trade we compute slippage. We then took a rolling sum of the results for the first 252 days (one business year) starting with June 26, 2009 and ending with July 9, 2010 (251 days before July 8, 2011) in order to compute annual slippage. It is on this annual data that we computed the statistical results shown in table 3. The notation used for each pair trade is in the form A/B and

[12]Mathematically, it is easy to check that this quantity is given by $(\mu_1 + \mu_2) + (\epsilon_t^1 + \epsilon_t^2)$. Where the indexes correspond to each LETF.

[13]We go long $|\beta|$ times the underlying and short the bullish LETF, and short $|\beta|$ times the underlying and short the bearish LETF.

should be read as going long A and shorting B. In the case where an underlying index exists, we use the same notation to mean that we long/short $|\beta|$ of the underlying index and short the corresponding LETF.

As can be seen from table 3 each of the 21 pair-trades has positive mean and median slippage. Furthermore, all of the 95 percent confidence intervals about the mean contain only positive slippage values, and in particular, they do not contain zero. Hence, the existence of slippage for these trades is unequivocal. We also performed the same data-analysis for monthly (20-days) running slippage, and found similar results, in that all of the 95-percent confidence intervals for each trade are stictly positive. Thus, there is very noticeable and significant slippage for this trade duration as well.

It is important to mention here that if the duration for these trades is just a single day, then there is no observable slippage. The reason for this comes from the ϵ_t term in the expression for slippage. Since ϵ_t is a mean-zero noise term, intraday the variance of ϵ_t is large and the trade will not be able to capture slippage since the mean-zero attribute of ϵ_t will not be utilized[14]. Hence, it is important to consider these trades over multiple days in order to capture slippage. In addition to these results, we also tested for serial correlation among the daily slippage values, and found that for each trade the data does not display autocorrelation[15]. In other words, the ϵ_t's for each trade are uncorrelated.

[14]Since we won't be adding them, the law of large numbers will not take effect.

[15]We used the Durbin-Watson Statistic to detect for the presence of autocorrelation.

Table 3: Slippage in basis points per year

ETF Pair	Mean	Median	Vol	90 percent CI	95 percent CI
EDC/EDZ	1235	1242	97	(1225, 1245)	(1223, 1247)
EEM/EDC	796	807	99	(786, 806)	(784, 808)
EEM/EDZ	439	441	67	(432, 446)	(431, 447)
UYG/SKF	399	400	25	(396.5, 401.5)	(396, 402)
IYF/UYG	108	108	24	(105.6, 110.4)	(105.2, 110.8)
IYF/SKF	291	290	17	(189.3, 292.7)	(289, 293)
URE/SRS	635	628	50	(630, 640)	(629, 641)
IYR/URE	87	90	35	(83.4, 90.6)	(84, 91)
IYR/SRS	548	546	54	(542.5, 553.5)	(541.5, 554.5)
TNA/TZA	824	834	74	(816.5, 831.9)	(815, 833)
IWM/TNA	151	152	80	(143, 159)	(141.3, 161.7)
IWM/TZA	673	675	31	(670, 676)	(669.2, 676.8)
AGQ/ZSL	2314	2045	662	(2246, 2382)	(2233.5, 2394.5)
SLV/AGQ	898	770	377	(859.5, 936.5)	(852.2, 943.8)
SLV/ZSL	1416	1259	429.3	(1372, 1459.8)	(1364, 1468)
DRV/DRN	1850	1151	1232	(1721, 1979)	(1696, 2004)
VNQ/DRN	1114	829	1302	(976, 1251)	(951, 1277)
VNQ/DRV	715	705	761	(635, 795)	(620, 810)
FXP/FXI	1645	1635	428	(1601, 1689)	(1593, 1697)
FAS/FAZ	901	909	46	(896.3, 905.7)	(895.4, 906.6)
BZQ/EWZ	1965	1863	737	(1890, 2040)	(1875, 2055)

Table reports the mean slippage for the sample data in basis points per year (252 business days), the median slippage amount for the same time period, the 90 percent confidence interval (CI) about the mean, and the 95 percent confidence interval (CI) about the mean. Here we see that not only are all of the 95 percent confidence intervals about the mean always positive, but they also contain large values. It is clear that there is unequivocal slippage for these trades. The full sample data ranges from June 26, 2009 until July 8, 2011.

