

Lecture 11: Quantitative Option Strategies Volatility Statistical Arbitrage

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G63.2936.001

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Long/Short Volatility

4550	0000	Zero Call	0	21/10/05	9	36.93	36.93	36.942	0.012
4550	0000	Zero Call	0	21/10/06	9	36.93	36.93	36.942	0.012
4550	0000	05 Oct 38.00 (0000 JM)	39	21/10/07	1	0.55	0.6	0.591	0.004
4550	0000	05 Oct 40.00 (0000 JN)	40	21/10/08	1	0.19	0.2	0	-0.17
4550	0000	Zero Call	0	21/10/10	9	36.93	36.93	36.947	0.01
4550	0000	05 Nov 37.00 (0000 JO)	37	21/10/11	1	0.7	0.75	2.707	0.36
4550	0000	05 Nov 38.00 (0000 JL)	38	21/10/11	1	1.55	1.6	1.96	0.36
4550	0000	Zero Call	0	21/10/12	9	36.93	36.93	36.947	0.01
4550	0000	05 Nov 40.00 (0000 KN)	40	21/10/13	1	0.45	0.5	0.367	-0.108
4550	0000	Zero Call	0	21/10/14	9	36.93	36.93	36.971	0.041
4550	0000	05 Dec 37.00 (0000 LK)	37	21/10/15	1	2.6	2.7	3.069	0.448
4550	0000	05 Dec 38.00 (0000 LL)	38	21/10/16	1	1.85	1.95	2.375	0.475
4550	0000	05 Dec 39.00 (0000 LM)	39	21/10/17	1	1.2	1.25	1.652	0.428
4550	0000	05 Dec 40.00 (0000 LN)	40	21/10/18	1	0.7	0.75	0.93	0.205
4550	0000	05 Dec 41.00 (0000 LO)	41	21/10/19	1	0.35	0.4	0.209	-0.167
4550	0000	Zero Call	0	21/10/20	9	36.93	36.93	36.973	0.043
4550	0000	06 Jan 36.525 (Y1Z A)	36.5	21/10/21	1	3.2	3.3	3.504	0.334
4550	0000	06 Jan 37.00 (0000 JH)	37	21/10/23	1	2.9	2.95	3.349	0.424
4550	0000	06 Jan 38.00 (0000 AL)	38	21/10/25	1	2.15	2.25	2.725	0.523
4550	0000	06 Jan 38.525 (Y1Z A)	38.5	21/10/26	1	1.75	1.8	2.332	0.557
4550	0000	06 Jan 39.00 (0000 AM)	39	21/10/27	1	1.4	1.45	1.937	0.522
4550	0000	06 Jan 39.525 (Y1Z A)	39.5	21/10/28	1	1.2	1.25	1.708	0.481
4550	0000	06 Jan 40.00 (0000 AN)	40	21/10/29	1	1	1.05	1.471	0.448
4550	0000	06 Jan 40.525 (Y1Z A)	40.5	21/10/30	1	0.75	0.8	1.079	0.304
4550	0000	06 Jan 41.00 (0000 AO)	41	21/10/31	1	0.6	0.65	0.944	0.219
4550	0000	06 Jan 41.525 (Y1Z A)	41.5	21/11/01	1	0.4	0.5	0.453	0.005
4550	0000	06 Jan 42.00 (0000 AP)	42	21/11/02	1	0.35	0.4	0.219	-0.158
4550	0000	Zero Call	0	21/11/03	9	36.93	36.93	36.956	0.026
4550	0000	06 Mar 36.00 (0000 CJ)	36	21/11/04	1	4.1	4.3	4.415	0.215
4550	0000	06 Mar 37.00 (0000 CK)	37	21/11/05	1	3.3	3.5	3.985	0.168

- Both long and short single stock option contracts
- Use a rich/cheap criterion for volatility which is reliable
- Manage the portfolio so as to be market-neutral and vol-neutral

The theory...

Factor analysis with ETFs and “stock proxies”

I_1, \dots, I_M

$(M \approx 15)$

Optionable indexes and ETFs

S_1, S_2, \dots, S_N

$(N \approx 500)$

Optionable stocks

$$\frac{dI_j}{I_j} = \sigma_j dW_j + v_j dt$$

Risk-neutral measure for ETFs

$$\frac{dS_i}{S_i} = \sum_{j=1}^M \beta_{ij} \frac{dI_j}{I_j} + \varepsilon_i$$

Historical regression of stocks
against ETFs or factors

Risk-Neutral Dynamics for Stock Prices

Use regression weights and the implied volatilities of the factors

$$\frac{dI_j}{I_j} = \sigma_j^I dW_j + v_j dt, \quad j = 1, \dots, M$$

$$\frac{dS_i}{S_i} = \underbrace{\sum_{j=1}^M \beta_{ij} \sigma_j^I dW_j}_{\text{Explained}} + \sigma_i^R dZ_i + \mu_i dt = \sigma_i d\bar{W}_i + \mu_i dt$$

$$\sigma_i^2 = \sum_{jk} \beta_{ij} \beta_{ik} \sigma_j^I \sigma_k^I \rho_{jk}^I + (\sigma_i^R)^2 \equiv (\sigma_i^E)^2 + (\sigma_i^R)^2$$

Explained

Unexplained

Model the unexplained (residual) volatilities

$$\sigma_i^R = \zeta \sigma_i \quad \therefore 0 < \zeta < 1$$

Unexplained vol is a random fraction of the total vol

$$\sigma_i^2 = (\sigma_i^E)^2 + \zeta^2 \sigma_i^2$$

$$\sigma_i^2 = \frac{(\sigma_i^E)^2}{1 - \zeta^2} \quad \therefore \quad \sigma_i = \sigma_i^E \cdot e^{X_i} \quad \therefore \frac{\sigma_i}{\sigma_i^E} = e^{X_i}$$

$$\frac{d\sigma_i}{\sigma_i} = \frac{d\sigma_i^E}{\sigma_i^E} + dX_i$$

Percent changes in vol modeled as changes in explained vol (market) + idiosyncratic shocks

Ansatz for Implied Volatilities

- We postulate the same model at the level of the short-term implied volatilities (say 30 days)

$$\sigma_{\text{impl},i} = \sigma_{\text{impl},i}^E \cdot e^{X_i}$$

- Hypothesis: X_i follows a mean-reverting process

1-Factor Model

$$\frac{dI}{I} = \sigma_I dW$$

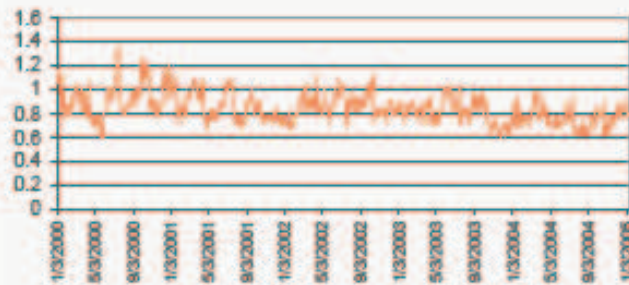
$$\frac{dS_i}{S_i} = \beta_i \sigma_I dW + \sigma_i^R dZ_i$$

$$\sigma_i = \beta_i \sigma_I e^{X_i}$$

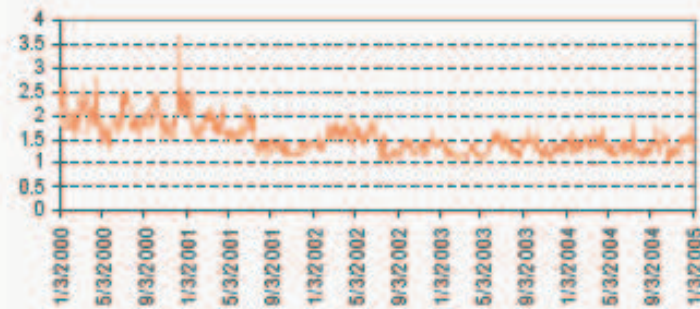
Test for mean-reversion of the ratio $\frac{\sigma_i}{\sigma_I}$

