Lecture 11:
Quantitative Option Strategies
Volatility Statistical Arbitrage

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Long/Short Volatility

- Both long and short single stock option contracts
- Use a rich/cheap criterion for volatility which is reliable
- Manage the portfolio so as to be market-neutral and vol-neutral

The theory…

Factor analysis with ETFs and ``stock proxies''

$I_1, \ldots\ldots, I_M$ $(M \approx 15)$ Optionable indexes and ETFs

$S_1, S_2, \ldots, S_N$ $(N \approx 500)$ Optionable stocks

$$\frac{dI_j}{I_j} = \sigma_j dW_j + \nu_j dt$$

Risk-neutral measure for ETFs

$$\frac{dS_i}{S_i} = \sum_{j=1}^{M} \beta_{ij} \frac{dI_j}{I_j} + \varepsilon_i$$

Historical regression of stocks against ETFs or factors

Risk-Neutral Dynamics for Stock Prices

Use regression weights and the implied volatilities of the factors

$$\frac{dI_j}{I_j} = \sigma_j^I dW_j + \nu_j dt, \qquad j = 1,\ldots,M$$

$$\frac{dS_i}{S_i} = \sum_{j=1}^{M} \beta_{ij}\sigma_j^I dW_j + \sigma_i^R dZ_i + \mu_i dt = \sigma_i d\overline{W_i} + \mu_i dt$$

$$\sigma_i^2 = \sum_{jk} \beta_{ij}\beta_{ik}\sigma_j^I\sigma_k^I\rho_{jk}^I + \left(\sigma_i^R\right)^2 \equiv \left(\sigma_i^E\right)^2 + \left(\sigma_i^R\right)^2$$

Explained

Unexplained

Model the unexplained (residual) volatilities

$$\sigma_i^R = \varsigma\sigma_i \qquad \therefore \quad 0 < \varsigma < 1$$

Unexplained vol is a random fraction of the total vol

$$\sigma_i^2 = \left(\sigma_i^E\right)^2 + \varsigma^2\sigma_i^2$$

$$\sigma_i^2 = \frac{\left(\sigma_i^E\right)^2}{1-\varsigma^2} \qquad \therefore \qquad \sigma_i = \sigma_i^E \cdot e^{X_i} \qquad \therefore \frac{\sigma_i}{\sigma_i^E} = e^{X_i}$$

$$\frac{d\sigma_i}{\sigma_i} = \frac{d\sigma_i^E}{\sigma_i^E} + dX_i$$

Percent changes in vol modeled as changes in explained vol (market) + idiosyncratic shocks

Ansatz for Implied Volatilities

- We postulate the same model at the level of the short-term implied volatilities (say 30 days)

$$\sigma_{\mathrm{impl},\,i} = \sigma^{E}_{\mathrm{impl},\,i} \cdot e^{X_i}$$

- Hypothesis: X_i follows a mean-reverting process

1-Factor Model

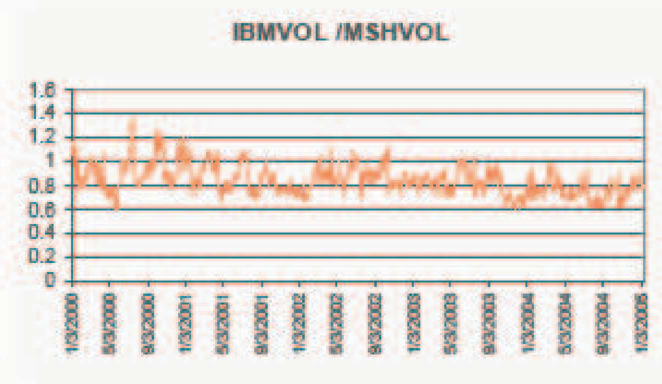
$$\frac{dI}{I} = \sigma_I dW$$

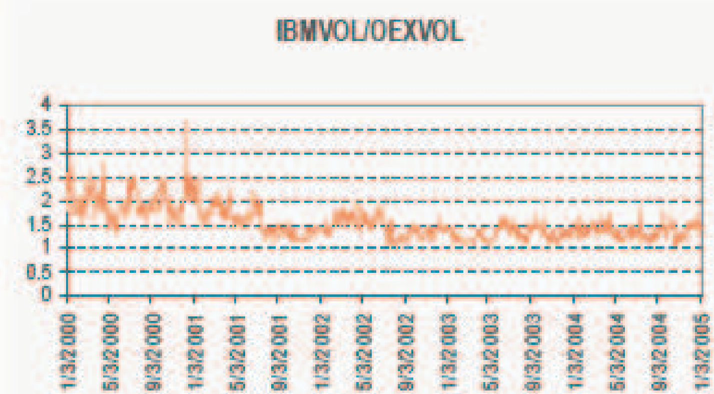
$$\frac{dS_i}{S_i} = \beta_i \sigma_I dW + \sigma_i^R dZ_i$$

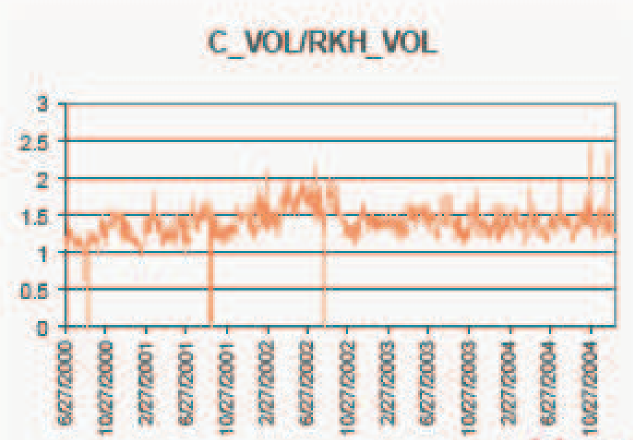
$$\sigma_i = \beta_i \sigma_I e^{X_i}$$

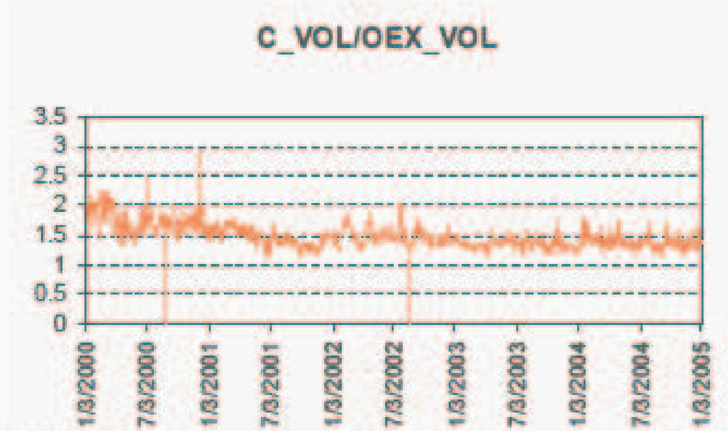
Test for mean-reversion of the ratio $\frac{\sigma_i}{\sigma_I}$

Implied Volatility Ratios: Strong mean reversion





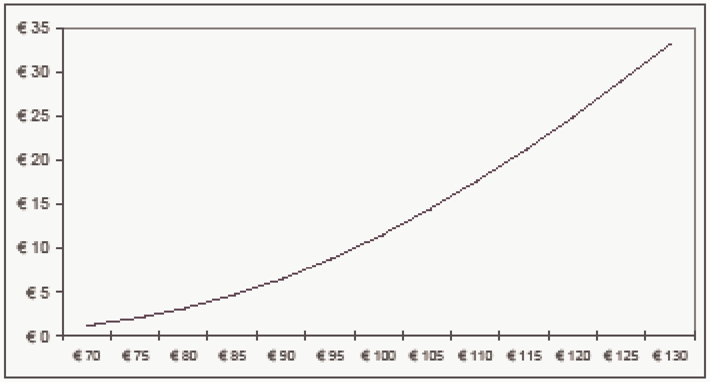




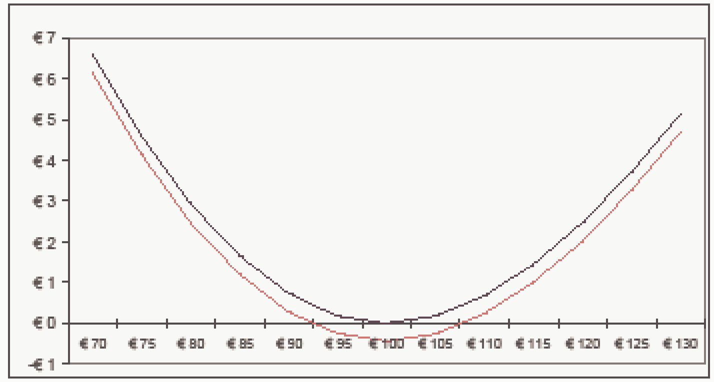
More on this later!

