Lecture 8:
Quantitative Option Strategies

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Equity Options Markets

- **Single-name options**

  Electronic trading in 6 exchanges, cross-listing of many stocks, penny-wide bid ask spreads for many contracts

- **Index Options**

  S&P 500, NDX, Minis. Traded on the Chicago Mercantile Exchange. VIX options & futures trade in CME as well.

- **ETF Options**

  Most of the large ETFs are optionable. Traded like stocks in multiple exchanges. SPY, QQQQ, XLF are among the most traded options in the US.

Options Markets
Halliburton (HAL) April 09

CALLS								PUTS						
Symbol	Last	Change	Bid	Ask	Volume	Open Int	Strike	Symbol	Last	Change	Bid	Ask	Volume	Open Int
HALDA.X	12.65	0	11.15	11.3	0	0	5	HALPA.X	0.03	0	N/A	0.04	100	210
HALDU.X	8.5	0	8.65	8.85	2	2	7.5	HALPU.X	0.05	0	0.01	0.06	1	2,237
HALDB.X	5.2	0	6.3	6.35	57	116	10	HALPB.X	0.15	0	0.1	0.12	25	3,775
HALDZ.X	4.2	0.15	4.05	4.15	20	944	12.5	HALPZ.X	0.4	0.12	0.39	0.4	185	10,482
HALDC.X	2.31	0.1	2.3	2.33	220	4,942	15	HALPC.X	1.06	0.33	1.09	1.11	52	10,592
HALDP.X	1.11	0.18	1.09	1.11	495	8,044	17.5	HALPP.X	2.42	0.34	2.36	2.37	196	8,482
HALDD.X	0.43	0.05	0.42	0.44	57	10,693	20	HALPD.X	4.59	0	4.15	4.25	250	12,440
HALDQ.X	0.15	0.02	0.14	0.16	23	7,646	22.5	HALPQ.X	7.25	0	6.4	6.45	25	2,770
HALDE.X	0.05	0.01	0.05	0.06	13	4,060	25	HALPE.X	9.95	0	8.8	8.85	4	1,111
HALDR.X	0.03	0	0.01	0.03	8	5,784	27.5	HALPR.X	12.35	0	11.25	11.35	18	977
HALDF.X	0.01	0	N/A	0.02	20	8,399	30	HALPF.X	14.8	0	13.7	13.9	18	5,772
HALDS.X	0.04	0	N/A	0.04	1	1,698	32.5	HALPS.X	15.5	0	16.2	16.4	20	150
HALDG.X	0.08	0	N/A	0.04	2	1,470	35	HALPG.X	18.93	0	18.7	18.9	5	514
HALDT.X	0.02	0	N/A	0.04	9	604	37.5	HALPT.X	20.59	0	21.2	21.35	40	151
HALDH.X	0.02	0	N/A	0.03	10	1,593	40	HALPH.X	20.6	0	23.7	23.85	10	139
HALDV.X	0.02	0	N/A	0.02	4	2,805	42.5	HALPV.X	26.1	0	26.2	26.4	752	311
HALDI.X	0.02	0	N/A	0.02	1	623	45	HALPI.X	28.6	0	28.7	29	152	0
HALDW.X	0.02	0	N/A	0.02	1	245	47.5	HALPW.X	31.1	0	31.2	31.4	52	13
HALDJ.X	0.02	0	N/A	0.02	7	733	50	HALPJ.X	24.55	0	33.7	33.9	0	0
HALDX.X	0.04	0	N/A	0.02	10	324	52.5	HALPX.X	14.8	0	36.2	36.4	0	0
HALDK.X	0.02	0	N/A	0.02	10	376	55	HALPK.X	19.1	0	38.7	39	0	

HAL= $16.36

Available expirations: Mar09, Apr09, Jul09, Oct09, Jan10, Jan11
2 front months, 2 LEAPS, quarterly cycle (*Jan cycle* for HAL).

Put-Call Parity

$$C - P = Se^{-dT} - Ke^{-rT}$$

Put-call parity holds for American options which are ATM, to within reasonable approximation.

CALLS				PUTS			(C-P+K*(1-r*40/252))/S	d_imp
HALDC.X	2.3	2.33	15	HALPC.X	1.09	1.11	0.988473167	7.26%
HALDP.X	1.09	1.11	17.5	HALPP.X	2.36	2.37	0.989451906	6.65%

Hal pays dividend of 9 cents at the end of Feb, May, Aug, Nov

There are no ex-dividend dates between now and April 20, 2009.

Option markets give an implied cost of carry for the stock (implied forward price), which may be different from the nominal cost of carry. This is due to stock-loan considerations.

