Complex Variables I PROBLEM SET 8 Due November 7, 2006

1. Problem 3, page 213 of text.

2. Show that

$$\frac{z}{(1-z)^2} = \sum_{n=1}^{\infty} n z^n, \quad |z| < 1.$$

3. Problem 6, page 213 of text.

4. Problem 8, page 220 of text.

5. Show that the function

$$f(z) = \begin{cases} \frac{e^{Nz}-e^z}{e^z-1} & \text{if } z \neq 0 \\ N-1 & \text{if } z = 0, \end{cases}$$

where N is a positive integer, is entire, and has a expansion

$$f(z) = \sum_{m=0}^{\infty} \frac{c_m}{m!} z^m,$$

where $c_m = \sum_{k=1}^{N-1} k^m$. Use your result to obtain the formula

$$\sum_{k=1}^{N-1} k^2 = \frac{N(N-1)(2N-1)}{6}$$

6. Show that the coefficients of the power series expansion

$$e^{2tz-z^2} = \sum_{n=0}^{\infty} \frac{H_n(t)}{n!} z^n$$

are polynomials of degree n, and derive the identity

$$H_{n+1}(t) - 2tN_n(t) + 2nH_{n-1}(t) = 0, \quad n \geq 1.$$

(Hint: For the latter, differentiate.The H_n are called *Hermite polynomials*.)