Complex analysis, homework 9 due April 11th.

Exercise 1. [18 points] Let C be the arc defined by

$$z(t) = \begin{cases} 3e^{i\pi t} & \text{if } 0 \leq t \leq 1, \\ -3 + 6(t-1) & \text{if } 1 \leq t \leq 2, \end{cases}$$

Evaluate the integral $\int_C f(z)\,\mathrm{d}z$ for the following functions f (give your answer in $x + iy$ form).

(1) $f(z) = \dfrac{\cos z}{(z+i)^2(z-4)}$;

(2) $f(z) = \dfrac{\cos z}{(z-i)^2(z-4i)}$;

(3) $f(z) = \dfrac{1}{(z-i)^2(z+2i)(z-2i)}$.

Exercise 2. [6 points] Let $M, R > 0$. Let f be an analytic on and within the circle centered at 0 with radius R. Assume $|f(z)| \leq M$ for any $|z| \leq R$. Let n be a nonnegative integer and $0 < \rho < R$. For $|z| \leq \rho$, find an upper bound for $|f^{(n)}(z)|$ which depends only on M, R, ρ, n.

Exercise 3. [6 points] Let f be an entire function. Assume there is a nonnegative integer n and a constant $M > 0$ such that $|f(z)| \leq M|z|^n$ for any $z \in \mathbb{C}$. Prove f is a polynomial.
Hint: You can first prove that $f^{(n+1)}(z) = 0$ using ideas similar to the proof of Liouville's theorem.