DIA Options Apr 18, 2009

Symbol	Last	Change	Bid	Ask	Volume	OpenInt	STRIKE	Symbol	Last	Change	Bid	Ask	Volume	Open Int
DIHDX.X	N/A	0	18.1	18.2	0	0	50	DIHPX.X	0.37	0	0.15	0.19	18	245
DIHDY.X	21	0	17.3	17.4	2	2	51	DIHPY.X	0.39	0	0.17	0.22	105	370
DIHDZ.X	16.3	0	16.3	16.4	1	93	52	DIHPZ.X	0.26	0.22	0.23	0.26	7	225
DIHDA.X	N/A	0	15.45	15.55	0	0	53	DIHPA.X	0.32	0.26	0.28	0.31	5	68
DIHDB.X	N/A	0	14.25	14.35	0	0	54	DIHPB.X	0.4	0.24	0.34	0.37	4	392
DIHDC.X	11.94	0	13.45	13.55	4	14	55	DIHPC.X	0.42	0.38	0.41	0.44	25	765
DIHDD.X	12.35	0.17	12.55	12.65	40	22	56	DIHPD.X	0.51	0.46	0.49	0.52	20	870
DIHDE.X	10.3	0.47	11.6	11.75	10	48	57	DIHPE.X	0.61	0.53	0.59	0.62	72	414
DIHDF.X	8.6	0	10.75	10.85	2	202	58	DIHPF.X	0.73	0.53	0.71	0.73	32	689
DIHDG.X	8.4	0	9.85	9.95	33	211	59	DIHPG.X	0.86	0.54	0.83	0.87	18	658
DIHDH.X	8.4	1.35	9	9.1	48	206	60	DIHPH.X	1	0.75	1	1.02	165	11,734
DIJDI.X	7.7	1.22	8.15	8.3	1	162	61	DIJPI.X	1.21	0.75	1.17	1.2	61	510
DIJDJ.X	7.2	0.8	7.4	7.45	34	228	62	DIJPJ.X	1.43	0.9	1.38	1.4	41	916
DIJDK.X	6.7	1.65	6.6	6.7	137	282	63	DIJPK.X	1.65	0.94	1.61	1.63	108	1,347
DIJDL.X	6	1.6	5.9	5.95	60	444	64	DIJPL.X	1.93	1.03	1.89	1.91	305	1,138
DIJDM.X	5.25	1.41	5.2	5.25	102	825	65	DIJPM.X	2.27	1.18	2.19	2.21	583	1,735
DIJDN.X	4.55	1.32	4.5	4.6	69	1,142	66	DIJPN.X	2.64	1.21	2.52	2.56	213	1,919
DIJDO.X	3.96	1.25	3.9	4	134	945	67	DIJPO.X	3.05	1.4	2.91	2.95	450	2,115
DIJDP.X	3.4	1.08	3.35	3.4	343	1,788	68	DIJPP.X	3.46	1.44	3.3	3.4	217	2,505
DIJDQ.X	2.85	0.91	2.84	2.87	168	1,709	69	DIJPQ.X	3.8	1.85	3.8	3.9	116	1,688
DIJDR.X	2.41	0.82	2.37	2.4	399	9,896	70	DIJPR.X	4.54	1.61	4.35	4.4	144	2,829
DIJDS.X	1.92	0.64	1.94	1.98	117	1,465	71	DIJPS.X	5.14	1.86	4.9	5	51	3,035
DIJDT.X	1.58	0.58	1.57	1.6	262	1,998	72	DIJPT.X	5.6	2.2	5.55	5.65	7	2,528
DIJDU.X	1.27	0.5	1.25	1.29	215	1,924	73	DIJPU.X	6.28	2.37	6.2	6.35	22	1,580
DIJDV.X	1	0.4	0.99	1.02	235	1,761	74	DIJPV.X	7.1	2.05	6.95	7.05	2	1,253
DIJDW.X	0.78	0.3	0.77	0.79	182	3,421	75	DIJPW.X	7.8	2.28	7.75	7.85	29	1,292
DIJDX.X	0.6	0.16	0.58	0.61	26	2,652	76	DIJPX.X	10.3	0	8.55	8.65	29	1,008
DIJDY.X	0.44	0.14	0.44	0.47	27	2,055	77	DIJPY.X	9.5	2.36	9.4	9.5	5	943
DIJDZ.X	0.32	0.05	0.32	0.35	81	1,800	78	DIJPZ.X	10.65	0.75	10.3	10.4	4	1,290
DIJDA.X	0.26	0.09	0.24	0.26	140	1,147	79	DIJPA.X	11.83	1.37	11.2	11.3	3	1,006
DIJDB.X	0.19	0.08	0.17	0.2	48	8,568	80	DIJPB.X	13.57	1.29	12.15	12.25	3	1,352
DIJDC.X	0.11	0	0.12	0.15	9	3,494	81	DIJPC.X	15.13	0	13.1	13.2	26	5,989
DAVDD.X	0.1	0	0.09	0.12	92	2,455	82	DAVPD.X	16.6	0	14.3	14.45	10	1,184
DAVDE.X	0.07	0.01	0.06	0.09	3	3,218	83	DAVPE.X	16.44	1.22	15.3	15.4	1	1,016
DAVDF.X	0.05	0	0.05	0.08	23	1,470	84	DAVPF.X	16.85	1.28	16.3	16.4	3	843
DAVDG.X	0.04	0	0.03	0.07	11	4,203	85	DAVPG.X	17.2	1.55	17.3	17.4	30	496
DAVDH.X	0.02	0	0.02	0.06	3	841	86	DAVPH.X	17.7	0	18.25	18.4	1	91
DAVDI.X	0.04	0	N/A	0.05	10	617	87	DAVPI.X	21.78	0	19.25	19.35	3	305
DAVDJ.X	0.04	0	N/A	0.05	8	748	88	DAVPJ.X	19.5	0	20.25	20.35	10	124
DAVDK.X	0.04	0.01	N/A	0.04	30	450	89	DAVPK.X	15.9	0	21.25	21.35	15	56
DAVDL.X	0.04	0	N/A	0.04	30	927	90	DAVPL.X	16.95	0	22.2	22.35	5	58
DAVDM.X	0.03	0	N/A	0.04	4	787	91	DAVPM.X	17.5	0	23.2	23.35	2	78

Implied Dividend Yield for DIA
April 18, 2009 Options

CALLS				PUTS			(C-P+K*(1-r*40/252))/S	d_imp
DIJDP.X	3.35	3.4	68	DIJPP.X	3.3	3.4	0.995267636	2.98%
DIJDQ.X	2.84	2.87	69	DIJPQ.X	3.8	3.9	0.994951292	3.18%

Dividend Yield from Yahoo.com= 3.30%

Actual payments are approx 15 cents / month ~ $1.80 ~ 2.60%

Step1 in understanding options markets: find the implied dividend from the market.