Risk/return in hedged option trading

Unhedged call option



Hedged option



Profit-loss for a hedged single option position (Black –Scholes)

$$P/L \approx -\theta \cdot \left(n^2 - 1\right) + NV \cdot \frac{d\sigma}{\sigma}$$

$$\theta = \text{time-decay (dollars)}, \qquad n = \frac{\Delta S}{S\sigma\sqrt{\Delta t}}, \qquad NV = \text{normalized Vega} = \sigma \frac{\partial C}{\partial \sigma}$$

n ~ standardized move

NASDAQ-100 Component Stocks 2000-2002

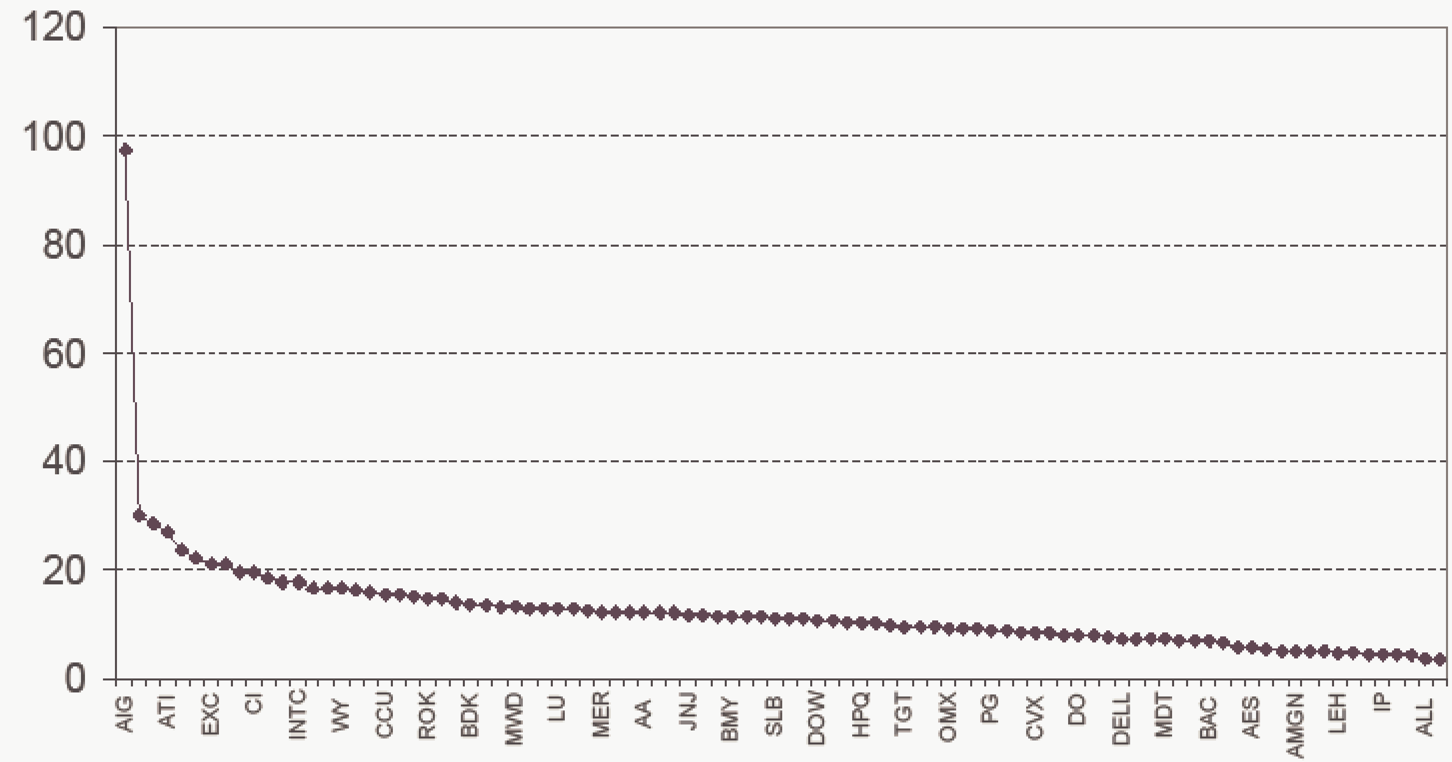
Mean reversion time (in days) :2000-2002						
industry	N	Mean	Median	Minimum	Maximum	Std Dev
Basic Mater	1	22.5742	22.5742	22.5742	22.5742	.
Health Care	14	25.398163	16.155747	7.4195302	73.968211	21.900874
Industry	1	23.792361	23.792361	23.792361	23.792361	.
Services	19	43.724024	34.101249	12.667757	151.63753	33.397578
Technology	49	28.164794	19.326816	6.2123701	289.88511	41.847069
consumer goods	3	26.500397	25.592741	10.428893	43.479557	16.544016
technology	1	47.70815	47.70815	47.70815	47.70815	.

