Complex analysis, homework 7 due March 14th.

Exercise 1.[12 points] For each of the following arc C, sketch it and say if it is a simple arc, a simple closed curve, a smooth arc and/or a contour (that is for each one of the 4 previous properties, say if it holds or not). No justification required.

(1) Let C be the arc defined by

$$z(t) = \begin{cases} 2t - it & \text{if } 0 \leq t \leq 2, \\ 8 - 2i - 2t & \text{if } 2 \leq t \leq 3, \\ 8 - 8i + 2(i-1)t & \text{if } 3 \leq t \leq 4. \end{cases}$$

(2) Let C be the arc defined by

$$z(t) = t + it^2, -2 \leq t \leq 2.$$

(3) Let C be the arc defined by

$$z(t) = 1 + e^{2it}, 0 \leq t \leq 2\pi.$$

Exercise 2.[6 points] Let C be the arc defined by

$$z(t) = \begin{cases} e^{-it} & \text{if } 0 \leq t \leq \pi, \\ t - 1 - \pi & \text{if } \pi \leq t \leq \pi + 2, \end{cases}$$

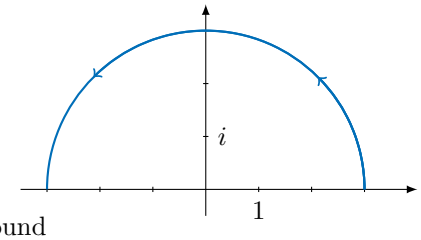
and $f(z) = 2\,\mathrm{Re}(z)$. Calculate the following integral

$$\int_C f(z)\,\mathrm{d}z.$$

Exercise 3.[6 points] Let C be the contour defined by $z(\theta) = e^{i\theta}$, $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$. Calculate the following integral

$$\int_C \mathrm{Log}(z)\,\mathrm{d}z.$$

Exercise 4.[6 points] Let C be the following arc (upper half circle centered at 0 with radius 3):



Prove the following bound

$$\left| \int_C \frac{z^2 - iz + 2}{z + 2}\,\mathrm{d}z \right| \leq 42\pi.$$