If the implied dividend is different from the nominal dividend then

-- check for HTB if $d_{imp} > d_{nom}$

-- check for dividend reductions if $d_{imp} < d_{nom}$

Calculation of d_{nom}, d_{imp}

$$d_{nom} = \frac{-1}{T} \ln\left(\frac{S - \sum_{i=1}^{n} D_i e^{-rT_i}}{S} \right)$$

Dividend payment dates

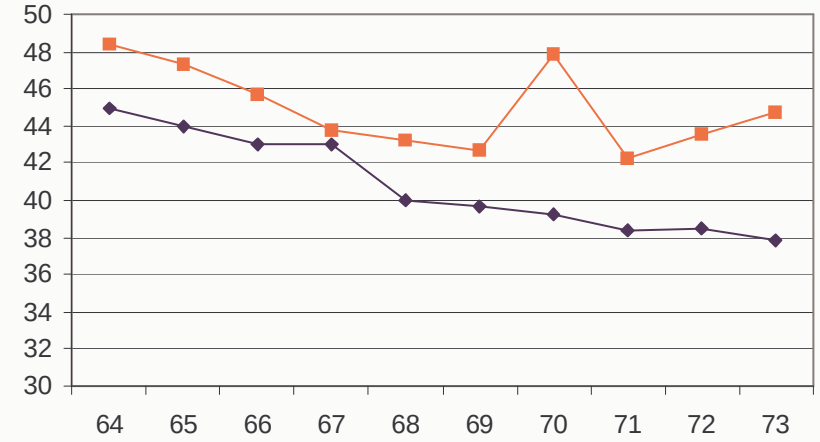
$$d_{imp} = \frac{-1}{T} \ln\left(\frac{C_{atm} - P_{atm} + K_{atm} e^{-rT}}{S} \right)$$

Implied Volatility
HAL April 09

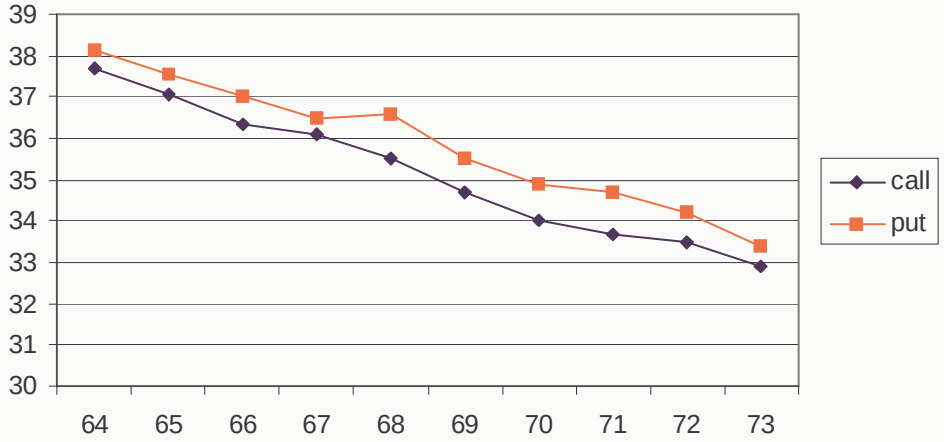
CALLS							PUTS					
Symbol	Last	Bid	Ask	IVOL	Delta	Strike	Symbol	Last	Bid	Ask	IVOL	Delta
HALDU.X	8.5	8.65	8.85	na	1.00	7.5	HALPU.X	0.05	0.01	0.06	211	0.00
HALDB.X	5.2	6.3	6.35	141	0.99	10	HALPB.X	0.15	0.1	0.12	144	-0.01
HALDZ.X	4.2	4.05	4.15	108	0.94	12.5	HALPZ.X	0.4	0.39	0.4	109	-0.05
HALDC.X	2.31	2.3	2.33	92.4	0.76	15	HALPC.X	1.06	1.09	1.11	93	-0.24
HALDP.X	1.11	1.09	1.11	85.1	0.36	17.5	HALPP.X	2.42	2.36	2.37	85	-0.63
HALDD.X	0.43	0.42	0.44	82.4	0.09	20	HALPD.X	4.59	4.15	4.25	84	-0.90
HALDQ.X	0.15	0.14	0.16	89.3	0.02	22.5	HALPQ.X	7.25	6.4	6.45	90	-0.97