Implied Volatility Ratios: Strong mean reversion

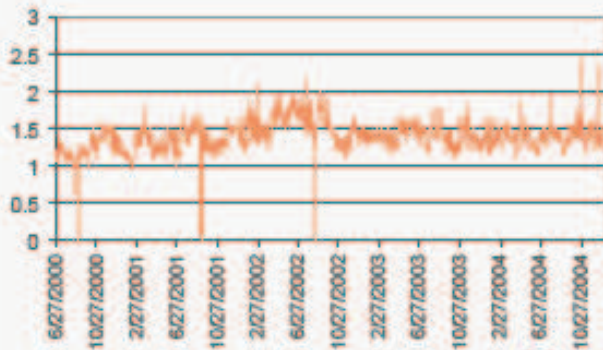
IBMVOL /MSHVOL



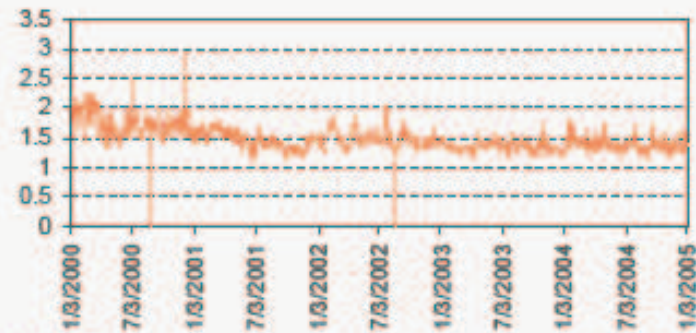
IBMVOL/OEXVOL



C_VOL/RKH_VOL



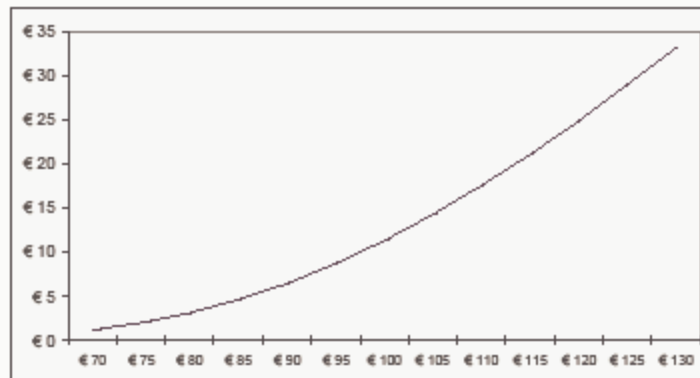
C_VOL/OEX_VOL



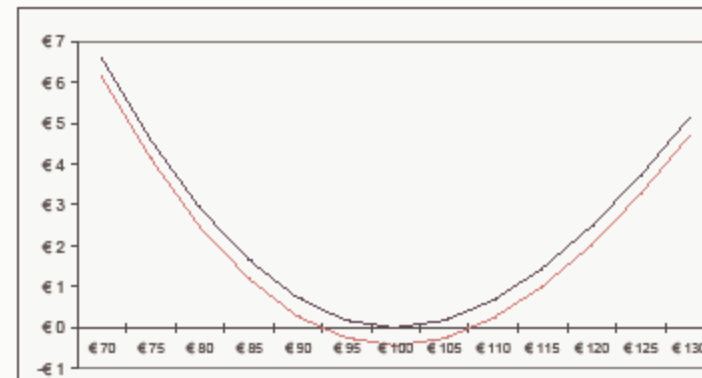
More on this later!

Risk/return in hedged option trading

Unhedged call option



Hedged option



Profit-loss for a hedged single option position (Black –Scholes)

$$P/L \approx -\theta \cdot (n^2 - 1) + NV \cdot \frac{d\sigma}{\sigma}$$

$$\theta = \text{time - decay (dollars)}, \quad n = \frac{\Delta S}{S\sigma\sqrt{\Delta t}}, \quad NV = \text{normalized Vega} = \sigma \frac{\partial C}{\partial \sigma}$$

$n \sim$ standardized move

NASDAQ-100 Component Stocks 2000-2002

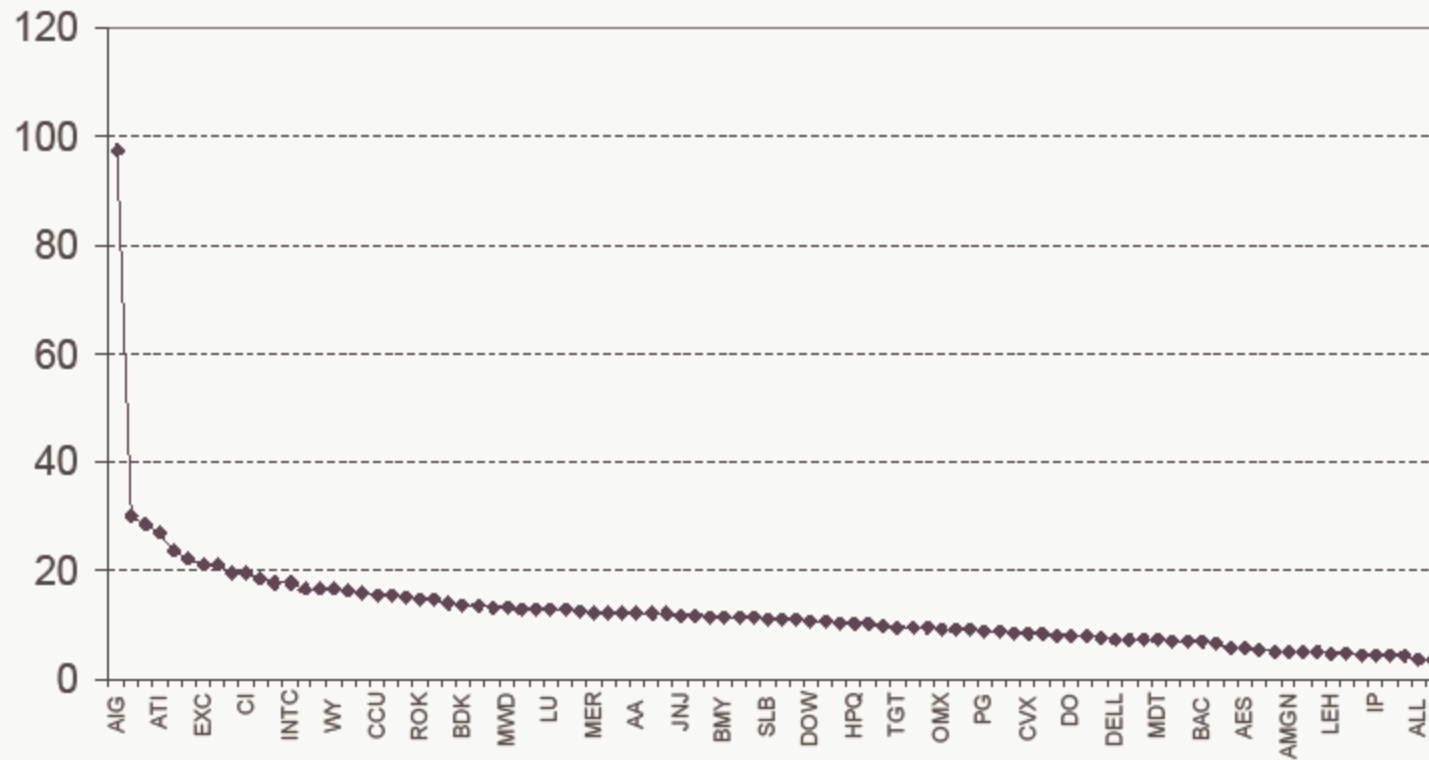
Mean reversion time (in days) :2000-2002						
industry	N	Mean	Median	Minimum	Maximum	Std Dev
Basic Mater	1	22.5742	22.5742	22.5742	22.5742	.
Health Care	14	25.398163	16.155747	7.4195302	73.968211	21.900874
Industry	1	23.792361	23.792361	23.792361	23.792361	.
Services	19	43.724024	34.101249	12.667757	151.63753	33.397578
Technology	49	28.164794	19.326816	6.2123701	289.88511	41.847069
consumer goods	3	26.500397	25.592741	10.428893	43.479557	16.544016
technology	1	47.70815	47.70815	47.70815	47.70815	.

