Complex analysis, homework 9 due April 13th.

**Exercise 1.** [18 points] Let $C$ be the arc defined by

$$z(t) = \begin{cases} 
3e^{it} & \text{if } 0 \leq t \leq 1, \\
-3 + 6(t - 1) & \text{if } 1 \leq t \leq 2,
\end{cases}$$

Evaluate the integral $\int_C f(z) \, dz$ for the following functions $f$ (give your answer in $x + iy$ form).

1. $f(z) = \frac{\cos z}{(z + i)^2(z - 4)}$;
2. $f(z) = \frac{\cos z}{(z - i)^2(z - 4i)}$;
3. $f(z) = \frac{1}{(z - i)^2(z + 2i)(z - 2i)}$.

**Exercise 2.** [6 points] Let $M, R > 0$. Let $f$ be an analytic on and within the circle centered at 0 with radius $R$. Assume $|f(z)| \leq M$ for any $|z| \leq R$. Let $n$ be a nonegative integer and $0 < \rho < R$. For $|z| \leq \rho$, find an upper bound for $|f^{(n)}(z)|$ which depends only on $M, R, \rho, n$.

**Exercise 3.** [6 points] Let $f$ be an entire function. Assume there is a nonnegative integer $n$ and a constant $M > 0$ such that $|f(z)| \leq M|z|^n$ for any $z \in \mathbb{C}$. Prove $f$ is a polynomial.

*Hint:* You can first prove that $f^{(n+1)}(z) = 0$ using ideas similar to the proof of Liouville’s theorem.