NASDAQ-100 Stocks 2003-2004

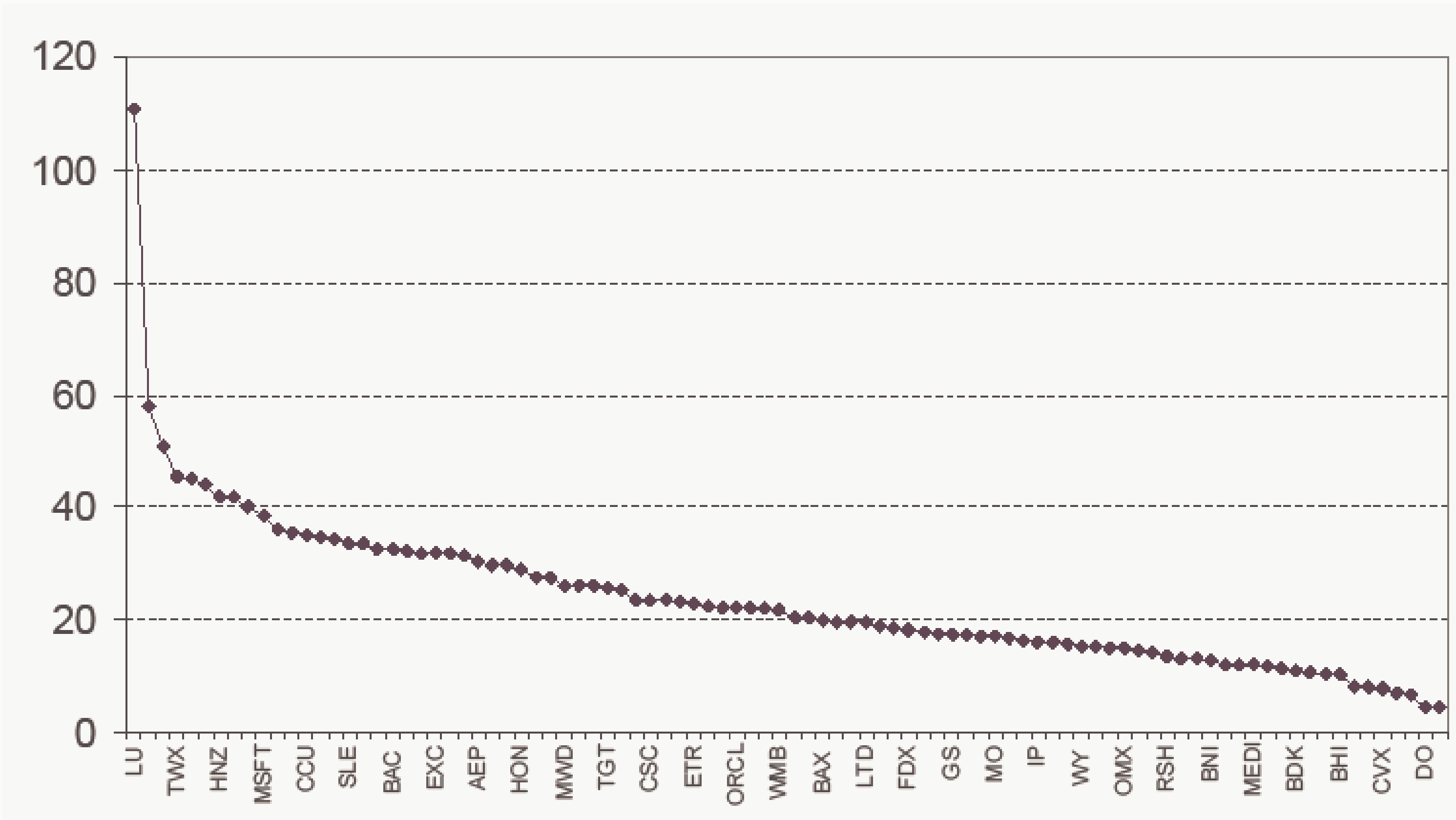
Mean reversion time (in days):2003-2004

Industry	N	Mean	Median	Minimum	Maximum	Std Dev
Basic Mater	1	43.33972	43.33972	43.33972	43.33972	.
Health Care	14	39.105986	16.121285	9.3171668	327.13404	83.178987
Industry	1	15.723726	15.723726	15.723726	15.723726	.
Services	22	33.464655	29.415571	3.5622151	91.695575	23.043199
Technology	49	33.131346	20.585755	3.9163219	322.94961	47.039319
consumer goods	3	112.48146	149.39033	30.747626	157.30641	70.89415
technology	1	28.01174	28.01174	28.01174	28.01174	.

OEX Components 2000-2002
Mean-reversion time in days



OEX Components: 2003-2004
Mean-reversion time in days



Weighted MC Approach

- Use a simple model for the dynamics of the single stock relative to its ETF
- Model the residual volatility as a fraction of the total implied (ATM) vol of the stock

$$\frac{dS_{it}}{S_{it}} = \beta_i \sigma_{I,atm} dW_t^{etf} + \gamma_i \sigma_{i,atm} dZ_t^S$$

$$\gamma_i = \sqrt{1 - R_i^2} \quad \text{in the sense of regression}$$

- Calibrate this to all options on ETF and to the forward for the stock under consideration, using Weighted Monte Carlo

Model Value vs. Market Value

$$C_{eur}(S,K,T) = e^{-rT} E^{WMC}\left(\max\left(S_T - K, 0\right)\right)$$

Solve for IVOL

$$BSCall\left(S,T,K,r,d,\sigma_{imp}(K,T)\right) = C_{eur}(S,K,T)$$

$$C_{\text{model}}(S,K,T) = AmericanBSCall\left(S,T,K,r,d,\sigma_{imp}(K,T)\right)$$

Compare:

$$C_{\text{model}}(S,K,T), \quad [\, C_{\text{bid}}(S,K,T), C_{\text{offer}}(S,K,T)]$$

Long/Short Options Portfolio

$$\text{P/L} = -\sum_i \theta_i \left(n_i^{\ 2} - 1\right) + \sum_i NV_i \frac{d\sigma_i}{\sigma_i}$$

$$= -\sum_i \theta_i \left(n_i^{\ 2} - 1\right) + \left(\sum_i NV_i\right) \frac{d\sigma_I}{\sigma_I} + \sum_i NV_i dX_i$$

$$= -\sum_i \theta_i \left(n_i^{\ 2} - 1\right) + \sum_i NV_i dX_i$$

The last equation holds if we are ``NV neutral''. In particular, this holds if net Theta=0 and all options have the same expiration.

Theta-neutral portfolio: expected return

$$\mathrm{P/L} = -\sum_i \theta_i \left(n_i^2 - 1\right) - 2 \cdot n_{\mathrm{days}} \sum_i \theta_i dX_i$$

$$dX_i = \kappa_i \left(\overline{X_i} - X_i\right) dt + \sigma_{X_i} dz_i$$

O-U process

$$E\{\mathrm{P/L} \mid X_1, \ldots, X_N\} = -2 \cdot n_{\mathrm{days}} \sum_i \theta_i \kappa_i \left(\overline{X_i} - X_i\right) dt$$

$$= -2 \cdot n_{\mathrm{days}} \sum_i \theta_i \frac{\left(\overline{X_i} - X_i\right) dt}{n_i^{MR} dt}$$

$$= -2 \sum_i \theta_i \left(\frac{n_{\mathrm{days}}}{n_i^{MR}} \right) \left(\overline{X_i} - X_i\right)$$

Positive for the right choice of theta

If X_i < equilibrium, buy option or sell theta. If X_i > equilibrium, sell option.

Theta-neutral portfolio: variance

$$\text{P/L} = -\sum_i \theta_i \left(n_i^2 - 1\right) - 2 \cdot n_{\text{days}} \sum_i \theta_i dX_i$$

$$Variance\{\text{P/L}\} = Var\left\{\sum_i \theta_i \left(n_i^2 - 1\right)\right\} + 4 n_{\text{days}}^2 Var\left\{\sum_i \theta_i dX_i\right\}$$

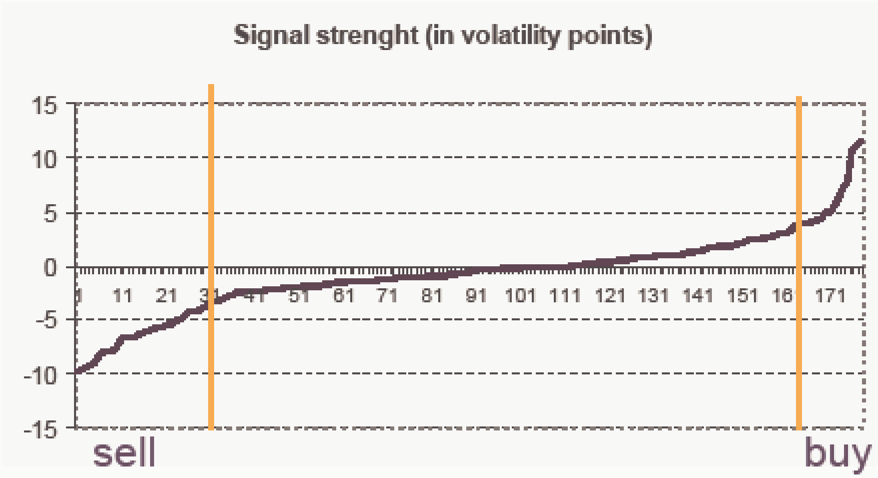
$$= 2 \sum_{ij} \theta_i \theta_j \rho_{ij}^2 + 4 T n_{\text{days}} \sum_i \theta_i^{\,2} \sigma_{X_i}^2$$

- Suggests a method for constructing efficient Theta-neutral portfolios with positive conditional expected return

Practical Implementation
Long-Short on 200 Stocks (NDX+OEX)

Pricing on Dec 2005

- Price short-dated options on 200 large-capitalization stocks
- 1200 option contracts priced, 178 considered
- 40 have significant rich/cheap signals