NASDAQ-100 Stocks 2003-2004

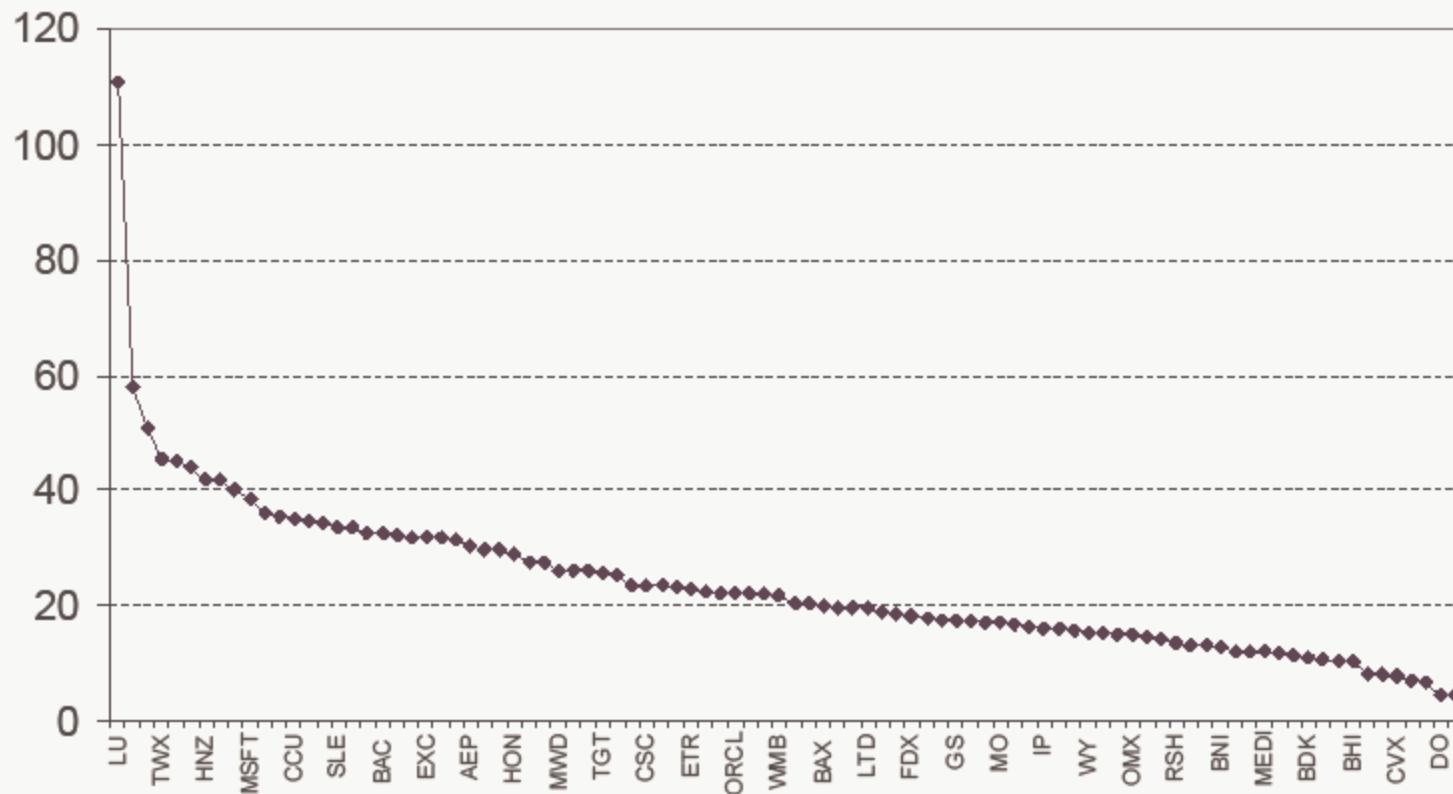
Mean reversion time (in days):2003-2004

Industry	N	Mean	Median	Minimum	Maximum	Std Dev
Basic Mater	1	43.33972	43.33972	43.33972	43.33972	.
Health Care	14	39.105986	16.121285	9.3171668	327.13404	83.178987
Industry	1	15.723726	15.723726	15.723726	15.723726	.
Services	22	33.464655	29.415571	3.5622151	91.695575	23.043199
Technology	49	33.131346	20.585755	3.9163219	322.94961	47.039319
consumer goods	3	112.48146	149.39033	30.747626	157.30641	70.89415
technology	1	28.01174	28.01174	28.01174	28.01174	.

OEX Components 2000-2002 Mean-reversion time in days



OEX Components: 2003-2004 Mean-reversion time in days



Weighted MC Approach

- Use a simple model for the dynamics of the single stock relative to its ETF
- Model the residual volatility as a fraction of the total implied (ATM) vol of the stock

$$\frac{dS_{it}}{S_{it}} = \beta_i \sigma_{I,atm} dW_t^{etf} + \gamma_i \sigma_{i,atm} dZ_t^S$$

$$\gamma_i = \sqrt{1 - R_i^2} \quad \text{in the sense of regression}$$

- Calibrate this to all options on ETF and to the forward for the stock under consideration, using Weighted Monte Carlo

Model Value vs. Market Value

$$C_{eur}(S, K, T) = e^{-rT} E^{WMC}(\max(S_T - K, 0))$$

Solve for IVOL

$$BSCall(S, T, K, r, d, \sigma_{imp}(K, T)) = C_{eur}(S, K, T)$$

$$C_{model}(S, K, T) = AmericanBSCall(S, T, K, r, d, \sigma_{imp}(K, T))$$

Compare :

$$C_{model}(S, K, T), [C_{bid}(S, K, T), C_{offer}(S, K, T)]$$

Long/Short Options Portfolio

$$\begin{aligned} P/L &= -\sum_i \theta_i (n_i^2 - 1) + \sum_i NV_i \frac{d\sigma_i}{\sigma_i} \\ &= -\sum_i \theta_i (n_i^2 - 1) + \left(\sum_i NV_i \right) \frac{d\sigma_I}{\sigma_I} + \sum_i NV_i dX_i \\ &= -\sum_i \theta_i (n_i^2 - 1) + \sum_i NV_i dX_i \end{aligned}$$

The last equation holds if we are "NV neutral". In particular, this holds if net Theta=0 and all options have the same expiration.

Theta-neutral portfolio: expected return

$$P/L = -\sum_i \theta_i (n_i^2 - 1) - 2 \cdot n_{\text{days}} \sum_i \theta_i dX_i$$

$$dX_i = \kappa_i (\bar{X}_i - X_i) dt + \sigma_{X_i} dz_i$$

O-U process

$$E\{P/L \mid X_1, \dots, X_N\} = -2 \cdot n_{\text{days}} \sum_i \theta_i \kappa_i (\bar{X}_i - X_i) dt$$

$$= -2 \cdot n_{\text{days}} \sum_i \theta_i \frac{(\bar{X}_i - X_i) dt}{n_i^{MR} dt}$$

$$= -2 \sum_i \theta_i \left(\frac{n_{\text{days}}}{n_i^{MR}} \right) (\bar{X}_i - X_i)$$

Positive for
the right choice
of theta

If $X_i < \text{equilibrium}$, buy option or sell theta. If $X_i > \text{equilibrium}$, sell option.

