Complex analysis, homework 11 due April 25th.

Exercise 1. [9 points] Give three Laurent expansions in powers of z for the function

$$f(z) = \frac{i-3}{(z-i)(z-3)} = \frac{1}{z-i} - \frac{1}{z-3}$$

and specify the annular domains in which those expansions are valid.

Exercise 2. [12 points] Find the radius of convergence of the following power series. Explain your answer.

(1) $\sum_{n=0}^{\infty} (n+1)^{2n} z^n$;

(2) $\sum_{n=0}^{\infty} (n2^n + 3^n)\, z^n$;

(3) $\sum_{n=0}^{\infty} (\rho e^{i\theta} z)^n$, for some $\theta \in \mathbb{R}$ and $\rho > 0$;

(4) $\sum_{n=0}^{\infty} \frac{nz^{2n}}{(4i)^n}$;

Exercise 3. [5 points] Show that the following function is entire

$$f(z) = \begin{cases} \dfrac{\sin(z)}{z-\pi} & \text{if } z \neq \pi, \\ -1 & \text{if } z = \pi. \end{cases}$$

Exercise 4. [4 points] Recall $\operatorname{Log} z = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(z-1)^n$, for $|z-1| < 1$. For $|z-1| < 1$, let C_z be a contour from 1 to z included in the open disk centered at 1 with radius 1. Write the following quantity as a power series in z around 1:

$$\int_C \operatorname{Log}(w)\, \mathrm{d}w.$$

Justify your answer.