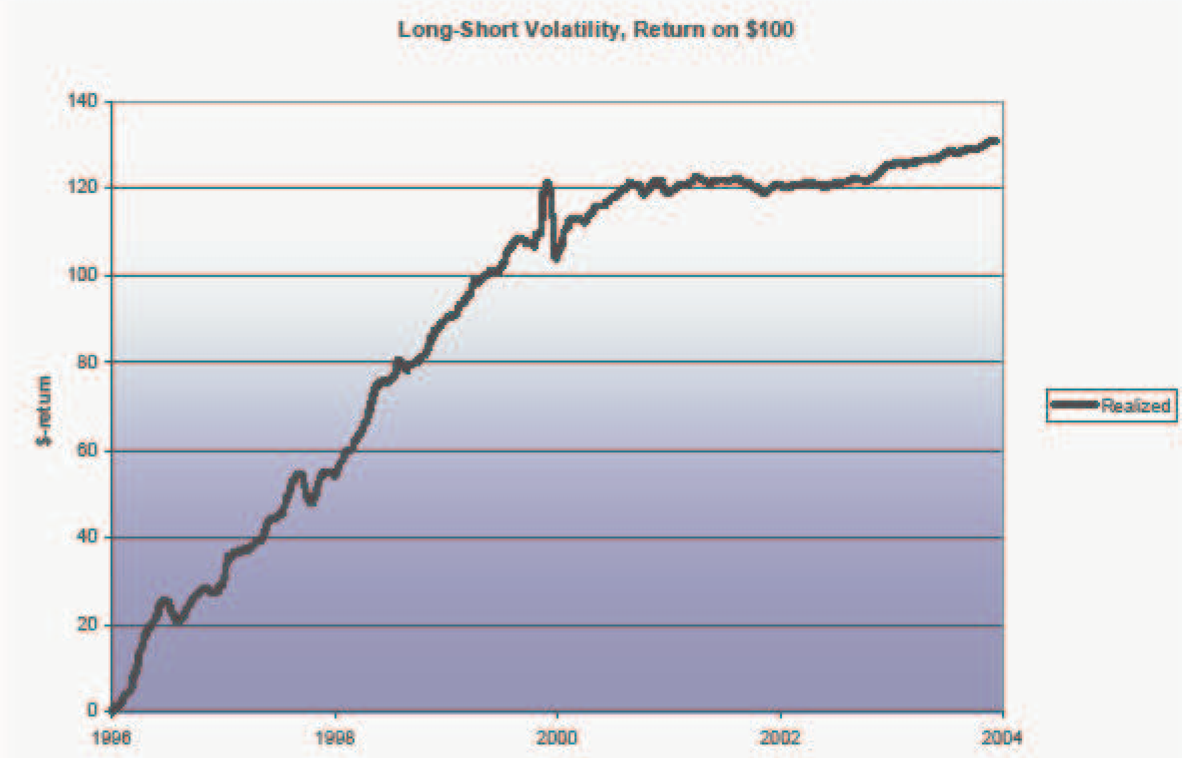
Simulation of P/L for $10MM 1% daily stdev

Year	1996	1997	1998	1999
PL	$2,783,545	$2,231,029	$4,803,604	$763,487
return (%)	28	22	48	8

Year	2000	2001	2002	2003
PL	$1,892,811	$540,803	$1,829,187	$2,560,862
return (%)	19	5	18	26

- Constant-VaR portfolio (1% stdev per day)
- Transaction costs in options/stock trading included

Back-Testing of Long-Short Strategy
Constant 1% Portfolio Standard Deviation

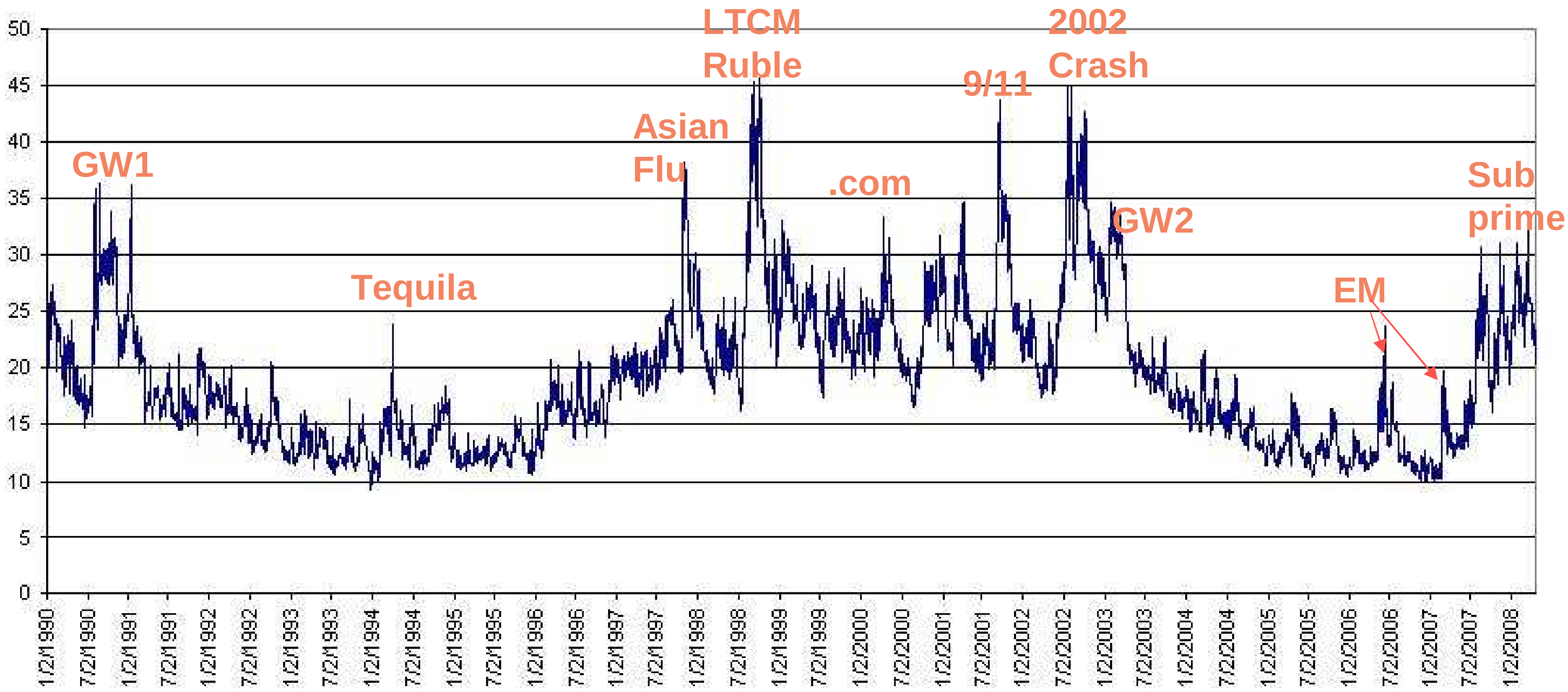


Portfolio Structure per 10MM invested in Long-Short Single Names

Year	2001	2002	2003
Av. # contracts/day	19,700	29,200	30,200
Decay collected /day	$89,500	$109,900	$90,000
Decay paid/day	$77,900	$98,600	$74,200
Net decay per day	$11,600	$11,300	$15,800
Net Theta/ Gross Theta	6.93%	5.42%	9.62%