Theta-neutral portfolio: variance

$$P/L = -\sum_i \theta_i (n_i^2 - 1) - 2 \cdot n_{\text{days}} \sum_i \theta_i dX_i$$

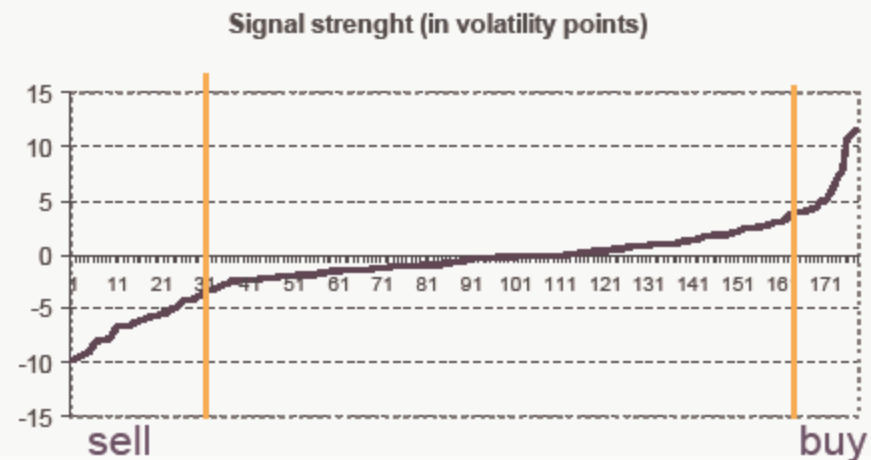
$$\begin{aligned} \text{Variance}\{P/L\} &= \text{Var}\left\{\sum_i \theta_i (n_i^2 - 1)\right\} + 4n_{\text{days}}^2 \text{Var}\left\{\sum_i \theta_i dX_i\right\} \\ &= 2\sum_{ij} \theta_i \theta_j \rho_{ij}^2 + 4Tn_{\text{days}} \sum_i \theta_i^2 \sigma_{X_i}^2 \end{aligned}$$

- Suggests a method for constructing efficient Theta-neutral portfolios with positive conditional expected return

Practical Implementation Long-Short on 200 Stocks (NDX+OEX)

Pricing on Dec 2005

- Price short-dated options on 200 large-capitalization stocks
- 1200 option contracts priced, 178 considered
- 40 have significant rich/cheap signals



Simulation of P/L for \$10MM 1% daily stdev

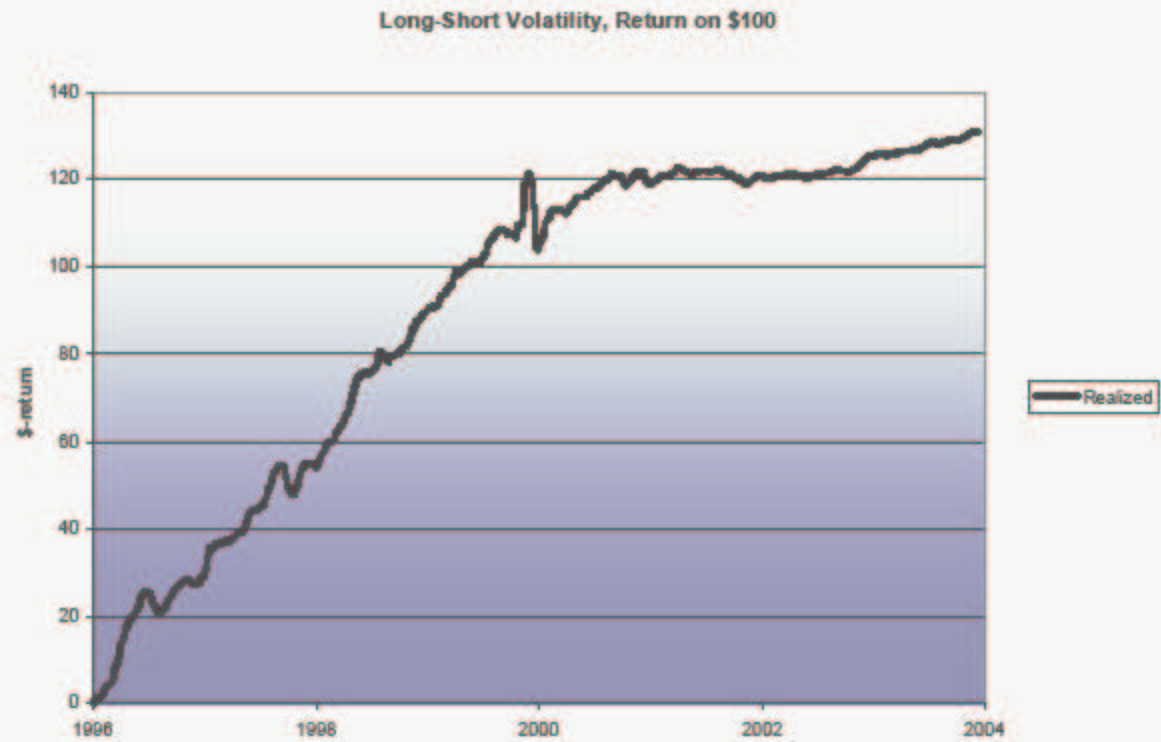
Year	1996	1997	1998	1999
PL	\$2,783,545	\$2,231,029	\$4,803,604	\$763,487
return (%)	28	22	48	8

Year	2000	2001	2002	2003
PL	\$1,892,811	\$540,803	\$1,829,187	\$2,560,862
return (%)	19	5	18	26

- Constant-VaR portfolio (1% stdev per day)
- Transaction costs in options/stock trading included