DIA Volatility Surface, March 10 2009, 12:00 noon

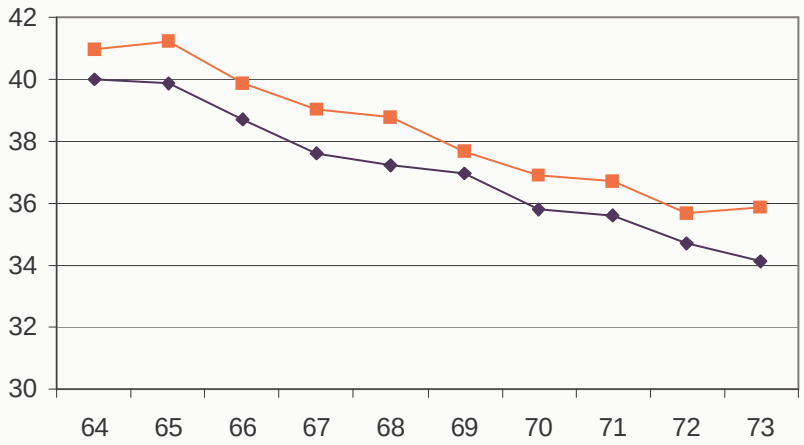
DIA, Mar09



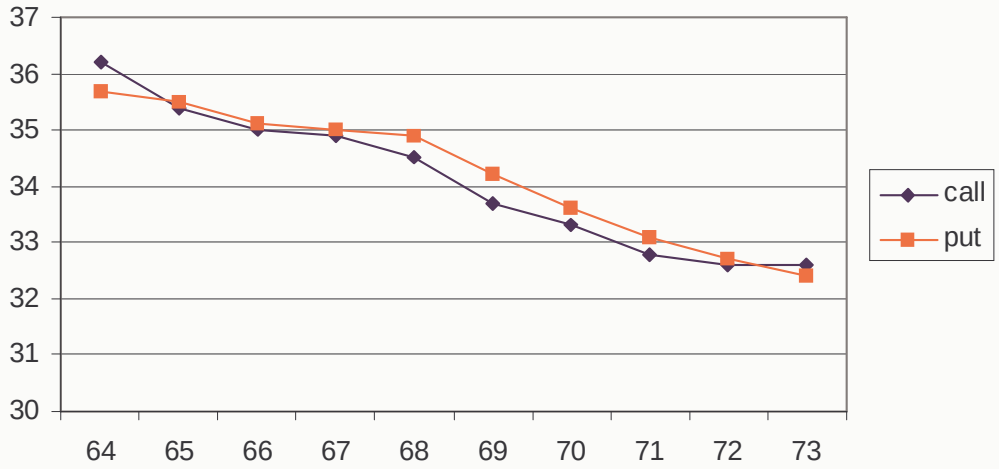
DIA, Jun 30, 09



DIA, Apr09



DIA, Sep 30, 09



These curves move and provide trading opportunities.

Many different trades possible

-- Carry trades using options (implied dividend vs. actual dividend, HTB)

-- Volatility surface trades (non-directional): trading different strikes on the same underlying asset

-- historical vol vs implied vol

-- Relative-value trades across names (non-directional)

    -- single-name option versus fair-value
    -- dispersion trading (index option versus components)

-- Directional volatility trades (long vol/ short vol, etc)

Skewness

-- For equities, the implied volatility curve is decreasing in the strike price around ATM

-- The effect is more pronounced for indices and etfs than for single names

Mechanics of option trading

-- Open position (long or short) and trade the stock so as to be delta-neutral.
-- Adjust the Delta of the option as the stock/option prices move

$$dC = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{\partial C}{\partial \sigma}d\sigma + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}dS^2 + \ldots$$

$$P\&L \approx dC - \Delta dS + \Delta S r dt - \Delta S d dt - rCdt$$

$$= \left(\frac{\partial C}{\partial S} - \Delta\right)dS + \frac{\partial C}{\partial \sigma}d\sigma + \frac{S^2}{2}\frac{\partial^2 C}{\partial S^2}\left(\frac{dS^2}{S^2} - \sigma^2 dt\right)$$

$$-\left(\frac{\partial C}{\partial S} - \Delta\right)S(r-d)dt$$

$$+\left(\frac{\partial C}{\partial t} + \frac{S^2\sigma^2}{2}\frac{\partial^2 C}{\partial S^2} + (r-d)S\frac{\partial C}{\partial S} - rC\right)dt$$

$$\approx \frac{\partial C}{\partial \sigma}d\sigma + \frac{S^2}{2}\frac{\partial^2 C}{\partial S^2}\left(\frac{dS^2}{S^2} - \sigma^2 dt\right)$$

