Probability, homework 1, due September 13th.

From *A first course in probability*, ninth edition, by Sheldon Ross.

Exercise 1. For years, telephone area codes in the United States and Canada consisted of a sequence of three digits. The first digit was an integer between 2 and 9, the second digit was either 0 or 1, and the third digit was any integer from 1 to 9. How many area codes were possible? How many area codes starting with a 4 were possible?.

Exercise 2. In how many ways can 8 people be seated in a row if

(i) there are no restrictions on the seating arrangement?
(ii) persons A and B must sit next to each other?
(iii) there are 4 men and 4 women and no 2 men or 2 women can sit next to each other?
(iv) there are 5 men and they must sit next to each other?
(v) there are 4 married couples and each couple must sit together?

Exercise 3. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if

(i) 2 of the men refuse to serve together?
(ii) 2 of the women refuse to serve together?
(iii) 1 man and 1 woman refuse to serve together?

Exercise 4. Prove that

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}.$$

Exercise 5. The following identity is known as Fermat's combinatorial identity:

$$\binom{n}{k} = \sum_{i=k}^{n} \binom{i-1}{k-1}, \ n \geq k.$$

Give a combinatorial argument (no computations are needed) to establish this identity. *Hint*: Consider the set of numbers 1 through n. How many subsets of size k have i as their highest-numbered member?