Back-Testing of Long-Short Strategy

Constant 1% Portfolio Standard Deviation

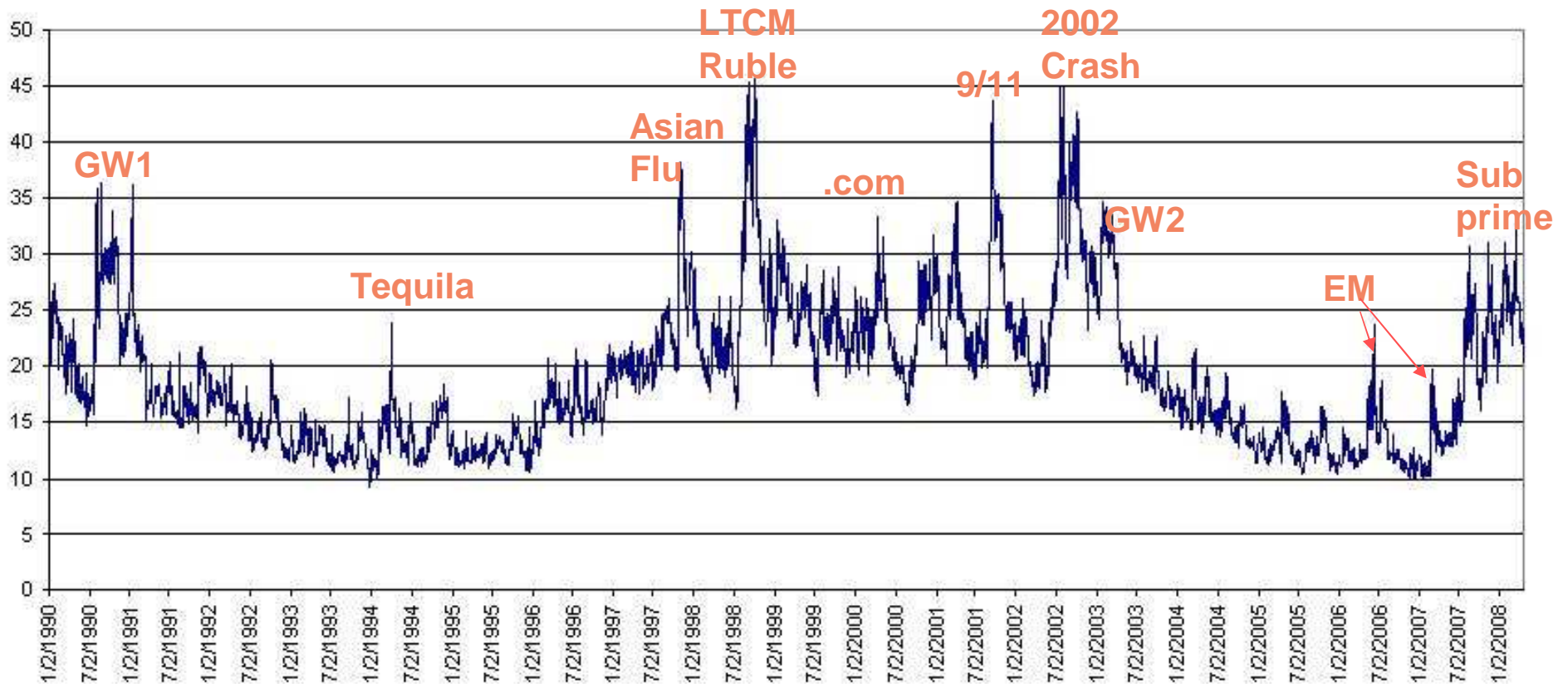


Portfolio Structure per 10MM invested in Long-Short Single Names

Year	2001	2002	2003
Av. # contracts/day	19,700	29,200	30,200
Decay collected /day	\$89,500	\$109,900	\$90,000
Decay paid/day	\$77,900	\$98,600	\$74,200
Net decay per day	\$11,600	\$11,300	\$15,800
Net Theta/ Gross Theta	6.93%	5.42%	9.62%

- 100 or more tickers
- Strategy is essentially Theta-neutral
- Excess Theta/Vega < 10%
- Per name, Theta < \$2000 dollars/day

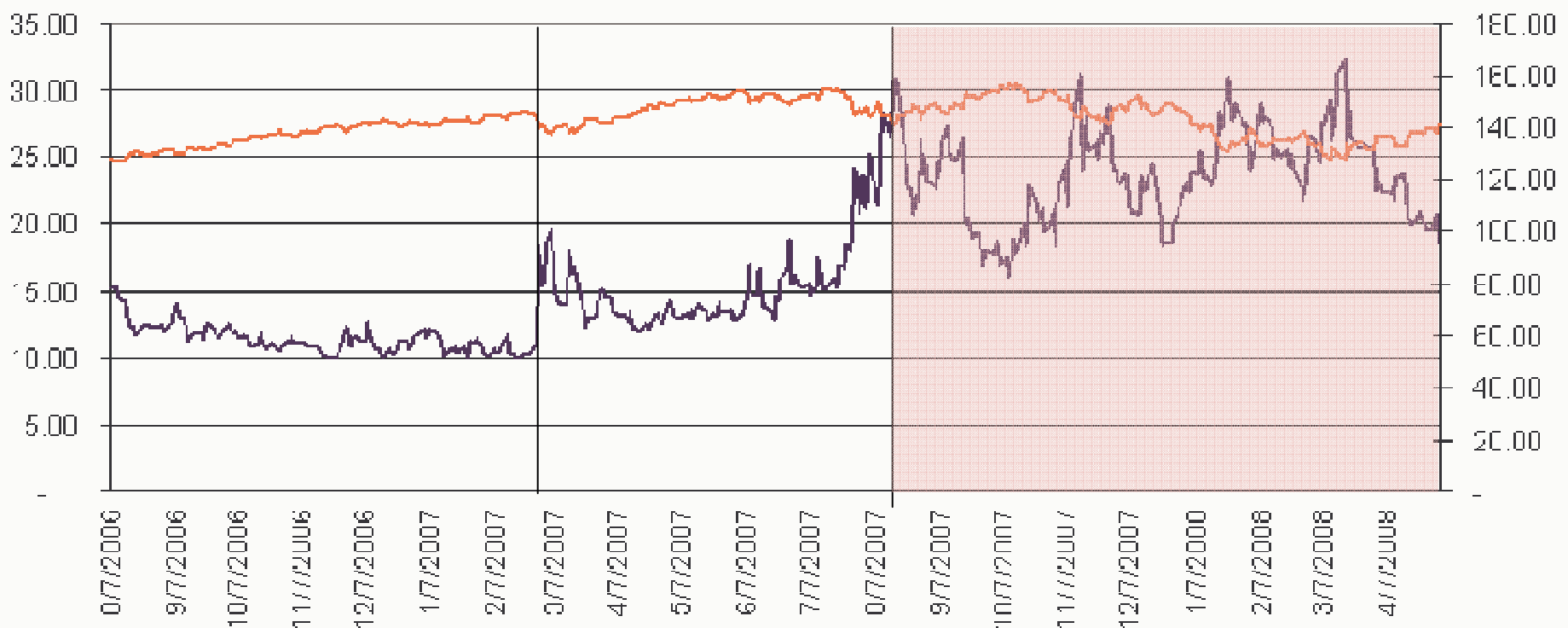
A Brief History of Volatility 1990 – 2008: the CBOE VIX Index



