1. (a) Find the modulus and the Arg of w where

$$w = \frac{(4-2i)(1+i)}{(1-i)(\sqrt{3}+i)}.$$

(b)Prove that, if $z = x + iy$ is a complex number, $|x| \leq |z| \leq |x| + |y|$.

2. (a) Prove that, if z and w are complex numbers and $|w| = 1$ (so that $w = \frac{1}{\bar{w}}$), then

$$\left| \frac{z-w}{1-\bar{z}w} \right| = 1.$$

(b) Prove that , if $|z| < 1, |w| < 1$, then

$$\left| \frac{z-w}{1-\bar{z}w} \right| < 1.$$

(Hint: Consider the square a product of the expression and its conjugate.)

3. The *Chebychev polynomials* $T_n(x)$ are defined by $T_n(x) = \cos(n \cos^{-1} x)$, $|x| \leq 1$ Use the de Moivre formula $(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$ to obtain $\cos(n\theta) = Re(\cos\theta + i\sin\theta)^n$ and, with $\theta = \cos^{-1} x$ show (using the expansion of $(a+ib)^n$) that the T_n are indeed polynomials, and that T_n is of order n. Write down the $T_n(x)$ through $n = 4$.

4. Find the square roots of (a)$-i$ and (b) $\sqrt{3} - i$, expressing them in the form $x + iy$. (c) Find the 5th roots of $1 - i\sqrt{3}$ in polar form.

5. Write in terms of complex variables $z, \bar{z}$: (a) The equation of an ellipse centered at origin with semi-axes a, b. (b) The equation of the straight line through (a, b) with slope m.

6. (a) Describe a set S of the complex plane such that the function $w = f(z) = z^3$ maps S into the entire complex plane such that every $w \neq \infty$ corresponds to a unique $z \neq \infty$. Prove that $f = z^3$ is continuous at $z = 1$.

(b) If $w = \frac{a+bz}{c+dz}, ad - bc \neq 0$, show that z is uniquely determined by the value of w, implying the inverse function is single-valued. (Assume

$$w = \frac{a+bz_1}{c+dz_1} = \frac{a+bz_2}{c+dz_2}, z_1 \neq z_2$$

and get a contradiction. This w defines the *linear fractional transformation*, an important class of maps we shall study later.)