1. Problem 4, page 181 of text.

2. Problem 7, page 189 of text.

3. Find the Taylor series of $f = \frac{1}{2-z}$ about the point $z_0 = i$. What is the radius R_0 of the largest disk with center at i where within which the Taylor series converges?

4. Problem 6, page 198 of text.

5. The Bessel function $J_n(z)$ is defined as the nth coefficient ($n \geq 0$) of the "generating function"

$$e^{\frac{z}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(z)t^n.$$

From the integral giving the coefficients of a Laurent series show that

$$J_n(z) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - z\sin\theta)d\theta.$$

6.* The Fibonacci numbers are defined by $c_0 = 0, c_1 = 1$ and $c_n = c_{n-1} + c_{n-2}, n = 2, 3, \ldots$. Show that, in a certain disk $|z| < R$, the series $\sum_{n=0}^{\infty} c_n z^n$ is the Maclaurin series of a rational function $f(z)$ of z (i.e. is a quotient of polynomials in z). Find an explicit expression for c_n. What is the largest possible value of R? (Hint: assuming that $\sum_{n=0}^{\infty} c_n z^n$ converges absolutely, show that $f(z) = z + zf(z) + z^2 f(z)$. Then use a partial fraction decomposition of $(1 - z - z^2)^{-1} = -(z - z_1)^{-1}(z - z_2)^{-1}$ to find an expression for the c_n. Here z_1, z_2 are the roots of $z^2 + z - 1 = 0$.)