Book-keeping: profit/loss from a delta-hedged option position

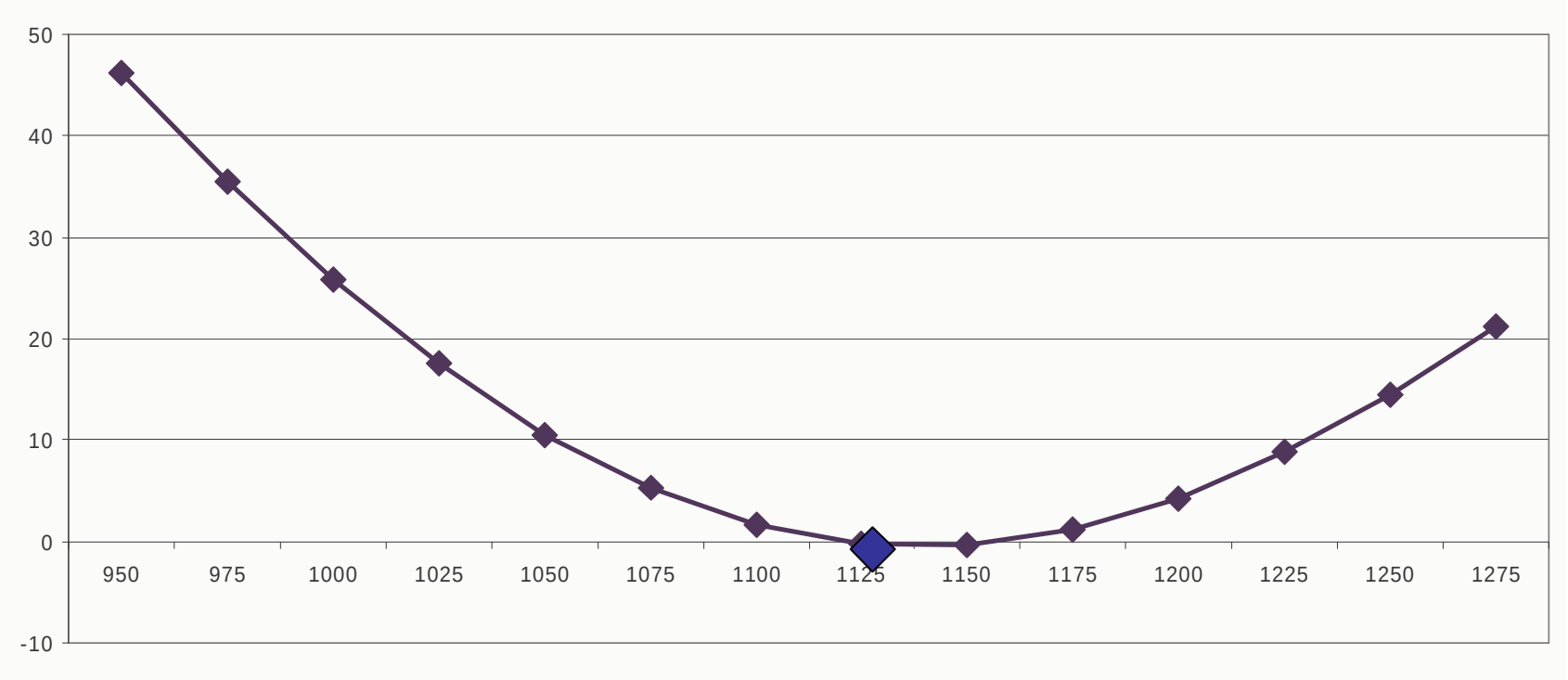
$$\mathrm{P/L} = \theta \cdot \left(n^2 - 1\right) + V \cdot d\sigma$$

or

$$\mathrm{P/L} = \frac{1}{2}\Gamma \cdot \left(\frac{\left(dI\right)^2}{I^2} - \sigma^2 dt \right) + V \cdot d\sigma$$

1-day P/L for Long Call/Short Stock

(Constant volatility=16%)

