Complex Variables I PROBLEM SET 6 Due October 24, 2006

1. Problem 2, page 163 of text.

2. If the points $z = a$ and $z = b$ are inside the domain bounded by a simple closed contour C, show that

$$\int_C \frac{e^{(z-a)(z-b)}dz}{(z-a)(z-b)} = 0.$$

3. We showed in class the the Legendre polynomial $P_n(z)$ has the representation

$$P_n(z) = \frac{1}{2\pi i} \oint_C \frac{(s^2-1)^n}{2^n(s-z)^{n+1}} ds.$$

Here z lies within the positively oriented contour C. Show, by taking C to be of center z and radius $\sqrt{|z^2-1|}$, that

$$P_n(z) = \frac{1}{\pi} \int_0^\pi (z + \sqrt{z^2-1}\cos\theta)^n d\theta.$$

This is a formula given by Laplace. (Hint: Write $s = z + \rho e^{i\phi}$, $0 \le \phi \le 2\pi$. Also let $\sqrt{z^2-1} = \rho e^{i\alpha}$. Then arrange the integrand so that $\theta = \phi - \alpha$ is the parameter of the circular contour.)

4. Problem 1 page 171 of text.

5. Probem 4, page 172 of text.

6.** Let $f(z)$ be an entire function (i.e. analytic in the entire complex plane), and let $M(R) = \max_{|z|=R} |f(z)|$, for $R > 0$. Suppose that $M(2R) \le 2^n M(R)$ for all $R > 0$, and for some positive integrer n. Show that then $f(z)$ is a polynomial of degree not exceeding n. (Note: "**" means this is a written comprehensive examination question just as presented there, with no hints. Try to solve it, but if you have problems go to the course web site to find a hint to one way to do it.)