

Probability, homework 10, due December 6th.

Some exercises are from *A first course in probability*, ninth edition, by Sheldon Ross.

Exercise 1 A die is continually rolled until the total sum of all rolls exceeds 300. Approximate the probability that at least 80 rolls are necessary.

Exercise 2 An insurance company has 10,000 automobile policyholders. The expected yearly claim per policyholder is 240 USD, with a standard deviation of 800 USD. Approximate the probability that the total yearly claim exceeds 2.7 million USD.

Exercise 3 Suppose that a fair die is rolled 100 times. Let X_i be the value obtained on the i th roll. Compute an approximation for

$$\mathbb{P}\left(\prod_{i=1}^{100} X_i \leq a^{100}\right), \quad 1 < a < 6.$$

Exercise 4 Let the X_ℓ 's be i.i.d. real random variables, uniform on $[0, 1]$. What is the limit of $(X_1^3 + \cdots + X_n^3)/(X_1 + \cdots + X_n)$ as $n \rightarrow \infty$? In which sense?

Exercise 5. Let $(X_i)_{i \geq 1}$ be a sequence of independent random variables, with X_i uniform on $[-i, i]$. Let $S_n = X_1 + \cdots + X_n$. Prove that $S_n/n^{3/2}$ converges in distribution and describe the limit.