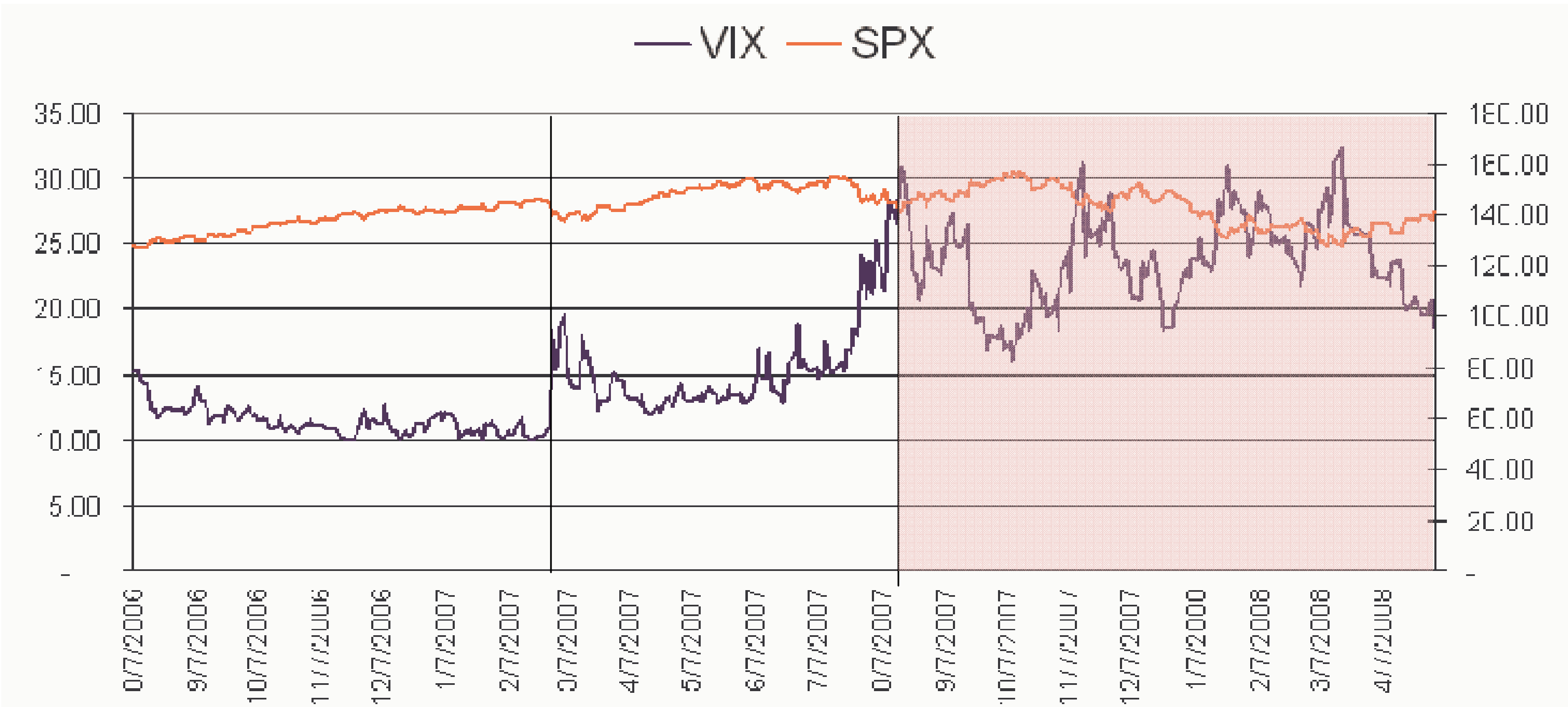
- 100 or more tickers
- Strategy is essentially Theta-neutral
- Excess Theta/Vega < 10%
- Per name, Theta < $2000 dollars/day

