Exercise 1. Let \((X_n)_{n \geq 1}\) be a sequence of random variables, on the same probability space, with \(E(X_\ell) = \mu\) for any \(\ell\), and a weak correlation in the following sense: \(\text{Cov}(X_k, X_\ell) \leq f(|k - \ell|)\) for all indexes \(k, \ell\), where the sequence \((f(m))_{m \geq 0}\) converges to 0 as \(m \to \infty\). Prove that \((n^{-1} \sum_{k=1}^n X_k)_{n \geq 1}\) converges to \(\mu\) in \(L^2\).

Exercise 2. A sequence of random variables \((X_i)_{i \geq 1}\) is said to be completely convergent to \(X\) if for any \(\varepsilon > 0\), we have \(\lim_{i \to \infty} P(|X_i - X| > \varepsilon) < \infty\). Prove that complete convergence implies almost sure convergence.

Exercise 3. Let \(X\) and \(Y\) be independent Gaussian random variables with null expectation and variance 1. Show that \(\sqrt{2} X + \sqrt{2} Y\) and \(\sqrt{2} X - \sqrt{2} Y\) are also independent \(N(0, 1)\).