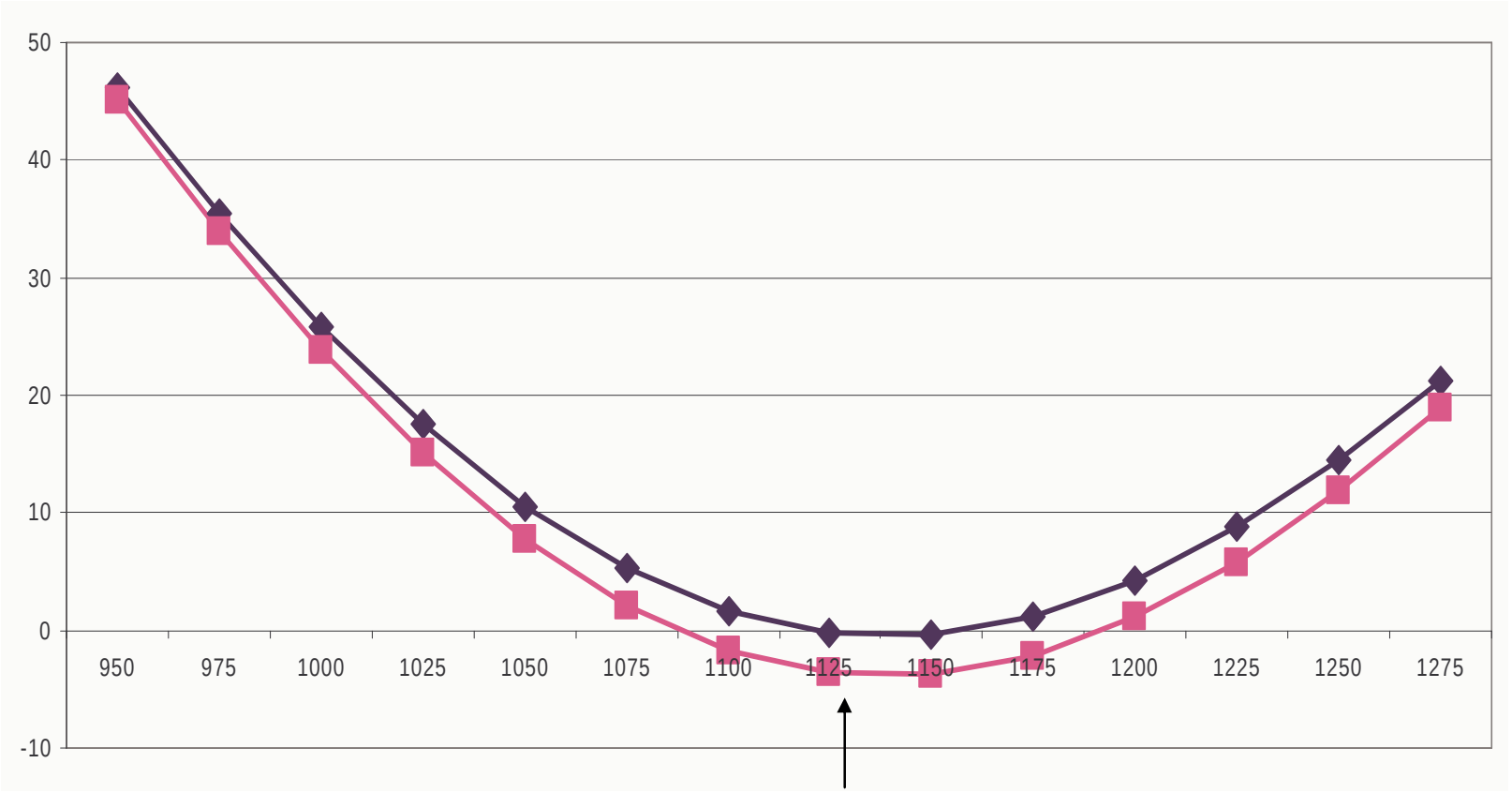


$$\text{P/L} \approx \theta \cdot \left(n^2 - 1\right)$$

$$\theta = \text{daily time-decay}, \quad n = \frac{\text{percent index change}}{\text{expected daily volatility}}$$

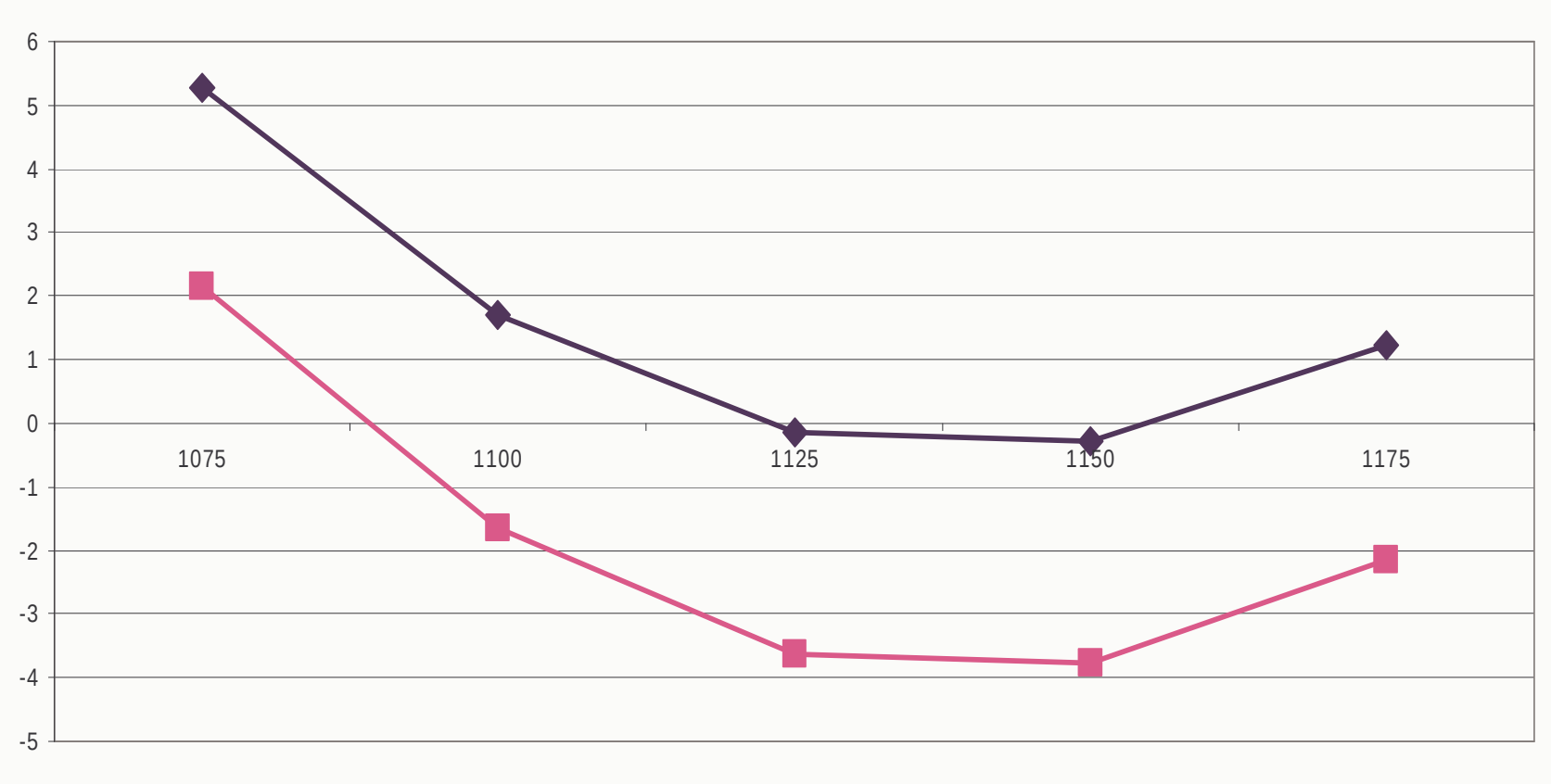
Assuming an implied volatility drop of 1%

Vol=15%



3.80 loss if stock does not move and volatility drops 1%

A closer look at the profit-loss due to a change in volatility



1% move in vol => 8% move in premium for a 6m ATM option

Measuring the Risk of a Portfolio (assuming delta neutrality)