A Brief History of Volatility 1990 – 2008: the CBOE VIX Index



Volatility decreases from 2003 to 2006-2007

S&P 500 Volatility 8/2006-4/2008

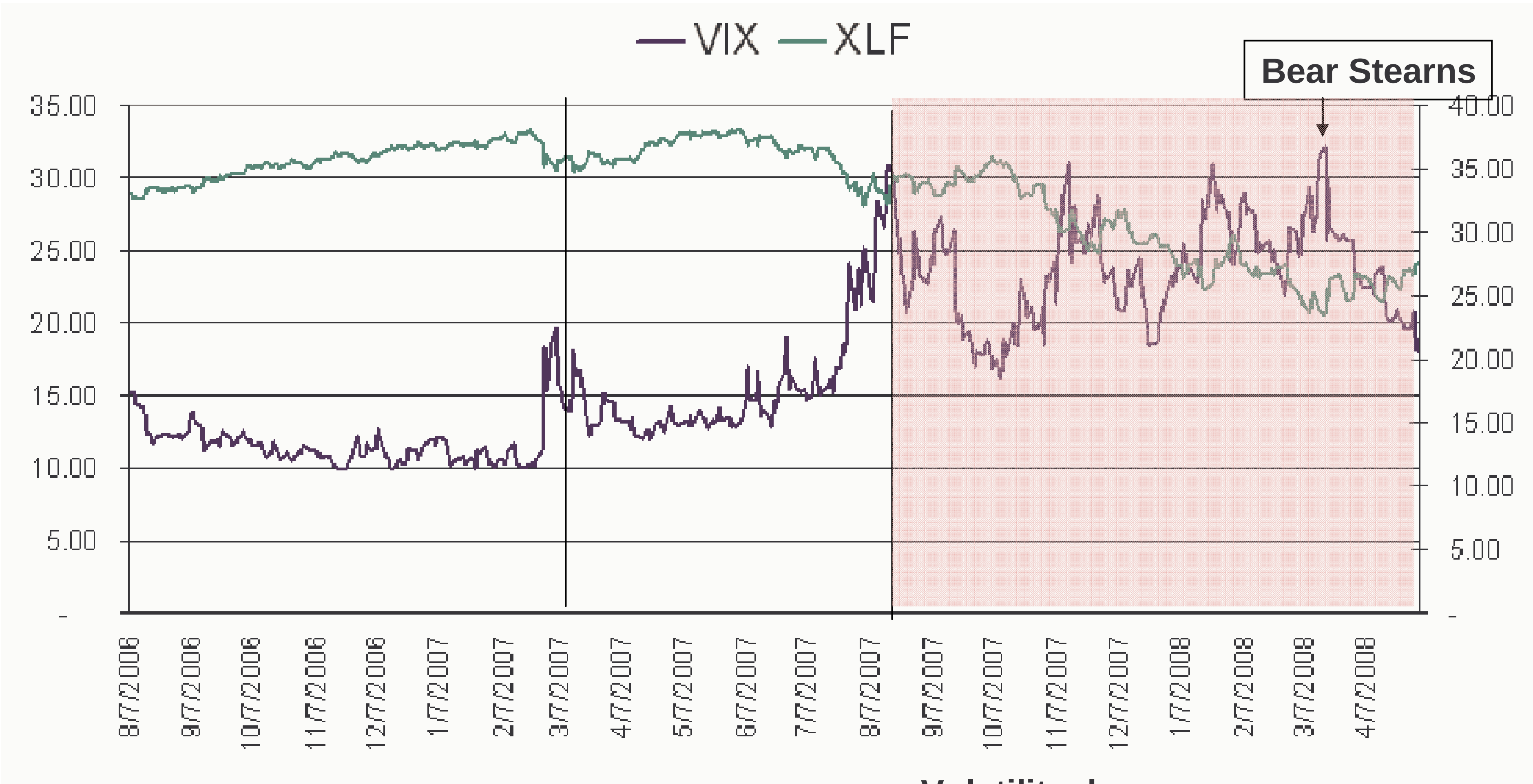


Volatility Continues to drop

Equity mkt. Ignores financials; Volatility doubles in at the end

Liquidity crunch Regime change in volatility

XLF (Financial S&P ETF) vs. VIX



Volatility drops are synchronous to Fed moves

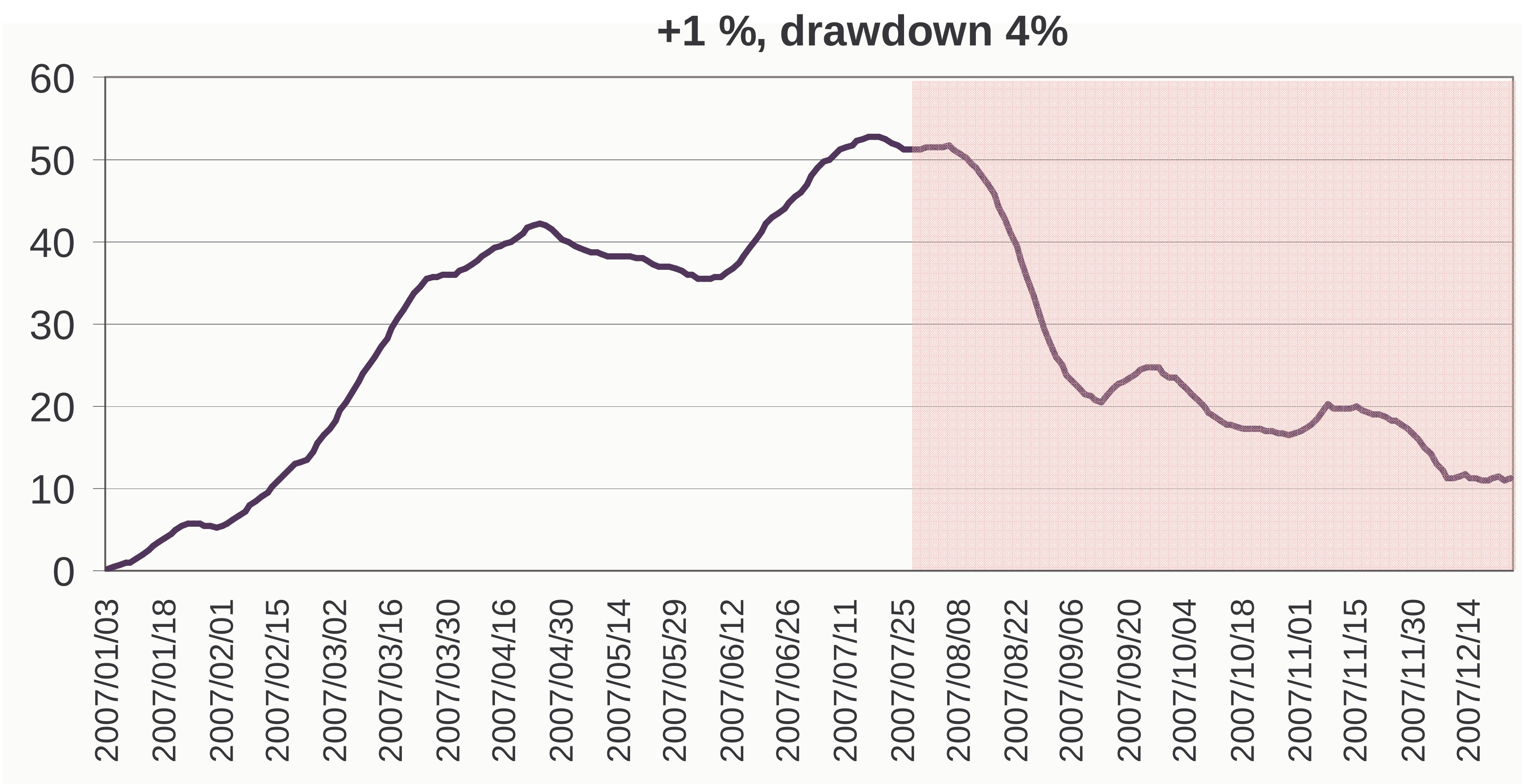
Implementation of L/S Strategy in 2007

- Few ``buy’’ signals in early 2007
- Buying volatility gave poor results 2004-2006
- Theta-neutral portfolios gave poor results in 2004-2006
- Suggested **selling volatility** on strong signals **in small amounts** per name
- Hedging with ETFs (sector neutral)
- Hedging with SPY (``globally neutral’’)
- Synthetic insurance company: selling protection (Gamma) on many single names
- Hedging is the key to maintain market neutrality and get desired results (as we will see)
- In back-testing we use **variance swaps** instead of options for simplicity

Market-Neutrality matching different Greeks

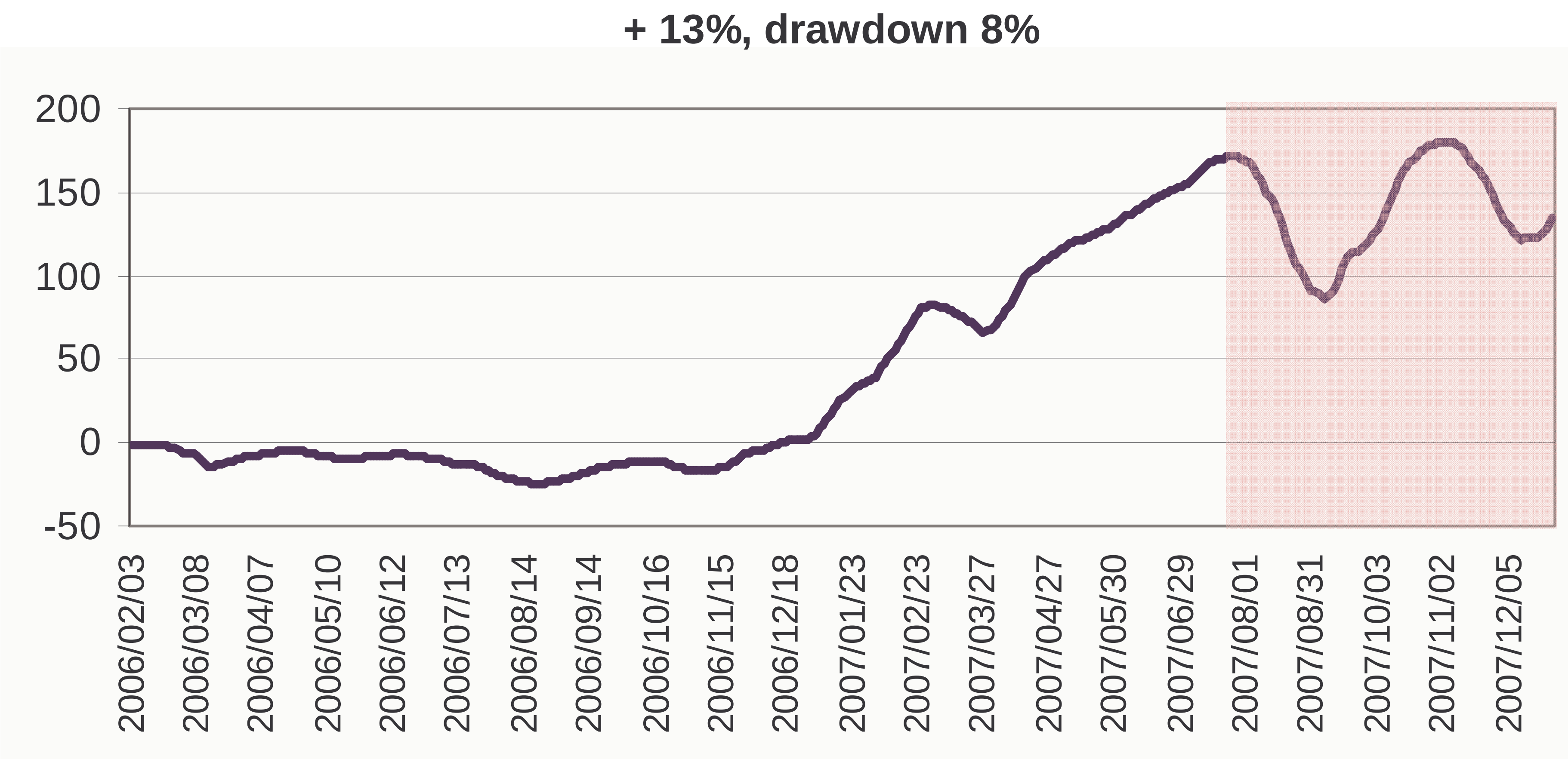
Portfolio profile	Vega Ratio	Exposure
Vega Neutral	$V_{etf} = V_{stock}$	**collect time decay** **net long gamma**
Theta Neutral	$V_{etf} = V_{stock} \cdot \frac{\sigma_{stock}}{\sigma_{etf}}$	**net long vega** **net long gamma**
Gamma Neutral	$V_{etf} = V_{stock} \cdot \frac{\sigma_{etf}}{\sigma_{stock}}$	**collect time decay** **net long vega**

