Complex Variables II PROBLEM SET 4 Due February 20, 2007

1. Problem 7, page 387 of text.(Take A a positive constant.)

2. Problem 9, page 387 of text.(Take A a positive constant.)

3. The (counter-clockwise) *moment* acting on a cylinder in 2D, due to the pressure force of a potential flow of a fluid of constant density, with uniform flow at infinity, is defined by

$$M = \oint_C p(xdx + ydy),$$

where C is the simple closed curve bounding the cylinder. Using the Bernoulli theorem ($p = -\frac{1}{2}\rho q^2 +$ constant. $\rho =$ fluid density) and the approach taken in class for proving Blasius' formula for the force on the cylinder, show that

$$M = -\frac{1}{2}\rho \oint_C \frac{dw}{dz}\overline{\frac{dw}{dz}} Re(z\overline{dz}) = -\frac{1}{2}\rho \, Re \oint_C \Big(\frac{dw}{dz}\Big)^2 z dz.$$

4. The complex potential for a uniform flow past a circular cylinder of radius a and center at the origin, having a circulation Γ, is

$$w = U\Big(z + \frac{a^2}{z}\Big) - i\frac{\Gamma}{2\pi} Log z.$$

Using residue theory and the moment formula of problem (3) above, show that the moment on the circular cylinder is zero.

5. A flat plate $x = 0, |y| \leq 2a$ is place in a uniform flow $(u, v) = (\cos\alpha, \sin\alpha)$, and the Kutta conition is satisfied at the trailing edge. Determine from the complex potential (using $dw/dz = dw/d\zeta \; d\zeta/dz$ the velocity components (u, v) on the top side of the plate. Show that $v = 0$ there and that u is singular at $z = -2a$ but is finite a $z = 2a$.