Volatility decreases from 2003 to 2006-2007

S&P 500 Volatility 8/2006-4/2008

— VIX — SPX



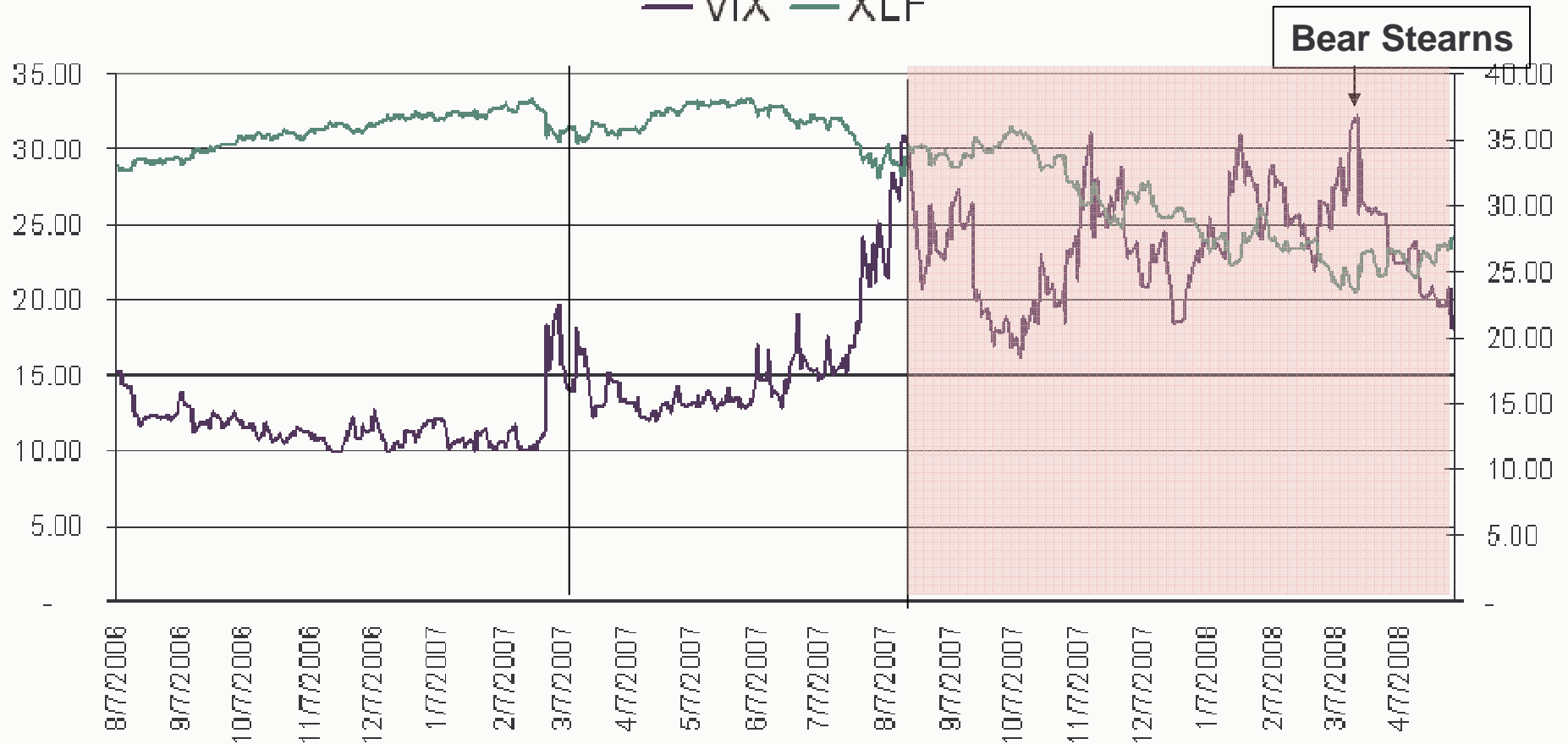
Volatility Continues to drop

Equity mkt. Ignores financials; Volatility doubles in at the end

Liquidity crunch Regime change in volatility

XLF (Financial S&P ETF) vs. VIX

— VIX — XLF



Volatility drops are synchronous to Fed moves

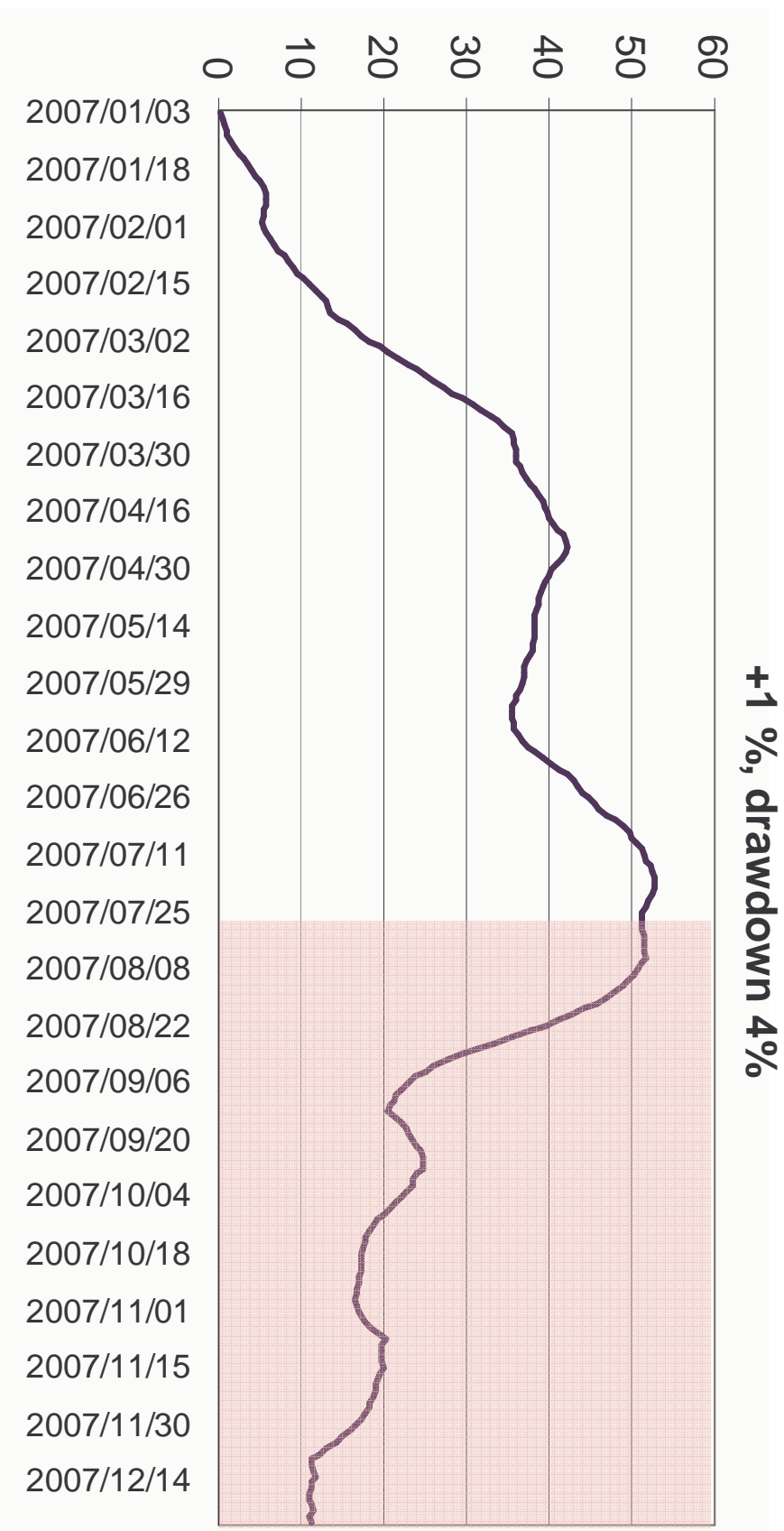
Implementation of L/S Strategy in 2007

- Few ``buy'' signals in early 2007
- Buying volatility gave poor results 2004-2006
- Theta-neutral portfolios gave poor results in 2004-2006
- Suggested **selling volatility** on strong signals **in small amounts** per name
- Hedging with ETFs (sector neutral)
- Hedging with SPY (``globally neutral'')
- Synthetic insurance company: selling protection (Gamma) on many single names
- Hedging is the key to maintain market neutrality and get desired results (as we will see)
- In back-testing we use **variance swaps** instead of options for simplicity

Market-Neutrality matching different Greeks

Portfolio profile	Vega Ratio	Exposure
Vega Neutral	$V_{etf} = V_{stock}$	collect time decay net long gamma
Theta Neutral	$V_{etf} = V_{stock} \cdot \frac{\sigma_{stock}}{\sigma_{etf}}$	net long vega net long gamma
Gamma Neutral	$V_{etf} = V_{stock} \cdot \frac{\sigma_{etf}}{\sigma_{stock}}$	collect time decay net long vega

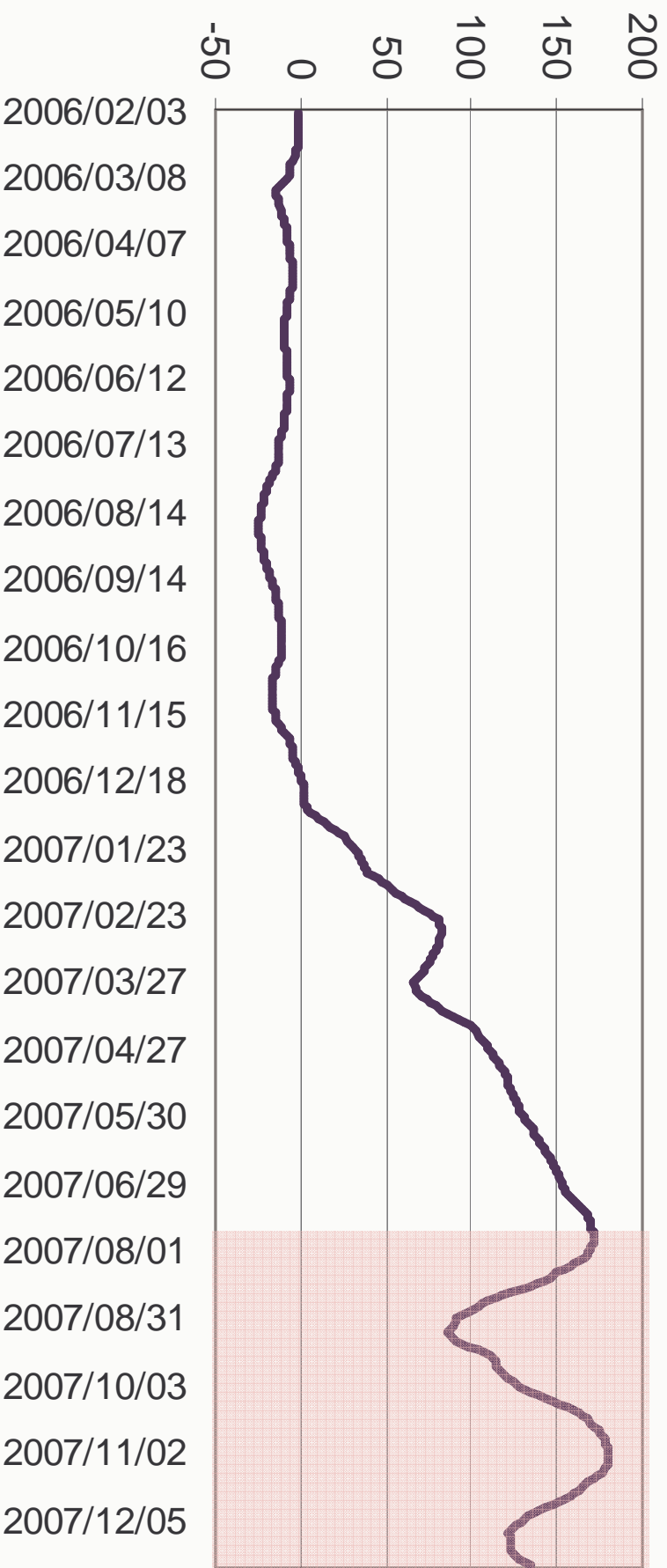
Vega-Neutral Per Sector 2007 Capital ~ \$1000 (all sectors)