Vega-Neutral Per Sector 2007
Capital ~ $1000 (all sectors)



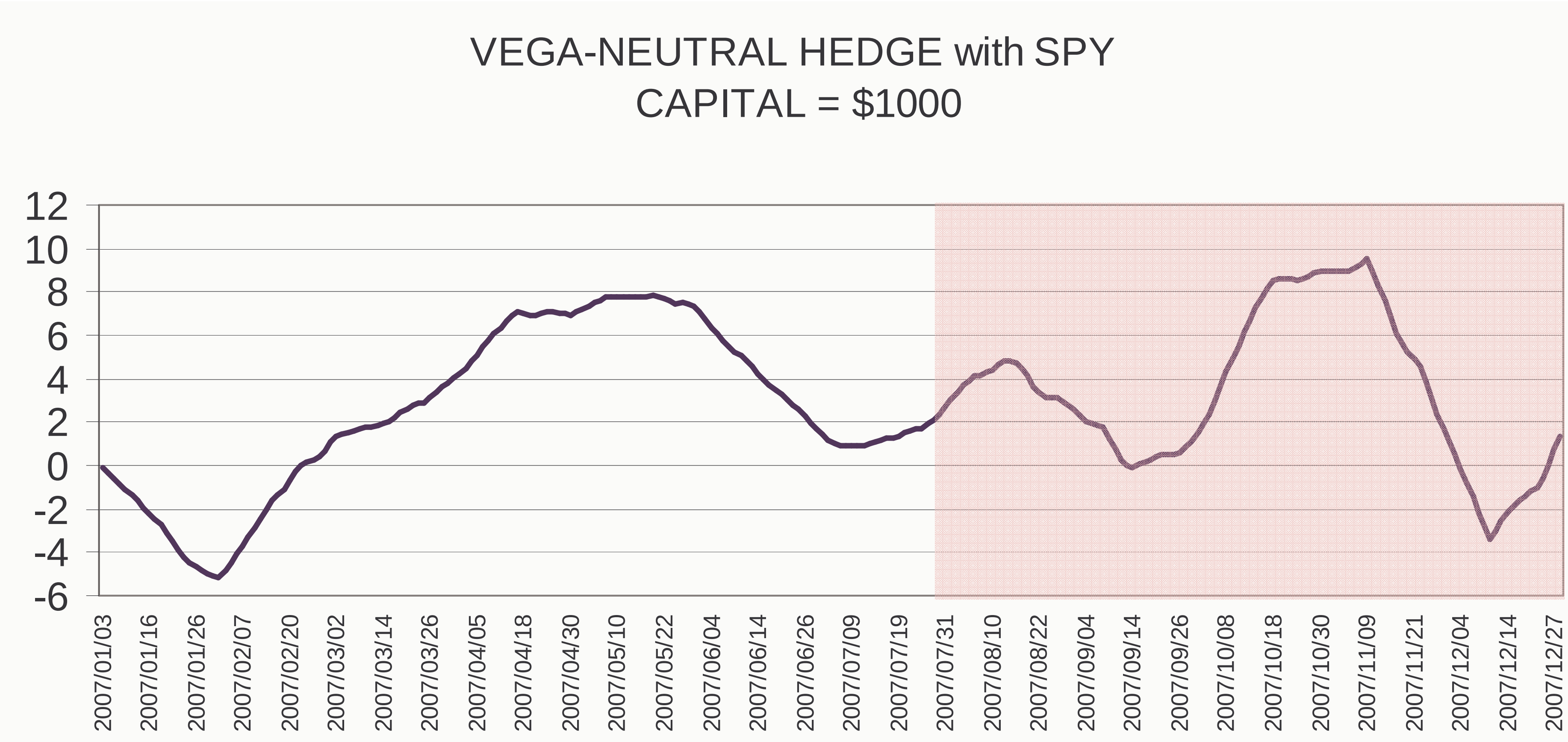
Bias: long etf gamma, collect decay (“etf light”)

Vega-Neutral 2006-2007
Capital ~ $1000 (all sectors)



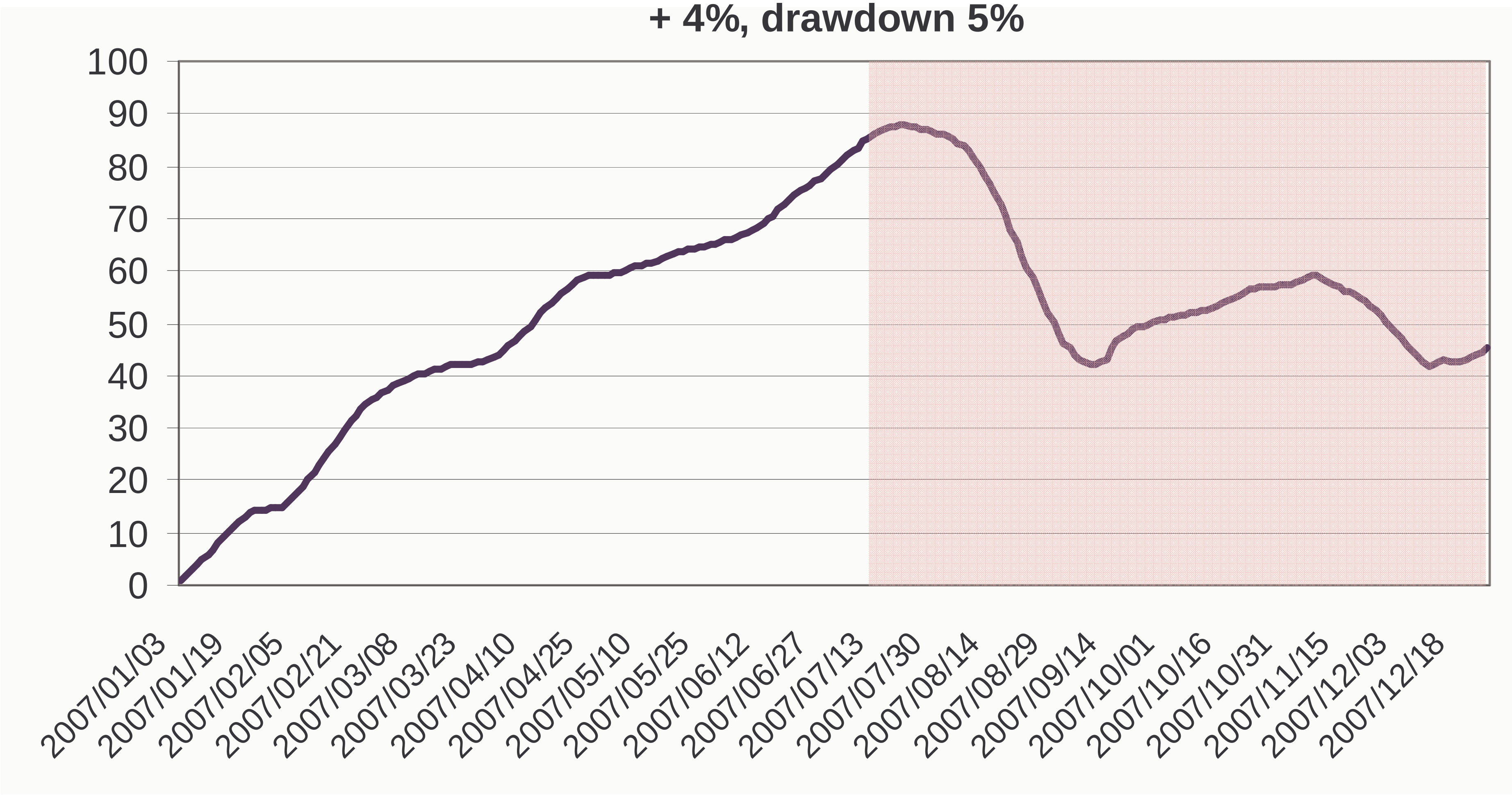
Bias: long etf gamma, collect decay ("etf light")

