Case Study 1: The bucket brigade production line

My friend Mike owns a small manufacturing operation in the Garment District of Manhattan. He called me recently to describe the following problem with his company:

One of the products Mike's company manufactures, the "widget", is made on a line of 20 machines, in linear order from 1 on the far left to 20 on the far right. Each machine is dedicated to a particular operation. To make a widget a worker starts with a "blank" drawn from a bin, then works at machine 1 to complete the first step, then moves to machine 2 for the second step, and so on until the widget is completed at machine 20, and placed in the "finished" bin. Because of the nature of the widget, these operations must be performed in order- one cannot skip around. One could imagine hiring 20 workers, one for each machine, but the company is small, and anyway workers do not like to be performing always the same task. Mike has three workers, worker 1 on the left of the line, worker 2 in the middle, and worker 3 on the right. They work in what he calls a "bucket-brigade production line" (of BB line). If we were to look in on this line in operation, we would see each worker with a partially completed widget (PCW). The workers move left to right from machine to machine, but they cannot cross over each other. So when a fast worker arrives at a machine where a slower worker is operating, the faster one gets "blocked" and must move at the pace of the slower worker.

When worker 3 on the right gets to the last machine a drops the completed widget into the finished bin, the production line *resets*. Mike explained the reset as follows: worker 3 moves to the right to worker 2, takes the PCW from 2 and continues to the right. 2 immediately moves left to worker 1, takes the PCW and continues production on it. Worker 1 moves to the left to the bin of blanks and begins production of another widget.

Mike's problem is this: he generally lets workers take any position they want, even though they are not at the same skill level, but once in place they keep the same ordering for the day. But some days the production rate is much higher than others. He tried using the least skilled worker at the end of the line for quality control and packaging, but that seems to reduce the production rate. Sometimes the least skilled worker in the center or even first in the line, where the most critical operations are performed, gives very good production.

So his question is this: is there an optimal arrangement of n workers on a BB production line of $m >> n$ machines, based upon their skill level?

How would you model a BB production line?

Notes and hints: This problem arose about 10 years ago and a model of the process was devised by Bartholdi *et al.*. I have put a link to one of their early papers on the class web site. I have also put my own notes on their model there. However I ask that you first make an effort to model this on your own. Keep it simple. You need to make some assumptions about movement on the line, about the nature of the "reset", and decide how you would treat the system quantitatively. A useful hint is to think of the worker positions just after the kth reset as a vector $v^{(k)}$, and ask how you would find $v^{(k+1)}$ knowing $v(k)$. I will go through the Bartholdi model in detail on Wednesday.

NOTE: Problems will henceforth be due and handed back in the Wednesday class.