5 Hard-to-Borrow and Empirical Results

From the previous section we see that shorting LETFs produces slippage and thus makes money. In order to see the full picture, and to determine the economics of these funds, we consider the cost of shorting thse LETFs. We consider the borrowing rates for each pair-trade in basis points per year[16]. Table 4 displays the same statistics for borrowing rates as table 3 does for slippage. From the slippage and borrowing rates we calculate the difference between the two. Table 5 displays our findings.

As can be seen from table 4, the median annual borrowing rates range from about 140-890 basis points per year. From table 3 we see that median annual slippage values range from about 90-2045 basis points, so that even though the lower bounds for borrowing rate and slippage are comparable, there is quite a bit of discrepancy among the upper bound values of the range. In table 5 we consider annual slippage minus borrowing rates for each of the 21 pair-trades. Here we see that the median values range from about -302 basis points per year, to 1413 basis points per year. In the cases where the borrowing rate is higher than slippage, the result is negative, but there are trades where the borrowing rate is much lower than slippage, and it is these trades that give large positive results. From table 5 we see that for 16 out of the 21 pair-trades the 95 percent confidence interval about the mean is strictly positive. So for these trades the borrowing rate needed to short the LETFs had not caught up to the resulting slippage from the pair-trade.

[16]As before, borrowing rate = fed fund rate - shorting rate. We obtained our borrowing rates data from the Interactive Brokers terminal.

Table 4: Borrowing Rates in basis points per year

ETF Pair	Mean	Median	Vol	90 percent CI	95 percent CI
EDC/EDZ	633	631	31	(630, 636)	(629, 637)
EEM/EDC	300	281	43	(296, 304)	(295, 305)
EEM/EDZ	500	480	46	(195.3, 504.7)	(404.4, 505.6)
UYG/SKF	347	338	26	(344.4, 349.6)	(344, 350)
IYF/UYG	150	143	19.6	(148, 152)	(147.2, 152.8)
IYF/SKF	486	452	78	(478, 494)	(476, 496)
URE/SRS	430	423	20	(428, 432)	(428, 432)
IYR/URE	268	270	10	(267, 269)	(267, 269)
IYR/SRS	385	382	22	(383, 387)	(382, 388)
TNA/TZA	680	650	66	(673, 687)	(672, 688)
IWM/TNA	480	459	57	(474, 486)	(473, 487)
IWM/TZA	526	516	40	(522, 530)	(521, 531)
AGQ/ZSL	765	811	101	(755, 775)	(753, 777)
SLV/AGQ	241	243	9	(240, 242)	(240, 242)
SLV/ZSL	612	653	116	(600, 624)	(598, 626)
DRV/DRN	879	884	31	(876, 882)	(875, 883)
VNQ/DRN	510	516	25	(507, 513)	(507, 513)
VNQ/DRV	562	554	27	(559, 565)	(559, 565)
FXP/FXI	419	413	24	(417, 421)	(416, 422)
FAS/FAZ	676	624	122	(664, 688)	(662, 690)
BZQ/EWZ	473	467	46	(468, 478)	(467, 479)

Table reports the mean borrowing rate for the sample data in basis points per year (252 business days), the median borrowing rate for the same time period, the 90 percent confidence interval (CI) about the mean, and the 95 percent confidence interval (CI) about the mean. The full sample data ranges from June 26, 2009 until July 8, 2011.