Vega-Neutral With SPY hedge



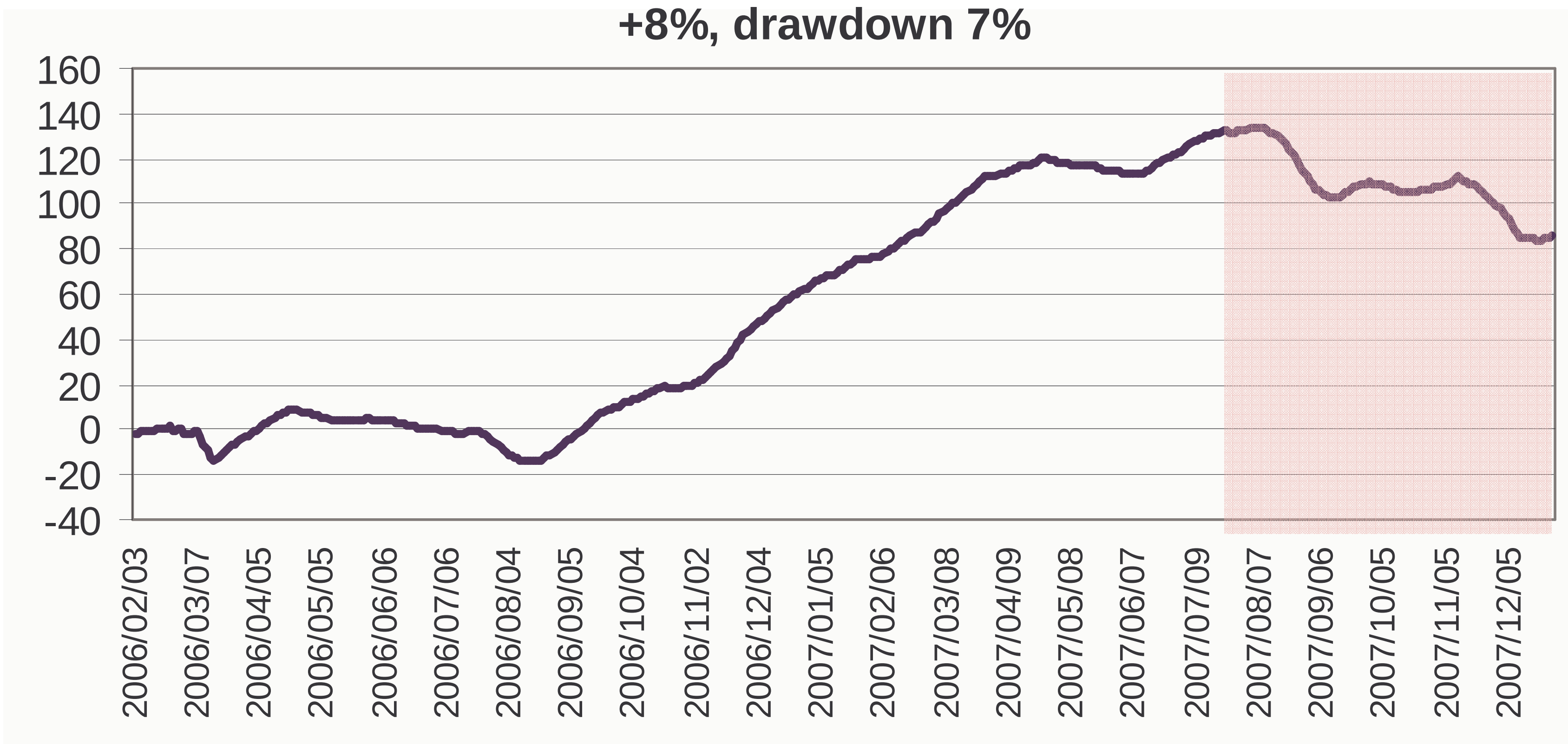
Global vega hedging does not work: too much decay, imprecise

Gamma-Neutral/Sector 2007
Capital=1000



Bias: collect decay, long vega

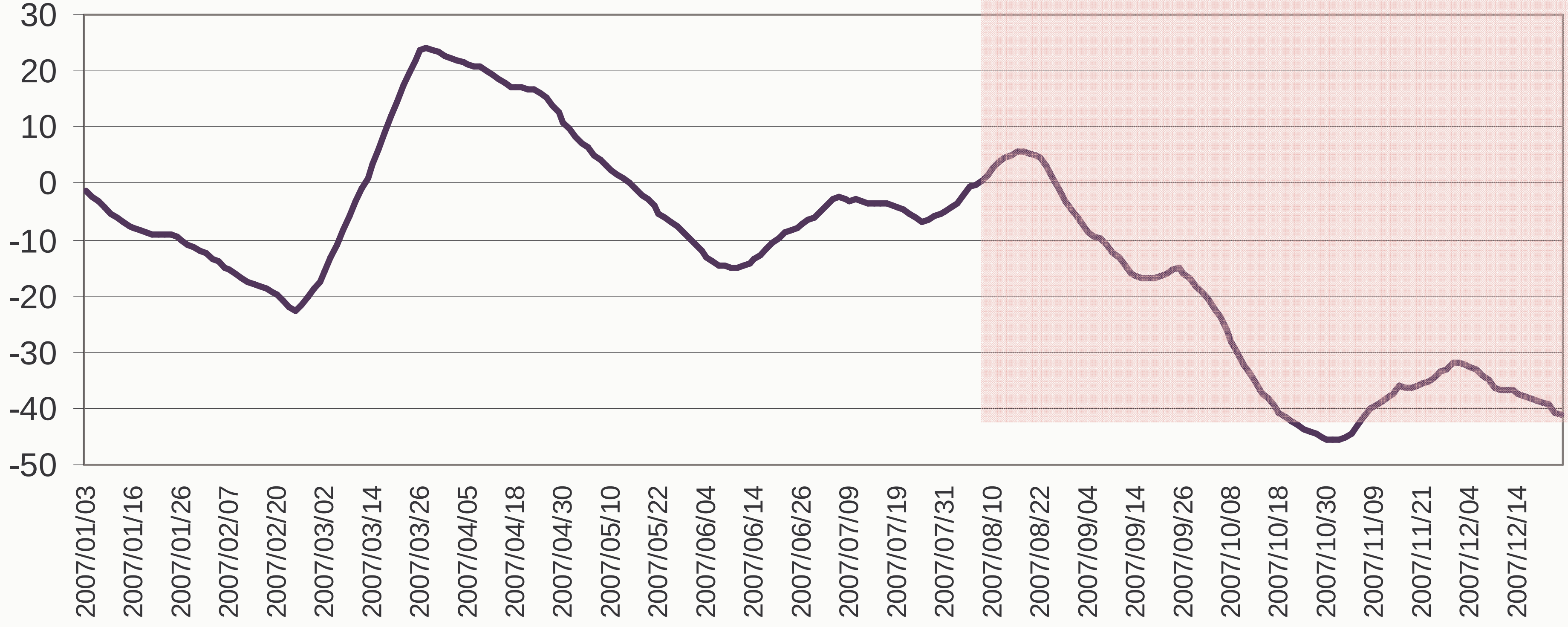
Gamma-Neutral 2006-2007 Capital=1000



Bias: collect decay, long vega

Theta-Neutral/Sector
2007 Capital=1000

-4%, drawdown 6%



Bias: long gamma, long vega

Not enough collected decay and liquidity crunch