Bias: long etf gamma, collect decay ("etf light")

Vega-Neutral 2006-2007 Capital ~ \$1000 (all sectors)

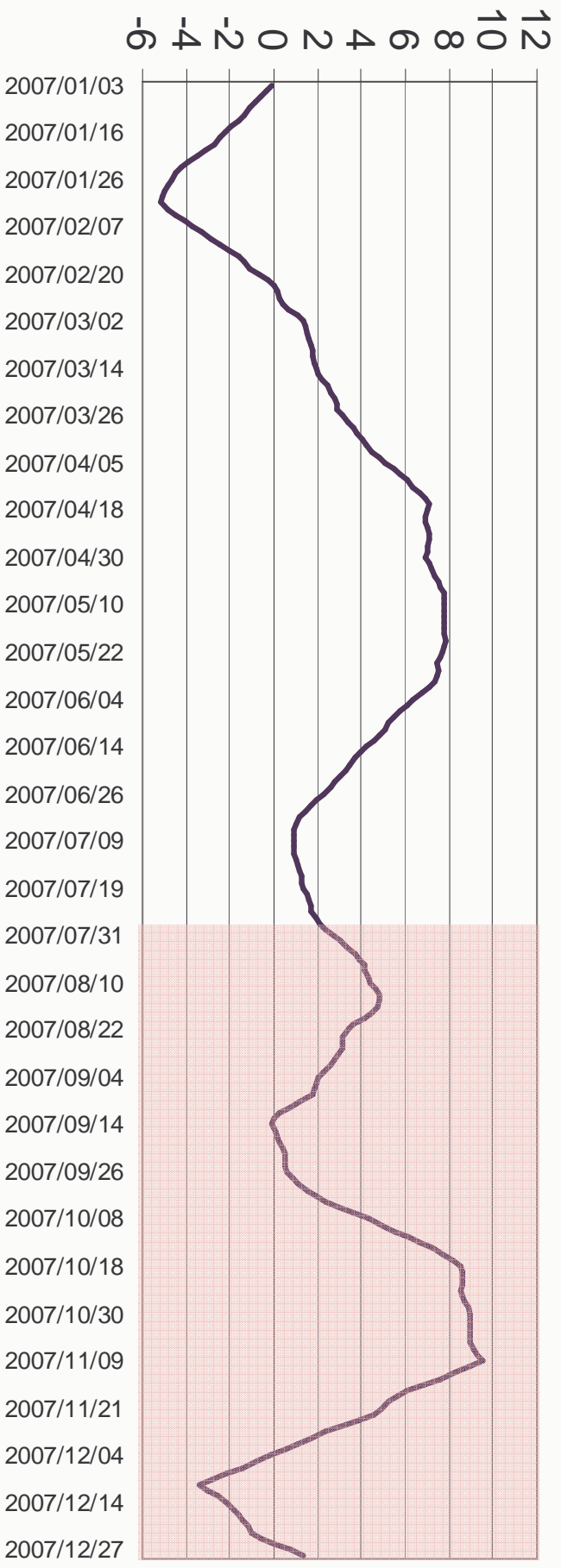
+ 13%, drawdown 8%



Bias: long etf gamma, collect decay ("etf light")