Portfolio of options on N stocks

n_{ij} contracts of option with underlying stock i, expiration T_j, volatility σ_{ij}

$$\Delta\Pi = \sum_{ij} n_{ij}\left(C\left(S_i + \Delta S_i, T_j, K_{ij}, \sigma_{ij} + \Delta\sigma_{ij}\right) - C\left(S_i, T_j, K_{ij}, \sigma_{ij}\right) - \frac{\partial C_{ij}}{\partial S_i}\Delta S_i \right)$$

$$= \sum_{ij} n_{ij}\left(C\left(S_i(1+R^{S_i}, T_j, K_{ij}, \sigma_{ij}\left(1+R^{\sigma_{ij}}\right)\right) - C\left(S_i, T_j, K_{ij}, \sigma_{ij}\right) - \frac{\partial C_{ij}}{\partial S_i} S_i R^{S_i} \right)$$

Need to define a joint distribution of stock returns and volatility returns to calculate statistics of PNL

Factor Model

Consider only parallel vol shifts and use 30-day ATM volatilities

$$R^{S_i} = \sum_{k=1}^{m} \beta_{ik} F_k + \varepsilon_i$$

$$R^{\sigma_i} = \sum_{k=1}^{m} \gamma_{ik} F_k + \varsigma_i$$

Extract factors from PCA of augmented matrix

$$C_{ij} = \left\langle R^{S_i} R^{S_j} \right\rangle, \quad D_{ij} = \left\langle R^{S_i} R^{\sigma_j} \right\rangle, \quad E_{ij} = \left\langle R^{\sigma_i} R^{\sigma_j} \right\rangle$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{C} & \mathbf{D} \\ \mathbf{D}' & \mathbf{E} \end{pmatrix} \qquad \mathbf{M} \in R^{2N \times 2N}$$

Alternative Approach using ETFs

$$\frac{d\sigma_i}{\sigma_i} = \beta_i \frac{dS_i}{S_i} + \gamma_i \frac{d\sigma_{ETF(i)}}{\sigma_{ETF(i)}} + \varsigma_i,$$

$$ETF(i) = \quad \text{ETF associated with stock } i$$

Model the ATM volatility returns as a function of the stock return and changes in the volatility of the sector.

Conjecture: there are fewer systematic factors that explain volatility returns than in the case of stock returns. (m<20)

Possible project: do the PCA on the Nasdaq 100 optionable stocks analyzing the matrix **M** for this case.

Modeling the Volatility Skew

$$x = \ln(K / S)$$

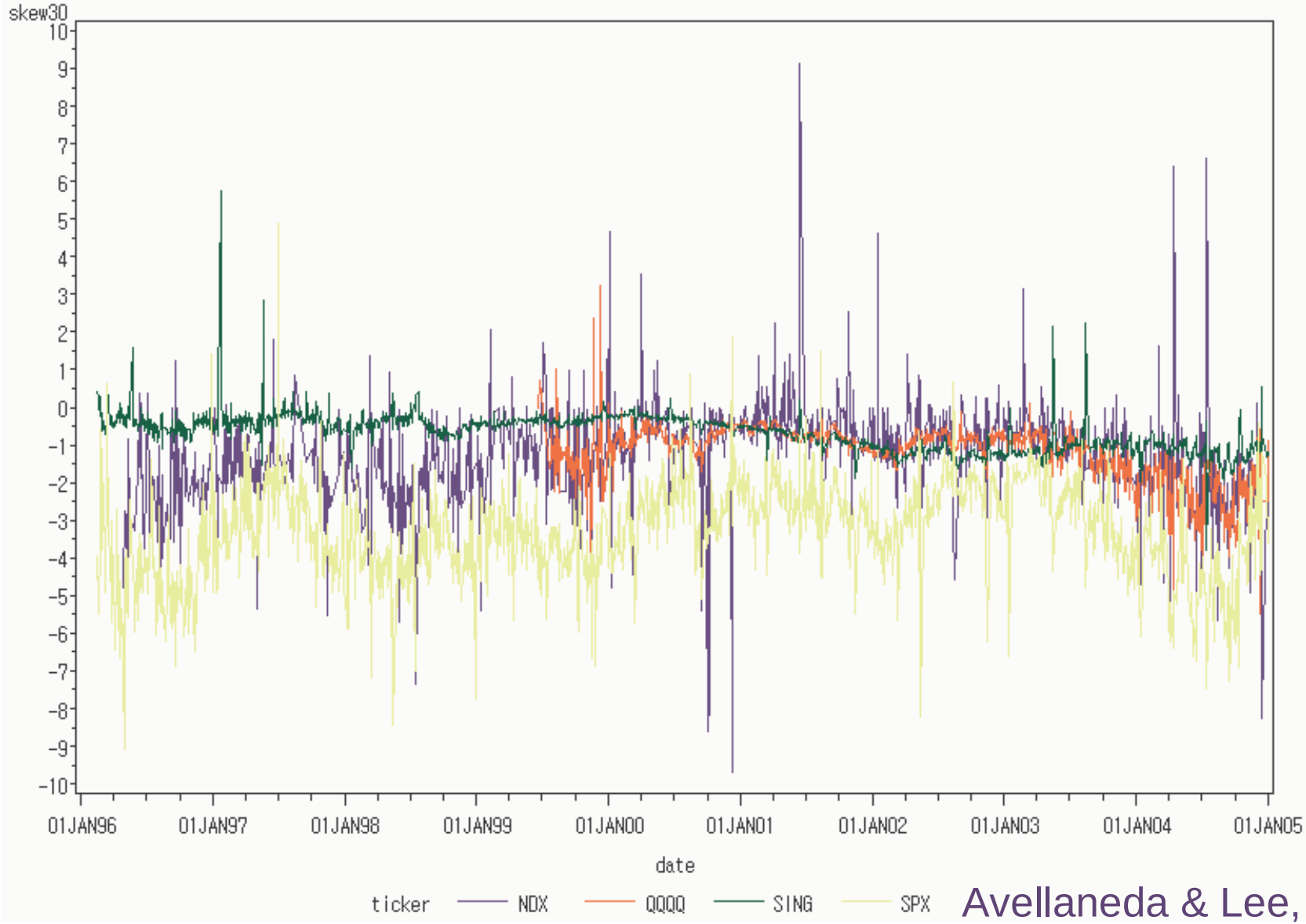
$$\sigma_{imp}(x,t) = \sigma_{imp}(0,t) \cdot \left(1 + \gamma x + \delta x^2 + \ldots\right)$$

