Exercise 1. [5 points] Prove the function defined by 
\[ f(z) = x^2 - y^2 + y + 2 + ix(2y - 1) \]
for \( z = x + iy \) is entire and find \( f'(z) \).

Exercise 2. [5 points] Compute the following quantities (that is express them in \( x + iy \) form):
1. \( \exp(2 + i \frac{5\pi}{6}) \);
2. \( \log((-e + ei)/\sqrt{2}) \) and \( \text{Log}((-e + ei)/\sqrt{2}) \).

Exercise 3. [3 points] Let \( z \in \mathbb{C} \). Prove that \( \exp(z) = \exp(\overline{z}) \).

Exercise 4. [4 points] Solve the equation \( e^{2z} + 1 = i \).

Exercise 5. [6 points] Prove that
1. \( \text{Log}((1 - i)^2) = 2 \text{Log}(1 - i) \);
2. \( \text{Log}((1 + i\sqrt{3})^4) \neq 4 \text{Log}(1 + i\sqrt{3}) \).

Exercise 6. [7 points] Recall that for any \( z \neq 0 \), we define \( \text{Log}(z) = \ln|z| + i \arg(z) \). Let \( D = \{ z \in \mathbb{C} : \text{Im}(z) > 0 \} \).
1. Using a geometric argument, express \( \arg(z) \) for \( z = x + iy \in D \) in terms of \( \cos^{-1}, x \) and \( y \). Explain why this formula does not work for all \( z \neq 0 \).
2. Using the theorem of Section 23, prove that \( \text{Log} \) is analytic on \( D \) and that \( \text{Log}'(z) = 1/z \) for any \( z \in D \).

Reminder: \( \frac{d}{dt} \cos^{-1}(t) = -\frac{1}{\sqrt{1-t^2}} \).