Table 5: Slippage-Borrowing Rate in basis points per year

ETF Pair	Mean	Median	Vol	90 percent CI	95 percent CI
EDC/EDZ	602	622	117	(590, 614)	(588, 616)
EEM/EDC	495	516	129	(482, 508)	(479, 511)
EEM/EDZ	-61	-41	108	(-72, -50)	(-74, -48)
UYG/SKF	52	60	37	(48, 56)	(48, 56)
IYF/UYG	-43	-41	25	(-46, -40)	(-46, -40)
IYF/SKF	-194	-159	81	(-202, -186)	(-204, -184)
URE/SRS	205	205	61	(199, 211)	(198, 212)
IYR/URE	-181	-180	36	(-185, -177)	(-185, 177)
IYR/SRS	164	174	49	(159, 169)	(158, 170)
TNA/TZA	143	178	134	(129, 157)	(127, 159)
IWM/TNA	-330	-302	57	(-343, -317)	(-346, -314)
IWM/TZA	147	153	44	(142, 152)	(142, 152)
AGQ/ZSL	1549	1278	633	(1484, 1614)	(1472, 1626)
SLV/AGQ	657	531	382	(618, 696)	(611, 703)
SLV/ZSL	804	646	401	(763, 845)	(755, 853)
DRV/DRN	971	269	1258	(839, 1103)	(814, 1128)
VNQ/DRN	604	354	1299	(468, 740)	(442, 762)
VNQ/DRV	152	162	758	(73, 231)	(57, 247)
FXP/FXI	1226	1218	436	(1182, 1270)	(1173, 1279)
FAS/FAZ	225	262	97	(215, 235)	(213, 237)
BZQ/EWZ	1492	1413	721	(1418, 1566)	(1404, 1580)

Table reports the mean slippage minus borrowing rate for the sample data in basis points per year (252 business days), the median slippage minus borrowing rate amount for the same time period, the 90 percent confidence interval (CI) about the mean, and the 95 percent confidence interval (CI) about the mean for these values. Note that for 16 trades the 95 percent confidence interval is positive. For these 16 trades borrowing rates had not yet caught up to slippage. The full sample data ranges from June 26, 2009 to July 8, 2011.

6 Conclusion

LETFs are unique in that the fund managers must always rebalance their positions. These funds use total return swaps at near market close each day in order to maintain the proper leverage. Fund managers rebalance their positions in the same direction as the underlying index. This predictable daily rebalancing by managers causes frictions in the market which in turn negatively impact the returns of LETFs. Since LETFs mandates' are publicly known, and are usually very large, the market absorbs this information, and managers get poorer prices for the assets and contracts they have to buy.

We constructed shorting trades in order to capture any existing slippage. Our data confirmed our expectations and unequivocally showed that the cumulative effect of rebalancing costs cannot be ignored. For holding periods greater than one day, even after accounting for compounding costs, LETFs fail to perform as expected. This is especially true for longer holding periods and for periods of high volatility.

In order to gain a full understanding of the performance of these trades, it was crucial to take into account the borrowing rates for shorting LETFs. We calculated the difference between the slippage and the borrowing rates. In 16 out of the 21 trades we found that the mean value of the difference is strictly positive with 95 percent confidence. Precisely because LETFs have negative expected returns with respect to their benchmark index, if they are not offset by high borrowing costs, then systematic shorting yields an arbitrage opportunity[17].

[17] At the time of this writing, borrow rates have increased, and LETF are becoming increasingly harder to borrow.

A Appendix

A.1 The Mechanics of Hedging Equation

We begin with the following result:

$$\Delta E_t = n_t(\Delta S_t - rS_t\Delta t) + E_t r\Delta t. \tag{5}$$

Expanding and using the fact that $E_t = \frac{n_t S_t}{\beta}$ gives:

$$E_{t+1} - E_t = \frac{n_{t+1}S_{t+1} - n_t S_t}{\beta} = n_t((S_{t+1} - S_t) - rS_t\Delta t) + \frac{n_t S_t}{\beta} r\Delta t.$$

Solving the last equality for n_{t+1} gives an expression for the number of shares of the underlying index that must be held on day $(t+1)$ in order to maintain a constant leverage:

$$n_{t+1} = n_t\left(1 + (\beta - 1)\left(1 - \frac{(1 + r\Delta t)S_t}{S_{t+1}}\right)\right). \tag{6}$$

Or equivalently,

$$\frac{\Delta n_t}{n_t} = (\beta - 1)\left(1 - \frac{(1 + r\Delta t)S_t}{S_{t+1}}\right). \tag{7}$$

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