Proposition: Under reasonable assumptions on price process (stoch. vol),

If $$\frac{d\sigma_{atm}}{\sigma_{atm}} = \beta \frac{dS}{S} + \varepsilon$$

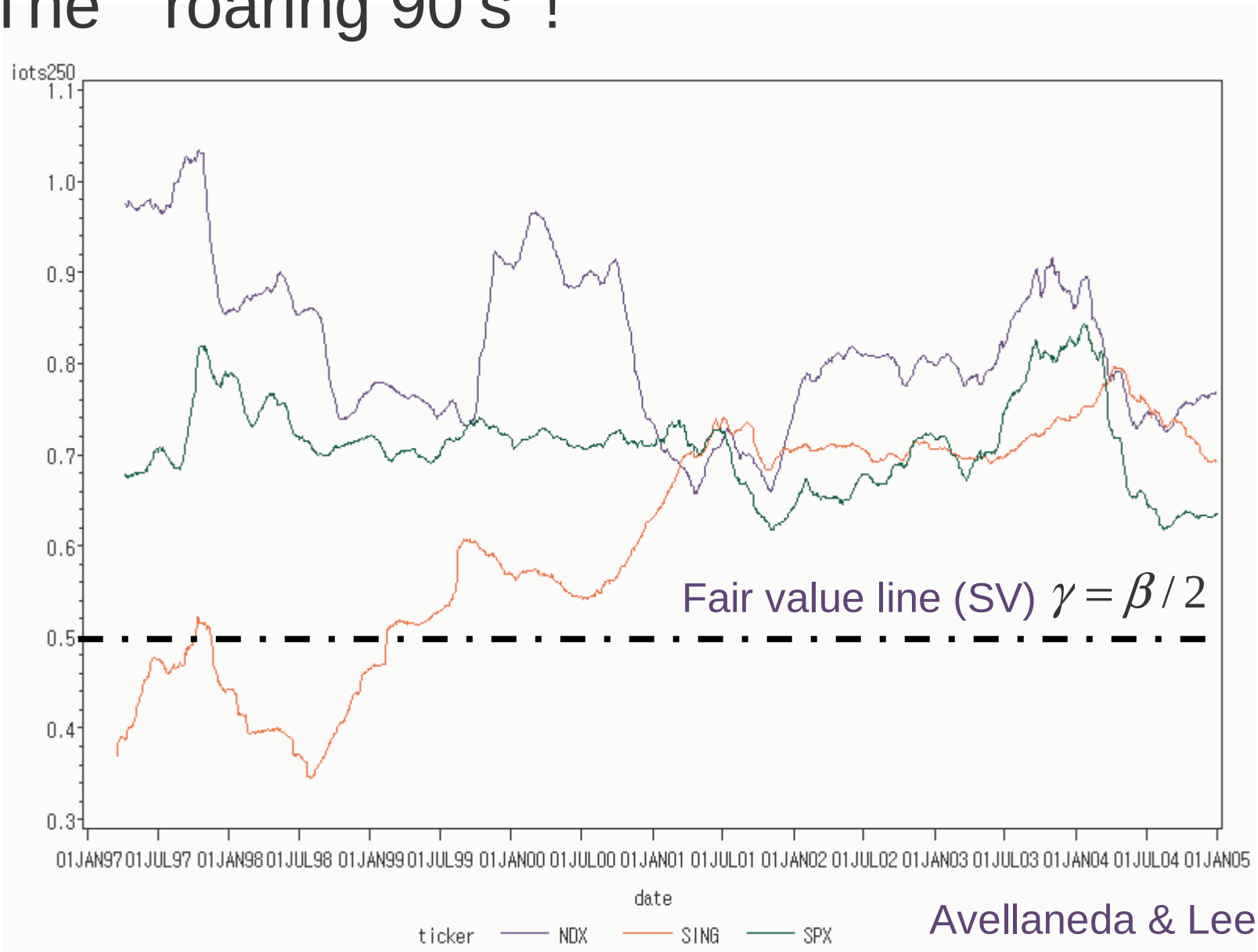
Then $$\gamma = \frac{\beta}{2}$$

Evolution of the slope of the 30-day implied volatility curve, 1996-2004



Avellaneda & Lee, 2005

Evolution of ratio [slope/leverage coefficient] The ``roaring 90's''!



Avellaneda & Lee, 2005