Vega-Neutral With SPY hedge

VEGA-NEUTRAL HEDGE with SPY
CAPITAL = \$1000

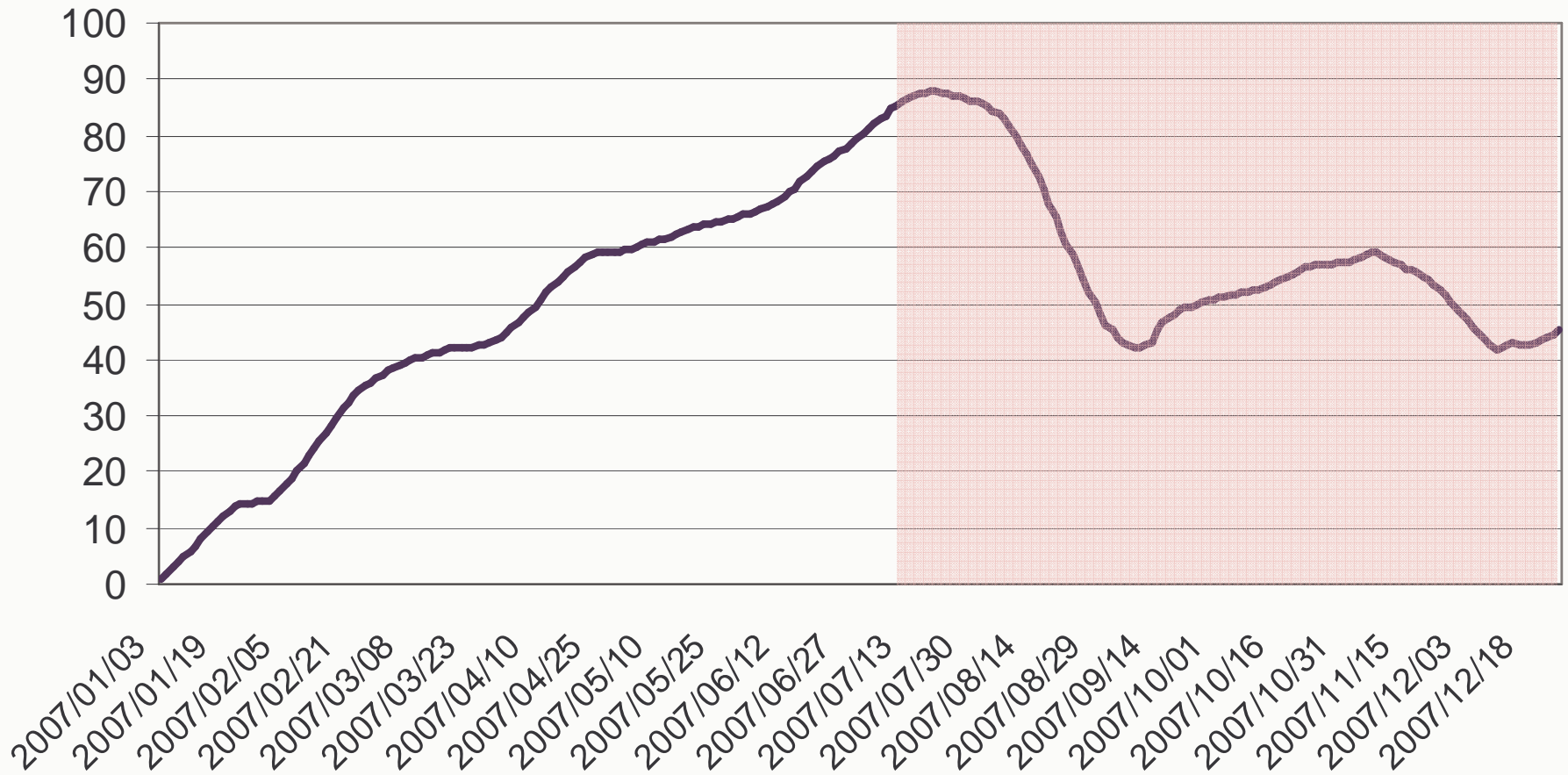


Global vega hedging does not work: too much decay, imprecise

Gamma-Neutral/Sector 2007

Capital=1000

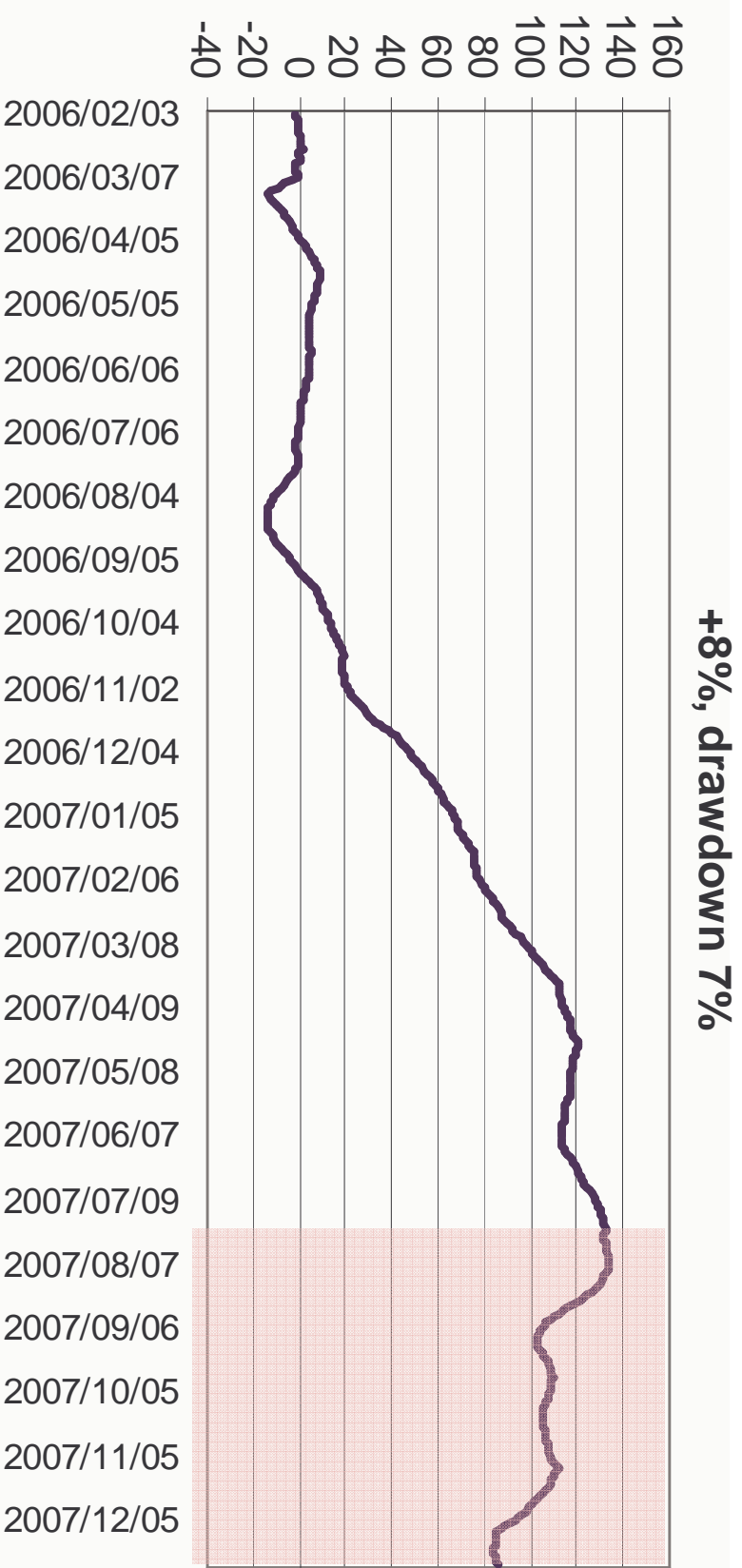
+ 4%, drawdown 5%



Bias: collect decay, long vega

Gamma-Neutral 2006-2007

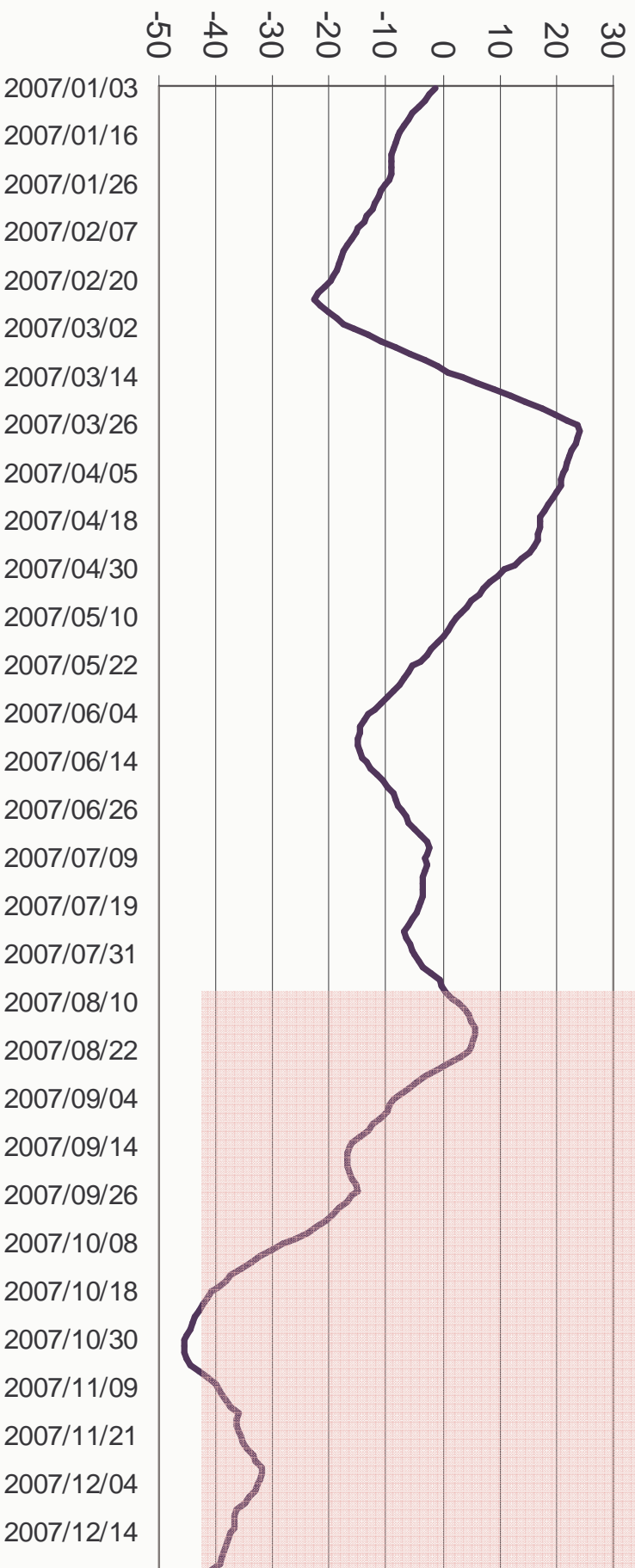
Capital=1000



Bias: collect decay, long vega

Theta-Neutral/Sector 2007 Capital=1000

-4%, drawdown 6%



Bias: long gamma, long vega

Not enough collected decay
and liquidity crunch